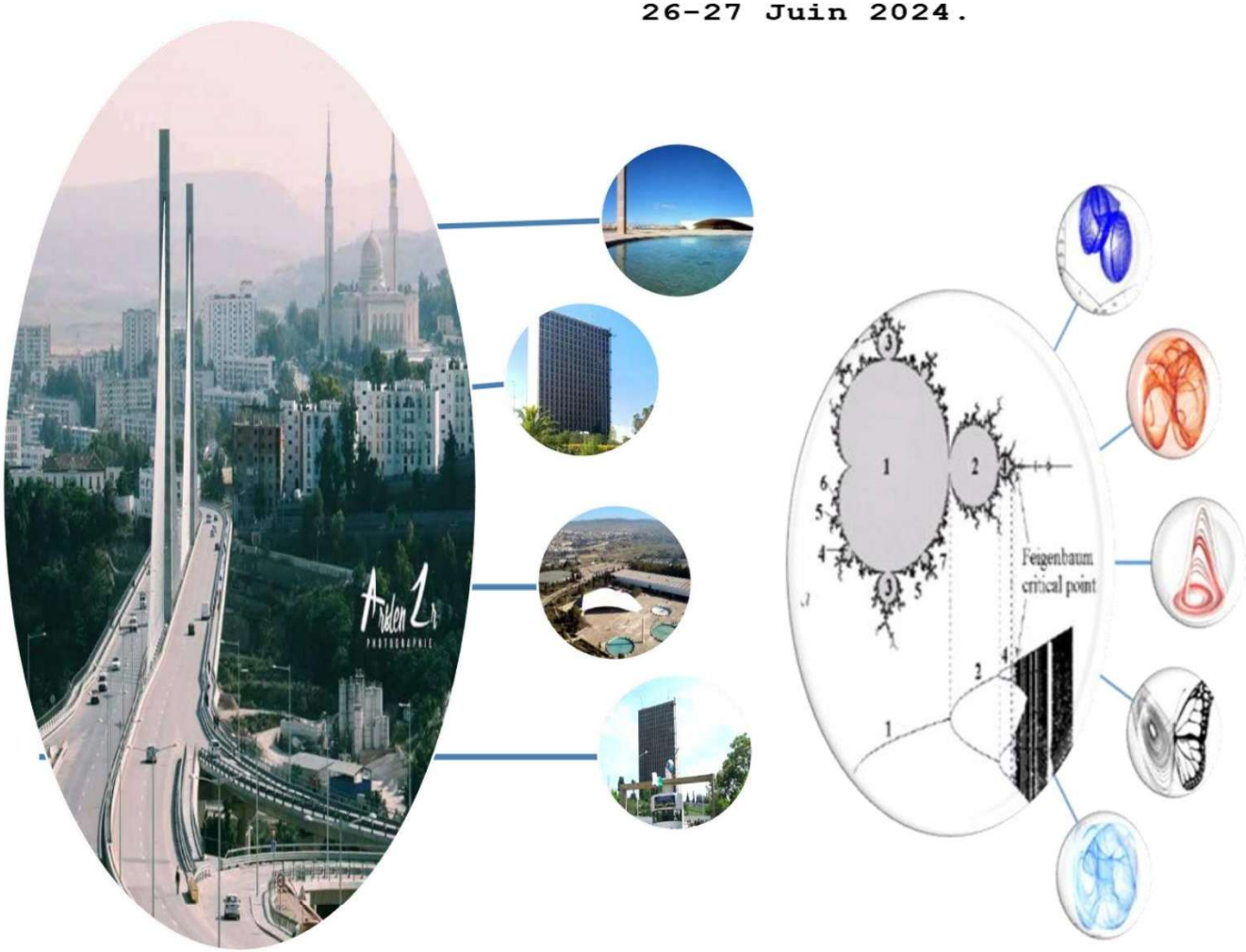
National Conference: Constantine -1-

Mathematical Modeling for Dynamic Systems (M^2DS)

26-27 Juin 2024.



Objectifs

Cette conférence nationale vise à consolider les ponts entre les théoriciens et les utilisateurs des systèmes dynamiques, en réunissant des spécialistes de différents horizons et d'expertises variées (génie, sciences, mathématiques) et en cherchant à identifier les directions émergentes de recherche de la discipline. Les utilisateurs des domaines du génie et des sciences appliquées y trouveront une occasion unique de s'approprier les plus récents développements de la théorie et des résultats mathématiques connexes, alors que les mathématiciens seront exposés aux applications les plus novatrices et les plus récentes.

Book Of Abstracts

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On Chaos for a Discrete Memristive Map with commensurate Fractional Order

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Abstract

In this paper, we develop a new three-dimensional (3D) fractional Duffing model with hidden dynamics. This model combines the classic Duffing system with a sinusoidal discrete memristor and a fractional Caputo-like difference operator varying fractional orders. We examine the system's main properties through equilibrium points, bifurcation diagrams, and phase diagrams. The inclusion of discrete sinusoidal memristors broadens the chaotic region and enriches the dynamic behaviors of the fractional Duffing model.

Keywords: Discrete fractional calculus, commensurate order, Chaos.

1 Introduction

The field of fractional calculus has evolved to include both continuous and discrete-time systems, focusing on fractional derivatives and fractional difference operators, respectively. Since 1974, there has been growing interest in discrete fractional calculus, leading to the development of various non-integer order difference operators.

Recently, research has shifted towards exploring fractional maps with unique dynamic behaviors and properties related to fixed points. These maps exhibit complex dynamics, including hyperchaos, which has sparked significant interest in the mathematical and engineering communities. Notably, fractional memristor-based discrete systems have emerged as a topic of investigation, with researchers examining the chaotic properties of these systems. This paper contributes to this field by introducing a novel 3D fractional-order memristor-based Duffing map. By combining a sinusoidal discrete memristor with the Duffing map, this system exhibits special characteristics, including multiple chaotic hidden attractors. The dynamic behavior of this system is analyzed using numerical methods like phase diagrams, bifurcation diagrams, and Lyapunov exponents.

2 Main results

This paper presented a sinusoidal discrete memristor in the Duffing map with commensurate fractional order. Numerical results revealed that the new fractional map can exhibit multiple coexisting attractors and chaos. Additionally, the memristor's inclusion has increased the complexity of the fractional Duffing map.

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Periods of the generalized Fibonacci functions modulo m

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Abstract
In this paper, we show the existence of primitive period of the $(k : (\lambda_1, \alpha_1), (\lambda_2, \alpha_2), ..., (\lambda_k, \alpha_k))$-step Fibonacci functions modulo m.

Keywords: Fibonacci functions, k-Fibonacci function, $(k; \alpha_1, \alpha_2, ..., \alpha_k)$-step Fibonacci function, primitive period modulo m.
2020 Mathematics Subject Classification: Primary 11B39, 11B83.

1 Introduction

In 1967, Elmore [2], defined the Fibonacci function $f : \mathbb{R} \to \mathbb{R}$ suche that

$$f(x) = f(x-1) + f(x-2), \tag{1.1}$$

for any real number x.
In 2019, Thongngam and Chinram [3] stadied periods of a Fibonacci function $f : \mathbb{Z} \to \mathbb{Z}$ modulo m and gave the following theorem

Theorem 1.1. *Let $f : \mathbb{Z} \to \mathbb{Z}$ be a Fibonacci function.*
(1) *If $f(0) = 0$, then $f(-n) = (-1)^{n+1} f(n)$ for any positive integer n.*
(2) *If $f(0) = 2f(1)$, then $f(-n) = (-1)^n f(n)$ for any positive integer n.*

Recently, in [4, 5], 2022 and 2023 respectively, the authors generalized the previous theorem, by definig the k-step and $(k : \alpha_1, \alpha_2, ..., \alpha_k)$-step Fibonacci functions, and proving the existence of primitive periods modulo m of these functions.

2 Main results

Definition 2.1. Let k be an integer $k \geqslant 2$, and $\alpha_1, \alpha_2, \ldots, \alpha_k$ be nonnegative integers such that $\alpha_1 \neq 0$, and $\lambda_1, \lambda_2, ..., \lambda_k$ be integers with $\lambda_k = \pm 1$. We define the $(k : (\lambda_1, \alpha_1), (\lambda_2, \alpha_2), ..., (\lambda_k, \alpha_k))$-step Fibonacci function $f : \mathbb{Z} \to \mathbb{Z}$ as follows

$$\begin{aligned} f(n) = \lambda_1 f(n-\alpha_1) + \lambda_2 f(n-\alpha_1-\alpha_2) + \cdots &+ \lambda_{k-1} f(n-\alpha_1-\alpha_2-\cdots-\alpha_{k-1}) \\ &+ \lambda_k f(n-\alpha_1-\alpha_2-\cdots-\alpha_k), \end{aligned} \tag{2.1}$$

for any integer n.

Theorem 2.1. *Let $f : \mathbb{Z} \to \mathbb{Z}$ be a $(k : (\lambda_1, \alpha_1), (\lambda_2, \alpha_2), ..., (\lambda_k, \alpha_k))$-step Fibonacci function $(k \geqslant 2)$, and m be a positive integer $m > 1$. Then there exists an integer $1 \leq \ell \leq m^{\alpha_1+\alpha_2+\cdots+\alpha_k}$ such that $f(n+\ell) \equiv f(n) (\mathrm{mod} m)$ for any integer n.*

Definition 2.2. Let $f : \mathbb{Z} \to \mathbb{Z}$ be a $(k : (\lambda_1, \alpha_1), (\lambda_2, \alpha_2), ..., (\lambda_k, \alpha_k))$-step Fibonacci function $(k \geqslant 2)$, and m be a positive integer $m > 1$. A positive integer ℓ such that $f(n+\ell) \equiv f(n) (\mathrm{mod} m)$ for any integer n is called a period of f modulo m. If such integer ℓ is the smallest, then it is called the primitive period of f modulo m and write $\ell := \ell_f(m)$.

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Introduction of a $2D$-chaotic map to encrypt digital images

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Abstract
To ensure confidentiality and avoid human attacks on our data, we exchange keys for encryption and decryption. In this work, we will take advantage of the commutativity of the circular matrix composition to create a common encryption key by applying the Diffie-Hellman exchange protocol through a conventional channel. To increase the security level of our system, we will introduce the chaotic logistic maps twice.

Keywords: Sharing keys, Circulant matrices, Logistic maps, Diffie-Hellman protocil, BB84 protocol.
2020 Mathematics Subject Classification: Primary 37D45, 35K55, 94A60.

Who holds the information, holds the power, this quote leads us to ask the question: how can we protect our information, and especially how can we exchange this information (text or images) with those we want without it being hacked?
The answer to this question is to find a way to share one or more keys that will be used for encryption and decryption of transmitted messages.
Based on the same techniques of our previous work [1], we choose this time two logistic maps :

$$\begin{array}{ll} (x_0,\mu_x) : x_{n+1} = \mu_x x_n(1-x_n) & ; 3 < \mu_x < 4 \quad ; x_0 \ \textit{initially chooses} \\ (y_0,\mu_y) : y_{n+1} = \mu y_n(1-y_n) & ; 3 < \mu_y < 4 \quad ; y_0 \ \textit{initially chooses} \end{array} \tag{0.1}$$

and we create two circulant matrices of order n.
But before that, we give some definitions

Definition 0.1. [2, 3] Toeplitz matrix is a matrix $\mathcal{T} \in \mathcal{M}_n(\mathbb{R})$ or $\mathcal{M}_n(\mathbb{C})$ of the form

$$\mathcal{T} = \begin{bmatrix} t_0 & t_1 & \cdots & t_{-(n-1)} \\ t_1 & t_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & t_{-1} \\ t_{n-1} & \cdots & t_1 & t_0 \end{bmatrix} \quad ; \quad \textit{where } t_i \in \mathbb{R} \ (\ \textit{or}\ \ \mathbb{C}) \ \textit{ for all } \ i \in \{-(n-1), \cdots, n-1\}.$$

We can distinct the two particular cases in the fallowing remark

1. If $t_{n-j} = t_{-j}$ for $j \in \{0, \cdots, n\}$, the matrix $\mathcal{T}$ is said to be circular matrix.
2. If $t_{n-j} = -t_{-j}$ for $j \in \{0, \cdots, n\}$, so we are talking about anti-circular matrix.

In this work, we are interested by circulant matrices, so we give the definition:

Definition 0.2. A circular matrix is a Toeplitz matrix given by:

$$\mathcal{C} = \mathcal{C}_n(c_1, \cdots, c_n) = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ c_n & c_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & c_2 \\ c_2 & \cdots & c_n & c_1 \end{bmatrix} \in \mathcal{M}_n(\mathbb{R}) \tag{0.2}$$

Theorem 0.1. *Let $G_{\mathcal{C}}$ be the set of all circulant matrices, then for any A and B elements of $G_{\mathcal{C}}$, we have $AB = BA$.*

Proof. See [1, 3] □

1 The construction of the key

The idea is to create and share a high complexity key, we assume two interlocutors Brahim and Salah. Both individuals complete the following steps:

Brahim will fix the two logistic maps (x_0, μ_x) and $(x'_0, \mu_{x'})$. the n first successive terms of each of the two sequences form the components of the vectors $U = (x_1, x_2, \cdots, x_n)$ and $U' = (x'_1, x'_2, \cdots, x'_n)$.
We obtain two circulant matrices B and B' generated by the two vectors U and U':

$$B = \langle U \rangle \quad and \; B' = \langle U' \rangle \tag{1.1}$$

Salah will fix the two logistic maps (y_0, μ_y) and $(y'_0, \mu_{y'})$. the n first successive terms of each of the two sequences form the components of the vectors $V = (x_1, x_2, \cdots, x_n)$ and $V' = (x'_1, x'_2, \cdots, x'_n)$.
We obtain two circulant matrices S and S' generated by the two vectors V and V'

$$S = \langle V \rangle \quad and \; S' = \langle V' \rangle \tag{1.2}$$

Let P be a public circulant matrix, the two interlocutors Brahim and Salah proceed to the following operations :

1. Brahim create the matrix

$$K_B = BPB' \tag{1.3}$$

and send it to Salah.

2. Brahim receives the matrix K_S from Salah and repeats the same operation, so he gets

$$K'_B = BK_SB' \tag{1.4}$$

1. Salah create the matrix

$$K_S = SPS' \tag{1.5}$$

and send it to Brahim.

2. Salah also receives the matrix K_B from Brahim and do the operation :

$$K'_S = SK_BS' \tag{1.6}$$

Theorem 1.1. *The two keys K'_B and K'_S in (1.4) and (1.6) are equal.*

Proof. We know that the multiplication in $(G_{\mathcal{C}}, \times)$ the monoïde of circulant matrices is commutative, so

$$\begin{array}{rcll} K'_B & = & BK_SB' & from\ (1.4) \\ & = & BSPS'B' & from\ (1.5) \\ & = & SBPB'S' & from\ the\ theorem\ 0.1 \\ & = & SK_BS' & from\ (1.3) \\ & = & K'_S & from\ (1.6) \end{array}$$

Finally, if we note by $K := K'_S = K'_B$. K is the key that we will use for encryption and decryption of messages exchanged between the two interlocutors.

Remark 1.1. The key K must be invertible, for this the choice of the initial parameters (x_0, μ_x) and (y_0, μ_y) in (0.1) must be made in a specific way so that the invertibility is ensured.

2 Application

Let Im be a digital image of $n \times n$ pixel. (if the image size is $n \times n$ pixel, then the vector sizes in (1.1) and (1.2) are fixed by n too). and Brahim wont to send the image Im to Salah.

- Brahim converts Im to a square matrix $I \in \mathcal{C}_n(\mathbb{R})$ using Matlab, Scilab or Python. Then he encrypts I to get $I' = KIK^{-1}$ and send it to Salah.
- Salah decrypts I' to get $K^{-1}KIK^{-1}K = I$.

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Particle creation in cosmology

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Abstract
The effect of the electric field on the creation of spin 1/2 particles from vacuum in the (d+1) dimensional de-Sitter space-time is studied. The Dirac equation with a constant electric field is solved by introducing an unitary transformation. Then the canonical method based on Bogoliubov transformation is applied to calculate the pair creation probability and the density of created particles both for positive or negative wave vector. By doing summation over all allowed states, the number of created particles per unit of time per unit of length and the imaginary part of the Schwinger effective action are expressed in closed forms.

Keywords: de-Sitter space-time, Schwinger effect, Bogoliubov transformation
2020 Mathematics Subject Classification: 81Q05,81R20,81T20,81U20,83C47.

1 Introduction

This work aims to explore how an electric field influences the creation of fermions particles during the expansion of the universe in $(d+1)$dimensional de-Sitter space-time. Recent research, particularly in $(1+1)$ dimensional de-Sitter space, has extensively investigated this phenomenon [1, 2, 3, 4].The geometric origins of the Schwinger effect in de-Sitter space were discussed in[1] , while [3]focused on studying the Schwinger mechanism under a uniform electric field in dS_2 and AdS_2.The authors of [2] investigated particle production by a constant electric field in $(1+1)$ dimensional de-Sitter space, focusing on scalar particles and noting that the adiabatic "*in*"vacuum develops a space-like current expectation value, breaking de-Sitter invariance de-Sitter invariance. The case of fermions, where spin plays a crucial role, has been less studied,where the spin of the fermion could play an important role on the behavior of created particles.
The paper is organized as follows: First, we consider a spin$\frac{1}{2}$ fermion in an electric field in $(d+1)$ dimensional de-Sitter space-time. Then, we proceed to solve the Dirac equation. To investigate particle creation, the canonical method based on Bogoliubov transformation is applied, connecting the "*in*"and"*out*" states. This method allows for the determination of the probability of particle pair creation, the number density of created particles, and the vacuum persistence.

2 Dirac equation in an expanding universe

Let us consider a spin $\frac{1}{2}$ fermion of mass m and charge $(-e)$ moving on the background geometry of a $(d+1)$ dimensional expanding universe described by

$$ds^2 = a^2(\eta)\left(d\eta^2 - dx^2\right), \tag{2.1}$$

where $a(\eta) = a_0 f(\eta)$, in the presence of an electric field described by the vector potential $A_\mu = (0, 0, ..., A_d(\eta))$, with $A_d(\eta) = E_0 f(\eta)$ .Here $f(\eta)$ is an arbitrary function. The covariant Dirac equation with the metric (2.1) takes the form

$$\left[i\tilde{\gamma}^\mu(\eta)\left(\partial_\mu - ieA_\mu(\eta) - \Gamma_\mu(\eta)\right) - m\right]\psi(\eta, \vec{x}) = 0 \tag{2.2}$$

where the curvature-dependent Dirac matrices $\tilde{\gamma}^\mu(\eta)$ are given in diagonal tetrad gauge by

$$\tilde{\gamma}^0(\eta) = \frac{1}{a(\eta)}\gamma^0 \qquad\qquad \tilde{\gamma}^i(\eta) = -\frac{1}{a(\eta)}\gamma^i \tag{2.3}$$

where γ^0 and γ^i with $i = \overline{1, d}$ are the usual Dirac's matrices that can be written, in $(d+1)$ dimensional Minkowski space After several calculation steps, we can derive the following second-order equation: .

$$\left[\frac{d^2}{d\eta^2} + a_0^2\mathcal{M}^2 f^2(\eta) - 2eE_0 k_d f(\eta) + k^2 \mp i\mathcal{M}a_0 f'(\eta)\right]\varphi_{1,2} = 0 \tag{2.4}$$

3 Pair creation in de-Sitter space

In the context of de-Sitter space with a constant electric field, the line element describing the space-time geometry can be formulated as follows:

$$ds^2 = dt^2 - e^{2Ht}dx^2 = a^2(\eta)\left(d\eta^2 - dx^2\right), \tag{3.1}$$

where $a(\eta) = \frac{-1}{H\eta}$ and H is the Hubble's constant. A constant electric field in the comoving system of coordinates is described by the vector potential $A_d(\eta) = -\frac{E_0}{H^2\eta}$, with $a_0 = H$, $\mathcal{M} = \sqrt{m^2 + \frac{e^2E_0^2}{H^2}}$ and $f(\eta) = -\frac{1}{H^2\eta}$. In such a case, the equation (2.4) is similar to the well-known Whittaker equation [5]

$$\left[\frac{d^2}{d\rho^2} - \frac{1}{4} + \frac{\lambda}{\rho} + \frac{\left(\frac{1}{4} - \mu_s^2\right)}{\rho^2}\right]\tilde{\varphi}_s(\rho) = 0, \tag{3.2}$$

with $\rho = -2ik\eta$ and $\tilde{\varphi}_s(\rho) \equiv \varphi_s(\eta)$, $s = \overline{1, 2}$. The constants μ_s and λ are given by $\mu_1 = \frac{1}{2} - i\frac{\mathcal{M}}{H} = \mu$, $\mu_2 = \frac{1}{2} + i\frac{\mathcal{M}}{H} = \mu^*$and $\lambda = i\frac{eE_0}{H^2}\frac{k_d}{k}$.
The relation between those states can be obtained by the use of the relation between Whittaker functions

$$M_{\lambda,\mu}(z) = \frac{\Gamma(2\mu+1)e^{i\pi\lambda}}{\Gamma\left(\mu - \lambda + \frac{1}{2}\right)}W_{-\lambda,\mu}(-z) + \frac{\Gamma(2\mu+1)e^{-i\pi\left(\mu-\lambda+\frac{1}{2}\right)}}{\Gamma\left(\mu + \lambda + \frac{1}{2}\right)}W_{\lambda,\mu}(z), \tag{3.3}$$

with $-\frac{3\pi}{2} < \arg z < \frac{\pi}{2}$ and $2\mu \neq -1, -2, \cdots$. The functions $\xi^{+}_{out}(\eta)$ and $\xi^{-}_{out}(\eta)$ can be then expressed in terms of $\xi^{+}_{in}(\eta)$ and $\xi^{-}_{in}(\eta)$ as follows

$$\begin{array}{c} \xi^{+}_{s,\vec{k},out}(\eta) = \alpha_{s,\vec{k}}\, \xi^{+}_{s,\vec{k},in}(\eta) + \beta_{s,\vec{k}}\, \xi^{-}_{s,\vec{k},in}(\eta) \\ \xi^{-}_{out}(\eta) = \alpha^{*}_{s,\vec{k}}\, \xi^{-}_{s,\vec{k},in}(\eta) + \beta^{*}_{s,\vec{k}}\, \xi^{+}_{s,\vec{k},in}(\eta) \end{array} \tag{3.4}$$

where the Bogoliubov coefficients α and β are given by

$$\frac{\beta_{s,\vec{k}}}{\alpha_{s,\vec{k}}} = \sqrt{\left(\frac{k\mathcal{M} - k_d \frac{eE_0}{H}}{k\mathcal{M} + k_d \frac{eE_0}{H}}\right)} \frac{\Gamma\left(i\left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H}\frac{k_d}{k}\right)\right)}{\Gamma\left(i\left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H}\frac{k_d}{k}\right)\right)} e^{-i\pi\frac{\mathcal{M}}{H}}. \tag{3.5}$$

The probability to create a pair of fermions in the state k is then

$$\mathcal{P}_{s,\vec{k}} = \left|\frac{\beta_{s,\vec{k}}}{\alpha_{s,\vec{k}}}\right|^2 = \frac{\sinh\pi\left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H^2}\right)}{\sinh\pi\left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H^2}\right)} e^{-2\pi\frac{\mathcal{M}}{H}}. \tag{3.6}$$

For the number density of created particles we have

$$n(s,k) = \left|\beta_{s,\vec{k}}\right|^2 = \frac{\sinh\left(\pi\left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H}\frac{k_d}{k}\right)\right) e^{-2\pi\frac{\mathcal{M}}{H}}}{\sinh\left(\pi\left(\frac{\mathcal{M}}{H} - \frac{eE_0}{H}\frac{k_d}{k}\right)\right) + \sinh\left(\pi\left(\frac{\mathcal{M}}{H} + \frac{eE_0}{H}\frac{k_d}{k}\right)\right) e^{-2\pi\frac{\mathcal{M}}{H}}}. \tag{3.7}$$

The effective Schwinger action defines the vacuum-vacuum transition amplitude as follows

$$\langle vac \mid vac\rangle = \exp\left(iS_{eff}\right) = \exp\left(i\int d^D x \mathcal{L}_{eff}\right) \tag{3.8}$$

consequently, the effective Schwinger action given by

$$2Im\mathcal{L}_{eff} = (1+\delta_d)\frac{1}{(2\pi)^{d-1}} \frac{\left(\mathcal{M}^2H^2 + eE_0^2\right)^{\frac{d}{2}}}{H\left(eE_0\right)^{\frac{d-2}{2}}} \sum \frac{1}{n^{\frac{d}{2}}} \exp\left(-2n\pi\frac{\mathcal{M}}{H}\right) I_{\frac{d-2}{2}}\left(\frac{2n\pi eE_0}{H^2}\right) \tag{3.9}$$

4 Concluding remarks

In conclusion, we have studied in this work the influence of an electric field on the creation of spin $\frac{1}{2}$particle in the $(d+1)$dimensional de-Sitter space. We have found exact solutions for the corresponding Dirac equation. Then we have established the Bogoliubov transformation between the positive and negative energy states to obtain the Bogoliubov coefficients from which we calculated the probability to create a pair of particles and the density of the created particles. From these results we were able to extract the imaginary part of the Schwinger effective lagrangian.

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Structural Analysis and Existence of Quasi-Hyperbolic regime in Piecewise Linear Mappings

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Abstract
This work is such an interesting generalization of the obtained results about the existence of a quasi-hyperbolic regime in certain family of the 2-D piecewise smooth mappings [1]. We extended the analysis to higher-dimensional piecewise linear systems and investigated the implications for the existence and properties of quasi-hyperbolic attractors in multidimensional phase spaces.

Keywords: Piecewise smooth maps, normal form, hyperbolic attractor
2020 Mathematics Subject Classification: Primary 37D45, 37C05, 34A35.

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Hilfer-Hadamard Fractional Differential Equations; Existence and Attractivity

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Abstract
This work deals with a class of Hilfer-Hadamard differential equations. Existence and stability of solutions are presented. We use an appropriate fixed point theorem.
This work is devoted to the existence and attractivity of solutions of the following problem

$$\begin{cases} ({}^{H}D^{\tau,\theta}_{c^+}i)(t) = \chi(t, i(t)); \quad t \in [c, +\infty),\ c > 0, \\ ({}^{H}I^{1-\varrho}_{c^+}i)(c) = d, \quad \varrho = \tau + \theta(1-\tau), \end{cases} \tag{0.1}$$

where $d \in \mathbb{R}$, $\chi : [c, +\infty) \times \mathbb{R} \to \mathbb{R}$, ${}^{H}I^{1-\varrho}_{c^+}$ is the left-sided Hadamard fractional of order $\tau > 0$ and ${}^{H}D^{\tau,\theta}_{c^+}$ is the Hilfer-Hadamard derivative operator of order τ $(0 < \tau < 1)$ and type θ $(0 \leq \theta \leq 1)$.

Keywords: Hilfer-Hadamard fractional derivative; Schauder fixed-point Theorem; uniformly locally attracting
2020 Mathematics Subject Classification: 26A33, 34A08.

1 Introduction

The beginning of the fractional calculus in 1695, the fractional differential equation has been used in fields like mathematics, engineering, bioengineering, physics, etc., to see interesting results in the theory of fractional calculus and fractional differential equations, the reader may consult the monographs by; Abbas *et al.*, Kilbas *et al.* , Oldham *et al.* , Podlubny , Samko *et al.* , Zhou *et al.*, and the papers by Abbas *et al.*, Benchohra *et al.* , Lakshmikantham *et al.* .

2 Main results

Definition 2.1. A measurable function $i \in BC_\varrho$ is a solution of (2) if it verifies $({}^H I_{c^+}^{1-\varrho} i)(c) = d$, and the equation $({}^H D_{c^+}^{\tau,\vartheta} i)(t) = \chi(t, i(t))$ on $[c, +\infty)$.

We will give the following hypotheses:

(H_1) The function $t \mapsto \chi(t, i)$ is measurable on $[c, +\infty)$ for each $i \in BC_\varrho$, and $i \mapsto \chi(t, i)$ is continuous.

(H_2) There exists a continuous function $l : [c, +\infty) \to [0, +\infty)$ such that

$$|\chi(t, i)| \leq \frac{l(t)}{1 + |i|} \quad for\ a.e. \quad t \in [c, +\infty) \quad and \quad each \quad i \in \mathbb{R},$$

and

$$\lim_{t\to\infty} \left(\log \frac{t}{c}\right)^{1-\varrho} \left({}^H I_{c^+}^{\tau} l\right)(t) = 0.$$

Set

$$l^* = \sup_{t\in[c,+\infty)} \left(\log \frac{t}{c}\right)^{1-\varrho} \left({}^H I_{c^+}^{\tau} l\right)(t).$$

We present a theorem on the existence and attractivity of solutions of the problem ().

Theorem 2.1. *If (H_1) and (H_2) hold, then (2) has at least one solution which is uniformly locally attractive.*

ally attractive.

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Fixed point theorems for multivalued mappings

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Abstract
The purpose of this study is to present some fixed point theorems by combining the contractions of Geraghty and Hardy-Rogers with F-contraction and α-admissible concept in the setting of set-valued mappings under weaker conditions. We derive new fixed point results on a metric space endowed with a partial ordering/graph by using the results obtained herein. We give also an example and an application to support new theory.

Keywords: first keyword, second keyword, third keyword,.....
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

The multivalued fixed point theory has many and different applications as in integral or differential inclusions, economy, optimization, etc. The contraction principle due to Banach has been generalized in different directions and one of such generalizations is due to Nadler, where he used the Pompeiu-Hausdorff metric to establish some fixed point results of multivalued mappings in metric spaces. Later many authors established some results in nonlinear analysis concerning the multivalued fixed point theory and its applications using the Pompeiu-Hausdorff distance, for more details. Recently, Wardowski introduced a new type of contractions called as F-contraction to show the existence of fixed points for such contraction by more simple method of proof than Banach's one. After that, several authors studied on different variations of F-contraction for single-valued and multivalued mappings, for example, In this study, we combine the notion of α-admissible with Wardowski contraction and the contractions of Geraghty and Hardy-Rogers in order to introduce new types of multivalued contractions to establish some fixed point theorems in the setting of complete metric spaces. We derive new fixed point results on a metric space endowed with a partial ordering/graph by using the results obtained herein. Finally, we give an example and an application of the existence of solution for an integral inclusion to illustrate our results.

2 Main results

Definition 2.1. Let (X, d) be a metric space and $\alpha : X \times X \to \mathbb{R}$. A mapping $T : X \to CL(X)$ is called α-F-Geraghty contraction of Hardy-Rogers type if there exist $F \in \mathcal{F}$, $\beta \in \Omega$ and $\tau > 0$ and non-negative real numbers a_1, a_2, a_3, a_4, a_5 with $a_1 + a_2 + a_3 + 2a_4 = 1$ and $a_3 \neq 1$ such that

$$\tau + F(H(Tx, Ty)) \leq F(\beta(N(x, y))N(x, y)), \tag{2.1}$$

for all $x, y \in X$ with $\alpha(x, y) \geq 1$ and $H(Tx, Ty) > 0$ where

$$N(x, y) = a_1 d(x, y) + a_2 d(x, Tx) + a_3 d(y, Ty) + a_4 d(x, Ty) + a_5 d(y, Tx).$$

Theorem 2.1. *Let (X, d) be a complete metric space and $T : X \to K(X)$ be an α-F-Geraghty contraction of Hardy-Rogers type. Assume that the following conditions are satisfied:*

1. *T is an α-admissible;*
2. *There exist $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;*
3. *T is an α-lower semi-continuous mapping, or X is α-regular, that is, for every sequence $\{x_n\}$ in X such that $x_n \to x \in X$ and $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$, then $\alpha(x_n, x) \geq 1$ for all $n \in \mathbb{N}$.*

Then T has a fixed point.

Let $X = \{1, 2, 3, 4\}$ and $d(x, y) = |x - y|$. Define $T \colon X \to CB(X)$ and $\alpha : X \times X \to [0, \infty)$ by

$$Tx = \begin{cases} \{2\}, & x \in \{1, 2\} \\ \{1\}, & x = 3 \\ \{3\}, & x = 4 \end{cases}$$

and

$$\alpha(x, y) = \begin{cases} 0, & (x, y) \in \{(2, 3), (3, 2), (3, 4), (4, 3)\} \\ 1, & \text{otherwise.} \end{cases}$$

We claim that T is an α-F-Geraghty contraction of Hardy-Rogers type, by taking $F(x) = \ln x$, $\beta(t) = \frac{t}{1+t}$, $\tau = \frac{1}{4}$, $a_1 = 1$ and $a_2 = a_3 = a_4 = a_5 = 0$. For that, we need to show that

$$H(Tx, Ty) \leq e^{-\frac{1}{4}} \beta(N(x, y)) N(x, y), \quad \text{for all } x, y \in X \text{ with } H(Tx, Ty) > 0 \text{ and } \alpha(x, y) \geq 1.$$

Note that $H(Tx, Ty) > 0$ and $\alpha(x, y) \geq 1$ if and only if $(x, y) \notin \{(x, x) \colon x \in X\} \cup \{(2, 3), (3, 2), (3, 4), (4, 3)\}$. By the symmetry property of the metric, we have the following cases:

1. For $x = 1$ and $y = 2$, we have

$$H(T1, T2) = 0, \quad N(1, 2) = a_1 d(1, 2) = 1 \quad \text{and} \quad \beta(N(1, 2)) = \frac{1}{2}$$

which implies

$$H(T1, T2) \leq e^{-\frac{1}{4}} \beta(N(1, 2)) N(1, 2).$$

2. For $x = 1$ and $y = 3$, we have

$$H(T1, T3) = 1, \quad N(1, 3) = a_1 d(1, 3) = 2 \quad \text{and} \quad \beta(N(1, 3)) = \frac{2}{3}$$

which implies

$$H(T1, T3) \leq e^{-\frac{1}{4}} \beta(N(1, 3)) N(1, 3).$$

3. For $x = 1$ and $y = 4$, we have

$$H(T1, T4) = 1, \quad N(1,4) = a_1 d(1,4) = 3 \quad \text{and} \quad \beta(N(1,4)) = \frac{3}{4}$$

which implies

$$H(T1, T4) \leq e^{-\frac{1}{4}} \beta(N(1,4)) N(1,4).$$

4. For $x = 2$ and $y = 4$, we have

$$H(T2, T4) = 1, \quad N(2,4) = a_1 d(2,4) = 2 \quad \text{and} \quad \beta(N(2,4)) = \frac{2}{3}$$

which implies

$$H(T2, T4) \leq e^{-\frac{1}{4}} \beta(N(2,4)) N(2,4).$$

Corollary 2.1. *Let (X, d) be a complete metric space, $\alpha \colon X \times X \to [0, +\infty)$ be a function and $T \colon X \to CB(X)$ (resp. $K(X)$) be a multivalued mapping. Assume that the following assertions hold:*

(i) T is an α_*-admissible;

(ii) There exist $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0, x_1) \geq 1$;

(iii) T is α-lower semi-continuous or X is α-regular;

(iv) There exist $F \in \mathcal{F}_*$, $\beta \in \Omega$ and $\tau > 0$ and non-negative real numbers a_1, a_2, a_3, a_4, a_5 with $a_1 + a_2 + a_3 + 2a_4 = 1$ and $a_3 \neq 1$ such that

$$\tau + F(H(Tx, Ty)) \leq F(\beta(N(x,y)) N(x,y)),$$

for all $x, y \in X$ with $\alpha_*(Tx, Ty) \geq 1$ and $H(Tx, Ty) > 0$ where

$$N(x,y) = a_1 d(x,y) + a_2 d(x, Tx) + a_3 d(y, Ty) + a_4 d(x, Ty) + a_5 d(y, Tx).$$

Then T has a fixed point.

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Fractional Partial Differential Equations: Quantitative and Qualitative Study

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Abstract

At the very beginning of the theory of finite dynamical systems, it was discovered that some relatively simple systems, even those governed by ordinary differential equations (ODEs), can generate very complicated (chaotic) behaviors. Furthermore, these systems are extremely sensitive to perturbations, in the sense that trajectories with close but different initial data may diverge exponentially. Often, the trajectories of such chaotic systems are localized, up to some transient process, in some subset of the phase space, the so-called strange attractors. These subsets have a very complicated geometric structure and accumulate the nontrivial dynamics of the system.
For distributed systems, whose time evolution is usually governed by partial differential equations (PDEs), the phase space X is (a subset of) an infinite-dimensional function space. We thus speak of infinite-dimensional dynamical systems. Given that global existence and uniqueness of solutions have been proven for a large class of PDEs arising from various domains in Mechanics and Physics, it is natural to investigate whether the features, particularly the notion of an attractor, obtained for dynamical systems generated by systems of ODEs, generalize to systems of PDEs.
In the context of fractional differential equations (FDEs), and more specifically for the case of Caputo fractional derivatives, the objective is to explore the global existence and uniqueness of solutions for a class of fractional PDEs (FPDEs). This investigation aims to extend the well-established concepts of dynamical systems to the realm of fractional derivatives, addressing the unique challenges posed by the Caputo formulation and understanding the implications for the behavior and properties of these infinite-dimensional dynamical systems.

Keywords:.Fractional partial differential equations (FPDEs), Chaotic behavior, Caputo fractional derivative, Fractional calculus
2020 Mathematics Subject Classification: Primary 26A33, 34A08, 37L30.

1 Introduction

The study of dynamical systems, particularly those governed by differential equations, has unveiled a wealth of complex behaviors, including chaotic dynamics and sensitive dependence on initial conditions. These phenomena are well-documented for systems defined by ordinary differential equations

(ODEs), where chaotic trajectories often converge to intricate geometric structures known as strange attractors. These attractors, which localize the system's long-term behavior, highlight the rich interplay between order and chaos within finite-dimensional phase spaces.
In recent years, the exploration has extended to distributed systems described by partial differential equations (PDEs), where the phase space is inherently infinite-dimensional. This extension necessitates a rigorous examination of whether the established properties of dynamical systems, such as global existence, uniqueness of solutions, and the nature of attractors, hold in these more complex settings.
An intriguing and burgeoning area of this exploration is the realm of fractional partial differential equations (FPDEs), particularly those involving Caputo fractional derivatives. The fractional derivative introduces memory effects and nonlocality into the equations, offering a more accurate description of various physical and mechanical processes. The Caputo derivative, known for its ability to handle initial conditions in a manner similar to classical derivatives, is particularly suited for modeling real-world phenomena.
This introduction aims to delve into the qualitative and quantitative study of the general form of fractional PDEs. On the qualitative side, we investigate the existence and uniqueness of solutions, the stability of these solutions, and the structure of attractors in infinite-dimensional spaces. Quantitatively, we focus on the precise behavior of solutions, the rate of convergence to attractors, and the sensitivity to initial conditions. Through this dual approach, we aim to bridge the gap between the finite-dimensional theory of ODEs and the infinite-dimensional theory of FPDEs, providing a comprehensive understanding of the dynamic behavior of systems governed by fractional derivatives.

2 Problem Definition and Mathematical Formulation

2.1 Nonlinear Diffusion Equation

Consider the nonlinear diffusion equation:

$$\frac{\partial u(x,t)}{\partial t} + D_t^\alpha u(x,t) = \nabla \cdot (k(u(x,t))\nabla u(x,t)),$$

where D_t^α denotes the Caputo fractional derivative of order $\alpha \in (0,1)$, $u(x,t)$ is the unknown function, and $k(u)$ is a nonlinear diffusivity coefficient.

2.2 Caputo Fractional Derivative

The Caputo fractional derivative of order α for a sufficiently smooth function $u(t)$ is defined as:

$$D_t^\alpha u(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{u'(\tau)}{(t-\tau)^\alpha}\, d\tau,$$

where:
- Γ is the Gamma function.
- $u'(\tau)$ is the first derivative of $u(\tau)$ with respect to τ.

The Caputo derivative is particularly useful because it allows for the incorporation of initial conditions in a manner similar to classical derivatives.

2.3 Initial and Boundary Conditions

$$u(x,0) = u_0(x), \quad u(0,t) = u(L,t) = 0,$$

where $u_0(x)$ is the initial distribution.

3 Qualitative Analysis

3.1 Existence and Uniqueness

We first establish the existence and uniqueness of solutions.

Theorem 3.1 (Existence and Uniqueness)**.** *Assume $k(u)$ is continuous and satisfies certain growth conditions. Then there exists a unique solution $u(x,t)$ to the nonlinear diffusion equation.*

Proof. Use the Banach fixed-point theorem or semigroup theory. Define an appropriate function space and operator to apply the theorem. □

3.2 Stability Analysis

Theorem 3.2 (Stability)**.** *The steady-state solution u_s of the nonlinear diffusion equation is stable if the spectrum of the linearized operator $L = D_t^\alpha - \nabla \cdot (k'(u_s)\nabla \cdot)$ lies in the left half-plane.*

Proof. Linearize the equation around the steady-state solution and analyze the spectrum of the resulting operator. □

3.3 Attractor Analysis

Theorem 3.3 (Global Attractor)**.** *The nonlinear diffusion equation possesses a global attractor in the appropriate function space.*

Proof. Prove that the solution semigroup is asymptotically compact and apply the theory of infinite-dimensional dynamical systems. □

4 Quantitative Analysis

4.1 Numerical Methods

- **Discretization:** Use finite difference or finite element methods for spatial discretization. For the time fractional derivative, use schemes like the Grünwald-Letnikov approximation.
- **Algorithm Implementation:** Implement the numerical scheme in software such as MATLAB or Python.

4.2 Convergence and Error Analysis

Perform convergence analysis to ensure numerical schemes approximate the true solution. Calculate error norms $||u - u_h||$ to quantify accuracy.

4.3 Simulation and Computation

Simulate $u(x,t)$ for various initial conditions and parameters α and $k(u)$. Visualize solution profiles over time.

4.4 Parameter Studies

Conduct parametric studies to understand the impact of fractional order α and nonlinear diffusivity $k(u)$ on solution dynamics.

5 Applications and Case Studies

5.1 Real-World Application

Model the diffusion of a pollutant in a porous medium with a nonlinear diffusion rate. Compare results with observed data.

5.2 Case Study

Analyze anomalous diffusion in biological tissues, validating the model against experimental data.

5.3 Comparison with Classical Models

Compare solutions of the fractional model with classical integer-order models, highlighting differences in diffusion rates and solution profiles.

6 Future Work and Extensions

6.1 Further Research Directions

Extend the study to other types of fractional derivatives (e.g., Riemann-Liouville, Caputo-Fabrizio). Explore applications in fields such as finance, ecology, and materials science.

6.2 Collaborations

Collaborate with researchers in applied mathematics, physics, and engineering to extend the impact of the findings.

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On the optimal controllability of the quasi-linear implicit fractional systems in Hilbert spaces

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Abstract
In this paper, the representation of a mild solution to the quasi-linear implicit fractional systems in Hilbert spaces using the spectral theory of the pencil operator $\lambda A - B$ and semigroup theory. We study the implicit fractional systems with nonlinear state dependent terms and show that if the Sobolev type resolvent families describing the linear part of the stationary systems are exponentially stable, we also establish conditions for the optimal controllability and prove that the nonlinear implicit fractional systems is exponentially stabilizable when the stabilizer acts linearly of the control systems. Finally, an example is given to illustrate the obtained results..

Keywords: Operators Theory, implicit fractional systems, spectral theory,semigroup theory, optimal controllability .
2020 Mathematics Subject Classification: Primary 34L05, 20M10, 57R27.

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Numerical Simulation of Dynamic Fluid-Structure Interaction with Elastic StructureRigid Obstacle Contact

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Abstract
In this work, we proposed a scheme to solve a dynamic fluid- structure interaction problem in the case where the movements of the structure are limited by a rigid obstacle. For fluid equations (Sokes or Navier-Stokes), the fictitious domain method with penalization is used. The equality of the velocities of the fluid and the interface structure is obtained by the penalization technique. Surface forces at the fluid- structure interface are calculated using the fluid solution in the structure domain. A quadratic optimization problem with linear inequality constraints is solved to obtain the displacements of the structure. Numerical results are presented.

Keywords: fluid-structure interaction; contact mechanics; fictitious domain with penalization.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

The heart is one of the fundamental organs of the human body, it is a hollow muscle, it ensures the circulation of blood throughout the body, allowing cells to receive oxygen and nutrients. It consists of four chambers in the human body: the left and right ventricles and the left and right atria, although the atria function to receive blood from the veins, while both ventricles pump blood to parts of the body through the arteries. There are also four valves in the heart, two of which separate each of the atrium and the ventricle called atrioventricular valves and two others separate the arteries and the ventricles, called the semilunar valves, these valves act to allow blood to move to one side only without returning to the opposite side.
Pathologies of one or more valves can lead to a significant limitation of their functions and, consequently, of the pump function of the heart. The main problems exposed to the valves are the inability to close completely, causing leakage which we can call regurgitation, or the inability to open properly which we can call stenosis. Damage to the operation of any of the valves can be dangerous, leading to the need for replacement of one of them.
From the mathematical, physical and fluid mechanics point of view, with the aim of allowing the local coupling of the fluid which represents the blood and the structure which represents the valve or more simply the interaction between the fluid and the structure, the mesh of the domain Fluid

will need to be updated to follow the movement of the valve. One way to do this is the use of an arbitrary Lagrangian-Euler (ALE) method with a moving grid. This method was used in [3] to model an aortic valve valve moving in a pulsatile flow and in [1] for the analysis of the filling of a deforming axisymmetric ventricle, where no valve was present in the orifice mitral.
Recently, fictitious domain methods have been introduced in order to do the coupling on separate meshes. These methods allow the coupling of the velocities of a structure to the velocities of the uid, while the structural mesh moves arbitrarily through the uid mesh.
The applicability of a fictional domain method to simulate the movement of a prosthetic valve rigid was studied in [2].To this end, the model's predictions on the flow fields and the moment of movement of the valve are compared to the experimental results obtained in a quasi-two-dimensional experience.To evaluate the sensitivity of the calculation method to variations Characteristic fluid flow parameters, expressed in Reynolds and Strouhal numbers, a range of flow impulses were used in the calculation method and the results were compared to the respective experimental results [?].
In this part we will treat the case of evolution for the problem which was studied in the previous chapter, then we present a model of fluid-structure interaction which will be close to what happens in the heart between the valves when they are open due to incoming blood. We will study the behavior of the structure which is represented by the elasticity equations and the fluid which is represented by the Stokes equation with a fictitious domain method always using the fixed point method.

2 Main results

We have carried out the behavior of an elastic structure immersed in an incompressible viscous fluid. We used the Stokes equation to model the flow motion

$$\rho^F \frac{\partial \mathbf{v}}{\partial t} - \nabla \cdot \sigma^F(\mathbf{v}, p) = \mathbf{f}^F, \quad \text{dans } \Omega^F_{\mathbf{u},\mathbf{t}} \tag{2.1}$$

$$\nabla \cdot \mathbf{v} = 0, \quad \text{dans } \Omega^F_{\mathbf{u},\mathbf{t}} \tag{2.2}$$

$$\sigma^F(\mathbf{v}, p)\,\mathbf{n}^F = 0, \quad \text{sur } \Sigma_2 \times [0,T] - \tag{2.3}$$

The vector $\mathbf{u}$ represents the fluid velocity field, the scalar function p is the pressure associated with it. We further consider the force field $\mathbf{f}$ acting on the system and the dynamic viscosity coefficient μ. and the displacement of the structure with the linear elasticity equations.

$$\rho^S \frac{\partial^2 \mathbf{u}}{\partial t^2} - \nabla_{\mathbf{X}} \cdot \sigma^S(\mathbf{u}) = \mathbf{f}^S, \quad \text{dans } \Omega^S_0 \times [0,T] \tag{2.4}$$

$$\mathbf{u} = 0, \quad \text{sur } \Gamma_D \times [0,T] \tag{2.5}$$

where $\mathbf{u}$ is the displacement vector and the stress tensor $\sigma^S(\mathbf{u})$, with conditions relative to the obstacle on the boundary of the structure. Numerical examples were given where the numerical calculations were carried out using the FreeFem++ software.

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SOME GENERALIZATIONS OF FRACTIONAL INTEGRAL INEQUALITIES.

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Abstract
In this work by applying the extended Chebyshev functional for monotone functions, we obtain some new integral inequalities for fractional operators. Moreover some known integrals are generalized.
Inequalities play a very important role in different fields of mathematics and present a active and attractive area of research. As examples, we cite the field of integration which is dominated by inequalities involving operators and integrals.
The well known Chebyshev functional is defined as follows

$$T(h_1,h_2)=\frac{1}{b-a}\int_a^b h_1(x)h_2(x)dx-\frac{1}{b-a}\int_a^b h_1(x)dx\frac{1}{b-a}\int_a^b h_2(x)dx \tag{0.1}$$

where f and g are two integrable functions.

Keywords: Fractional integral, inequalities, The Chebyshev functional.
2020 Mathematics Subject Classification:00A11; 55B55 .

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Time Dependent Problems

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Abstract
In this paper, we explore the numerical solutions of a fourth-order spatial and second-order temporal differential equation. We begin by establishing theoretical error estimates that lay a foundation for understanding the precision and reliability of the numerical methods applied. For the numerical analysis, the finite difference method is employed to discretize the equation, providing a practical framework for computation. We supplement our discussion with several examples that demonstrate the efficacy of the finite difference approach in solving this class of differential equations. Furthermore, we include a series of graphs to visually illustrate the behavior of the solutions and the error dynamics.

Keywords: Numerical Solution, Finite Difference Methode, Time Dependent Problem.
2020 Mathematics Subject Classification: Primary 65L20 · 65L12 · 37N30.

1 Introduction

The study of fourth-order partial differential equations (PDEs) is pivotal in numerous fields of science and engineering, including fluid mechanics, elasticity, and plate theory. These equations, often complex and challenging to solve analytically, necessitate the use of efficient numerical methods for their resolution. Among these methods, the finite difference method stands out for its straightforward implementation and ability to handle various boundary conditions and geometries. This presentation focuses on the application of the finite difference method to a fourth-order PDE, exploring the specific challenges associated with the higher order of the equation and demonstrating how these challenges can be overcome to achieve accurate and reliable numerical solutions.

2 Main results

Our work demonstrates that the error estimations for the chosen partial differential equation are accurate. Numerical methods applied to an example yielded an approximate solution closely matching the exact solution, validated through graphical analysis.

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Improved the bias in kernel quantile function estimation

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Abstract
Most of existing quantile estimators have problems of inefficiency in extreme quantiles. To solve this problem, In this paper we suggested an alternative estimator and provided its asymptotic behaviour when quantile near the boundary value. A simulation studies and two real data applications were included to demonstrate the efficiency and reliability of our theoretical results.

Keywords: Kernel quantile estimation, Mean Square Error, Optimal Bandwidth, Boundary Quantiles.
2020 Mathematics Subject Classification: Primary MR4501156, MR4605957,MR3468201.

1 Introduction

Percentiles and more generally, quantiles are commonly used when a parametric form for the underlying distribution is not available. The quantile function approach it has been used in exploratory data analysis, applied statistics, reliability and survival analysis by several authors among them Falk [1], Parzen [3] and in the books of Galambos [2] Without loss of generality, for $X_1, ..., X_n$ be independent and identically distributed with an unknown density $f(.)$ and absolutely continuous distribution function $F(.)$, while $X_{(1)}, ..., X_{(n)}$ denote the corresponding order statistics. The quantile function Q is defined to be the left-continuous inverse of F as follows:

$$Q(p) = \inf\{x : F(x) \geq p\} = F^{-1}(p), \ 0 < p < 1.$$

Many nonparametric estimators of the quantile function have been proposed and studied extensively. Traditionally, the estimator of the distribution function is the empirical function $F_n(x)$, which is defined as

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} 1_{]-\infty,x]}(X_i),$$

where the indicator function $1_{]-\infty,x]}(X_i) = 1$ if $X_i \leq x$ and 0 otherwise.

A basic estimator of $Q(p)$ is the empirical quantile or the sample quantiles which is given by

$$Q_n(p) = \inf\{x : F_n(x) \geq p\} = X_{([np])},$$

where $[np]$ denotes the integer part of np.
Because of the variability of individual order statistics, the sample quantiles suffer from lack of efficiency. In order to reduce this variability, different approaches of estimating sample quantiles through weighted order statistics have been proposed. The kernel estimators of F at x

$$\tilde{F}_n(x) = \int_{-\infty}^{x} \tilde{f}_n(t)\, dt = \frac{1}{n}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right),$$

where $h = h_n$ is the smoothing parameter (or the bandwidth) since it controls the amount of smoothness in the estimator, and satisfy $h := h_n \to 0$ as $n \to \infty$, $k(.)$ is a kernel function which is a predetermined density function symmetric about 0 and the function K is defined from a kernel k as

$$K(x) = \int_{-\infty}^{x} k(t)\, dt.$$

The corresponding estimator of the quantile function $Q = F^{-1}$ is then defined by

$$Q_n^c(p) = \inf\{x : \tilde{F}_n(x) \geq p\}, 0 < p < 1.$$

Parzen [3] proposed the classical kernel quantile estimator as below:

$$\check{Q}_n(p) = \sum_{i=1}^{n}\left[\int_{\frac{i-1}{n}}^{\frac{i}{n}} \frac{1}{h} k\left(\frac{p-x}{h}\right) dx\right] X_{(i)}.$$

Under suitable conditions on F, Falk [1] proposed the following kernel type quantile estimator

$$\tilde{Q}_n(p) = \frac{1}{h}\int_0^1 Q_n(x)\, k\left(\frac{p-x}{h}\right) dx,$$

this kernel-type quantile estimate can then be approximated by $\check{Q}_n(p)$. The aim of this work is to propose a new estimator of the quantile function to reduce the order of the bias and improve its asymptotic properties when p in the interior region, based on convex combination technique of skewed estimators. This idea is inspired by the works of Choi et al.et Cheng et al. and Kim et al. In the following Section, we derive the explicit form of our estimator and its asymptotic properties are illustrated. A numerical simulation studies have been carried out to see numerical performance of the proposed estimator, and compared with the existing approaches mentioned previously, real data example is also given.

2 Main results

In order to reduce the bias in the case of estimating the quantile function, we propose a new estimator $\hat{Q}_n(p)$ of $Q(p)$ using a bias reduction technique in the kernel quantile function estimator based on the technique of convex combination of three estimates $\tilde{Q}_n$, $\bar{Q}_1$ and $\bar{Q}_2$ where for $j = 1, 2$, $\bar{Q}_j$ estimator represents the value at p, of the tangent line which meets $\tilde{Q}_n$ at $p + \varphi h$ and $p - \varphi h$ respectively. Our proposed estimator at p is

$$\begin{aligned}\hat{Q}_n(p) &= \frac{1}{(1+2\alpha)h}\int_0^1 Q_n(x)\left[k\left(\frac{p-x}{h}\right)+\alpha\left(k\left(\frac{p-x+\varphi h}{h}\right)-\varphi k^{(1)}\left(\frac{p-x+\varphi h}{h}\right)\right)\right.\\ &\left.+\alpha\left(k\left(\frac{p-x-\varphi h}{h}\right)+\varphi k^{(1)}\left(\frac{p-x-\varphi h}{h}\right)\right)\right]dx.\end{aligned}$$

The following theorem shows that the bias of $\hat{Q}_n(p)$ is of order $O(h^4)$, while that of mentioned estimators is $O(h^2)$, also, it shows that the variance of the proposed estimator remains at the same order as the kernel quantile function estimators and it gives the expressions for the bias and the variance of the proposed estimator.

Theorem 2.1. *Assume that Q has four bounded, continuous derivatives in a neighborhood of p and the kernel k is probability density function symmetric about zero, then if $0 < h \to 0$, $nh^2 \to \infty$, we have for all fixed $p \in]\varphi h, 1-\varphi h[$*

$$Bias\left(\hat{Q}_n(p)\right) = \frac{1}{24}h^4 Q^{(4)}(p)\left(\mu_4 - \frac{3(1+6\alpha)}{2\alpha}\mu_2^2\right) + o\left(h^4\right), \tag{2.1}$$

and

$$Var\left(\hat{Q}_n(p)\right) = \frac{p(1-p)}{n}\left(Q^{(1)}(p)\right)^2 - \frac{h}{n}\left(Q^{(1)}(p)\right)^2 U(\alpha) + o\left(\frac{h}{n}\right) + o(1), \tag{2.2}$$

As a results we reveal the superior performance of the proposed estimator, especially if we choose φ big enough. The estimator of the quantile function proposed here is not good at the tails.

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On a new version of grierer-meinhardt model using fractional discrete calculus

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Abstract
As mathematical models of biological pattern generation, this study investigates the dynamics of the fractional discrete Gierer-Meinhardt reaction-diffusion system. After deriving the discrete non-integer fractional variant of the Gierer-Meinhardt system and establishing that the system has a unique equilibrium, we analyze the system's local asymptotic behavior in both the presence and absence of diffusion. The conditions for the global stability of the steady-state solution are determined using relevant approaches and the Lyapunov method. Throughout the study, two comprehensive biological models and simulations are employed to validate the utility of the considered approach.

Keywords:. Discrete fractional-order reaction–diffusion Gierer-Meinhardt model, Second order difference operator, Local-global asymptotic stability, Lyapunov method
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

Nonlinear reaction-diffusion equations are widely used in chemistry, physics, and biology for pattern formation. These equations also have applications in various other research areas. The theoretical framework for these models is well-established.
Recently, fractional reaction-diffusion models have gained attention. These models introduce fractional derivative indices, enhancing the freedom in representing self-organization phenomena. However, both analytical and numerical methods for these models are more complex.
Discrete fractional calculus has emerged as an intriguing field, attracting mathematicians and researchers from various disciplines. It employs discrete fractional operators to address real-world problems in biology, ecology, and applied sciences. Despite the interest in partial difference equations for applications like population dynamics and chemical processes, discrete reaction-diffusion models remain under-researched.
The Gierer-Meinhardt model is a prominent biological pattern generation model. It includes a saturating factor to control activator intensity and interaction capacity, making it suitable for modeling size maintenance and structure spacing in biological systems. Various studies have explored the model, demonstrating solutions and applications in biological pattern formation [1-3].

2 The fractional discrete Gierer–Meinhardt model

The Gierer–Meinhardt reaction-diffusion system, as is well known, was proposed in as follows

$$\begin{cases} u_t = a_1\Delta u + \sigma - \mu u + \frac{u^p}{v^q}, \quad x \in \Omega, t > 0 \\ v_t = a_2\Delta v - \nu v + \frac{u^r}{v^s}, \quad x \in \Omega, t > 0 \\ \frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \quad , \quad x \in \partial\Omega, t > 0 \\ u(x,0) = \phi_0(x) > 0, \quad v(x,0) = \phi_1(x) > 0, \quad x \in \Omega \end{cases} \tag{2.1}$$

where $\Omega \subset \mathbb{R}^n$, is a bounded domain with smooth boundary $\partial\Omega$, $a_1, a_2 > 0$, $\mu, \nu, \sigma > 0$ and the indices p,q,r and s are non negative with $p > 1$.
Based on the model (2.1) and with the discretization used in [2]. Assuming that $x \in [0, L]$, we have $x_{i+1} = x_i + k, \quad i = 0, \ldots, m$ and using the central difference formula concerning x, $\frac{\partial^2 u(x,t)}{\partial x^2}$ and $\frac{\partial^2 v(x,t)}{\partial x^2}$ can be approximately expended as

$$\begin{cases} \frac{\partial^2 u(x,t)}{\partial x^2} \approx \frac{u_{i+1}(t) - 2u_i(t) + u_{i-1}(t)}{k^2} \\ \frac{\partial^2 v(x,t)}{\partial x^2} \approx \frac{v_{i+1}(t) - 2v_i(t) + v_{i-1}(t)}{k^2} \end{cases} \tag{2.2}$$

Using the definition of the second order difference operator of u_i and v_i we obtain

$$\begin{cases} \frac{\partial^2 u(x,t)}{\partial x^2} \approx \frac{\Delta^2 u_{i-1}(t)}{k^2} \\ \frac{\partial^2 v(x,t)}{\partial x^2} \approx \frac{\Delta^2 v_{i-1}(t)}{k^2} \end{cases} \tag{2.3}$$

We consider the following discrete-time reaction-diffusion fractional Gierer–Meinhardt system

$$\begin{cases} {}^{C}_{\hbar}\Delta^{\alpha}_{t_0} u_i(t) = \frac{a_1}{k^2}\Delta^2 u_{i-1}(t + \alpha\hbar) + \sigma - \mu u_i(t + \alpha\hbar) + \frac{u_i^p(t+\alpha\hbar)}{v_i^q(t+\alpha\hbar)}, \\ {}^{C}_{\hbar}\Delta^{\alpha}_{t_0} v_i(t) = \frac{a_2}{k^2}\Delta^2 v_{i-1}(t + \alpha\hbar) - \nu v_i(t) + \frac{u_i^r(t+\alpha\hbar)}{v_i^s(t+\alpha\hbar)}. \end{cases} \tag{2.4}$$

Where $0 < \alpha < 1, t \in (\hbar\mathbb{N})_{t_0}$. With the periodic boundary conditions

$$\begin{cases} u_0(t) = u_m(t), \quad u_1(t) = u_{m+1}(t) \\ v_0(t) = v_m(t), \quad v_1(t) = v_{m+1}(t) \end{cases} \tag{2.5}$$

and the initial condition

$$u_i(t_0) = \phi_1(x_i) \geq 0, \quad v_i(t_0) = \phi_2(x_i) \geq 0. \tag{2.6}$$

3 Local stability

Theorem 3.1. *System (10) is locally asymptotically stable if the following conditions hold:*

- *If* $\mu\nu^2 + s\mu\nu > (p(\nu + s) - rq)\nu^{1+\frac{q}{s+1}} u^{p+\frac{s-rq}{s+1}-1}$ *and* $p\nu^{\frac{q}{s+1}} u^{p-\frac{rq}{s+1}-1} < \nu^2 + s\nu + \mu$ ·
- *If* $\mu\nu^2 + s\mu\nu > (p(\nu + s) - rq)\nu^{1+\frac{q}{s+1}} u^{*p+\frac{s-rq}{s+1}1}$ *and* $p\nu^{\frac{q}{s+1}} u^{*p\frac{mq}{s+1}1} < \nu^2 + s\nu + \mu$

Theorem 3.2. *System (8) is asymptotically stable at (u*, v*) under the following states:* · *If we suppose that*

$$\left(\frac{1}{4}-\frac{a_1a_1}{k^4}\right)\left(p\nu^{\frac{q}{s+1}}u^{*p-\frac{rq}{s+1}-1}-\mu\right)^2<2\nu^{\frac{q}{s+1}-1}u^{*p-\frac{rq}{s+1}-1}(p(s+1)-2rq)+\nu(s+1)(\nu(s+1)-6\mu),$$

andwe have the following two cases:
- *If* $a_1<a_2$ *and* $\frac{a_1}{k^2}\theta_i-\mu+p\nu^{\frac{q}{s+1}}u^{*p-\frac{nq}{s+1}1}\geq 0$.
$-If a_1>a_2 and\ \frac{a_1}{k^2}\theta_i-\mu+p\nu^{\frac{q}{s+1}}u^{*p-\frac{mq}{s+1}}\geq 0$ *.In addition, the eigenvalues satisfy* $Arg(\lambda_j(\theta_i))>\frac{c\pi}{2}$, *where*

$$\lambda_j(\theta_i)=\frac{\mathrm{tr}(J_{i(u^*,v^*)})\pm\sqrt{\mathrm{tr}(J_{i(u^*,v^*)})^2-4\mathrm{det}(J_{i(u^*,v^*)})}}{2},\quad j=1,2.$$

· *If we suppose that*

$$\left(\frac{1}{4}-\frac{a_1a_1}{k^4}\right)\left(p\nu^{\frac{q}{s+1}}u^{*p-\frac{rq}{s+1}-1}-\mu\right)^2>2\nu^{\frac{q}{s+1}-1}u^{*p-\frac{rq}{s+1}-1}(p(s+1)-2rq)+\nu(s+1)(\nu(s+1)-6\mu),$$

and the eigenvalues are complex described as follows:

$$\lambda_j(\theta_i)=\frac{\mathrm{tr}(J_{i(u^*,v^*)})\pm i\sqrt{\mathrm{tr}(J_{i(u^*,v^*)})^2-4\mathrm{det}(J_{i(u^*,v^*)})}}{2},\quad j=1,2.$$

Besides, it must also satisfy $Arg(\lambda_j(\theta_i))>\frac{c\pi}{2}$.

4 Global stability

Theorem 4.1. *The system (8) is globally asymptotically stable if*

$$\min\left\{\mu u^*-\frac{pu^{*p}}{v^{*q}}-\frac{ru^{*r}}{v^{*q}},\nu v^*-\frac{u^{*r}}{v^{*s}}(1+s)-\frac{qu^{*p}}{v^{*q}}\right\}\geq 0,$$

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Stability of an axially moving with a logarithmic nonlinearity

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Abstract
We studied the stabilization of an axially moving viscoelastic beam with logarithmic source terms. To show the stabilization of these system, we use a multipliers technique..

Keywords: Moving structure, stabilization, multiplier technique.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

we take into account the initial boundary value problem that is

$$\begin{cases} \rho\left(\varphi_{tt}+2\mathcal{V}\varphi_{xt}+\mathcal{V}^2\varphi_{xx}\right)+EI\varphi_{xxxx}-EI\displaystyle\int_0^t \mathcal{Z}(t-s)\varphi_{xxxx}(s)ds=\kappa\varphi\ln|\varphi|, \\ \quad x\in(0,l),\ t\geq 0, \\ \varphi(0,t)=\varphi_x(0,t)=\varphi_{xx}(l,t)=0, \quad t\geq 0, \\ \rho\mathcal{V}^2\varphi_x(l,t)+EI\varphi_{xxx}(l,t)-EI\displaystyle\int_0^t \mathcal{Z}(t-s)\varphi_{xxx}(l,s)ds=y(\varphi(l,t)),\ t\geq 0, \\ \varphi(x,0)=\varphi_0(x),\ \varphi_t(x,0)=\varphi_1(x),\ x\in(0,l) \end{cases} \tag{1.1}$$

whereas $\varphi=\varphi(x,t)$ is the beam transversal displacement,
$\mathcal{V}$ is the axial speed ,
EI is the beam flexural rigidity,
ϱ is the beam mass per unit length,
$\mathcal{Z}$ nonnegative functions see [?],
$\varphi_0(x)$, $\varphi_1(x)$are the initial data,
y represents the nonlinear term,
κ ia a small postive real number.

2 Main results

we studied the stabilization of an axially moving viscoelastic beam with Logarithmic Source Terms. We obtained an asymptotic stability result of global solution, for certain class of relaxation functions.

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The $K_{p,q}$-Compactness and $K_{p,q}$-Null Sequences

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Abstract
In this presentation, we deal with one of the most important topics in functional analysis, namely $\mathcal{I}$-Compact set and $\mathcal{I}$-null sequence we introduce some results related to relatively $K_{p,q}$-compact sets and $K_{p,q}$-null sequence..

Keywords: Compact set , $\mathcal{I}$-Compact set , $\mathcal{I}$-null sequence, set $K_{p,q}$-Compactness, $K_{p,q}$-Null Sequence
2020 Mathematics Subject Classification: Primary 47A35, 60Fxx, 60G10.

1 Introduction

In 1955, A. Grothendieck [3] described relatively compact subsets of a Banach space as sets contained in the closed absolute convex hull of a norm null sequence. Nowadays this result is called the Grothendieck compactness principle. A linear operator is said to be compact if it maps bounded sets to relatively compact ones.

Carl and Stephani(1984)[2] introduced a new type of compact sets called *relatively $\mathcal{I}$ -compact set.* A subset B of a Banach space E is *relatively $\mathcal{I}$ -compact* if there is a Banach space G and an operator $S \in \mathcal{I}(G, E)$ such that $B \subseteq S(M)$, where M is a compact subset of G. The new class of sets is formed by $\mathcal{I}$-compact sets. A linear operator T between Banach spaces E and F is called *$\mathcal{I}$-compact* if $T(B_E)$ is a relatively $\mathcal{I}$-compact subset of F.
The operator ideal formed by all linear $\mathcal{I}$-compact operators is denoted by $\mathcal{K}_{\mathcal{I}}$ The new class of sets is formed by $\mathcal{I}$-compact sets.

Lassalle and Turco [5] introduced a way to measure the size of $\mathcal{I}$-compact sets. For an $\mathcal{I}$-compact subset K of X,

$$m_{\mathcal{I}}(K; X) := inf\left\{\|\mathcal{U}\|_{\mathcal{I}} : \mathcal{U} \in \mathcal{I}(Z, X), \text{relatively compact } C \subset B_Z, K \subset \mathcal{U}(C)\right\}.$$

where B_Z is the unit ball of a Banach space Z.

In 2002, a form of compactness was introduced by Sinha and Karn [8]. Let $1 \leq p \leq \infty$. A subset K of Banach space X is relatively p-compact if there exists $(x_n) \in \ell_p(X)$ such that

$$K \subset p - conv\{(x_n)_n\} = \left\{\sum_{n=1}^{\infty} \alpha_n x_n : (\alpha_n)_n \in B_{\ell p^*}\right\}$$

Galicer, Lassalle and Turcointroduced a notion to measure the size of p-compact sets. For a p-compact subset K of X:

$$m_p(K;X) := inf\{\|(x_n)_n\|_p : (x_n)_n \in \ell_p(X), K \subset p - conv\{(x_n)_n\}\}$$

and $m_p(K,X) = \infty$ if K is not p-compact. and it is shown that K is p-compact if and only if K is $K_{1,p}$-compact. In this case, $m_p(K;X) = m_{K_{1,p}}(K;X)$.

Let $0 < r \leq \infty, 1 \leq p,q \leq \infty$ and $1+1/r = 1/p+1/q$. A linear operator $T \in \mathcal{L}(E,F)$ is called (p,q)-compact if

$$T = \sum_{n=1}^{\infty} \lambda_n x_n^* \otimes y_n$$

with $(\lambda_n) \in \ell_r, (x_n^*)_n \in \ell_{q^*}^w(E^*)$, and $(y_n)_n \in \ell_{p^*}^w(F)$. In the case $r = \infty$ let us suppose that $(\lambda_i) \in c_0$.
The class of all (p,q)-compact operators is denoted by $K_{p,q}$ and it is a Banach linear operator ideal. It is well known that $K_{p,1}$ is the ideal of p-nuclear operators and K_{p,p^*} is the ideal of p-compact operators.
Using the ideal of $K_{p,q}$-compact operator Kim in [4] extended a new result to the $K_{p,q}$-compactness.

Let X be a Banach space, by $c_0(X)$ we denote the space of all null sequence that is

$$c_0(X) = \left\{(x_n)_n, x_n \in X : \lim_{n\to\infty} \|x_n\| = 0\right\}$$

Piñeiro and Delgado [7] introduced an extended concept of null sequences.
For $1 \leq p \leq \infty$, sequence $(x_n)_n$ in a Banach space, X is called p-null (in symbol $(x_n)_n \in c_{0,p}(X)$) if for every $\varepsilon > 0$, there exist a natural number n_ε and $(z_k)_k \in \varepsilon B_{\ell_p}(x)$ such that:

$$x_n \in p - conv(z_k)_k$$

for all $n \geq n_\varepsilon$.
For a Banach operator ideal $\mathcal{I}$ and a Banach space X, a sequence $(x_n)_n$ in X is called $\mathcal{I}$-null if there exist a Banach space Z and $S \in \mathcal{I}(Z,X)$ such that for every $\varepsilon > 0$, there is a natural number n_ε so that:

$$x_n \in \varepsilon S(B_Z)$$

for all $n \geq n_\varepsilon$. Let $c_{0,\mathcal{I}}(X)$ be the space of all $\mathcal{I}$-null sequences in X. Since $\mathcal{I}$-null sequences are $\mathcal{I}$-compact (see [2]), Lassalle and Turco defined a norm on $c_{0,\mathcal{I}}(X)$ by:

$$\|(x_n)_n\|_{c_{0,\mathcal{I}}(X)} := m_{\mathcal{I}}(\{x_n\}_n;X).$$

So the second goal of our work is to study the $K_{p,q}$-null sequences extended by Kim [4]

2 Main results

Theorem 2.1. *Let K be a subset of a Banach space X and let $1 \leq p,q,r \leq \infty$ with $1/r = 1/p + 1/q - 1$. Then, the following statements are equivalent:*
(a) K is $K_{p,q}$-compact.
(b) There exist $(x_n)_n \in \ell_{p^*}^u(X)$ and $(\lambda_n)_n \in \ell_r$ such that:

$$K \subset q - conv(\lambda_n x_n)_n.$$

(c) There exist a relatively compact $M \subset B_{\ell_{q^*}}(B_{c_0}$ when $q = 1)$ and an $S \in K_{p,q}(\ell_{q^*}, X)$ such that $K \subset S(M)$.

In this case:

$$\begin{aligned} m_{K_{p,q}}(K,X) &= inf\left\{\| (x_n)_n \|_{p^*}^{w} \| (\lambda_n)_n \|_r : (x_n)_n \in \ell_{p^*}^{u}(X), (\lambda_n)_n \in \ell_r, K \subset q - conv(\lambda_n x_n)_n\right\} \\ &= inf\left\{\|S\|_{Kp,q} : relatively\ compact\ M \subset B_{\ell q^*}, S \in K_{p,q}(\ell_{q^*}, X), K \subset S(M)\right\}. \end{aligned}$$

Using the ideal of $K_{p,q}$-compact operator Kim in [4] extended a new result to the $K_{p,q}$-null sequence In the following theorem.

Theorem 2.2. *Let $(x_n)_n$ be a sequence in a Banach space X and let $1 < p, q, r < \infty$ with $1/p+1/q-1 = 1/r$. Then, the following statements are equivalent:*

(a) $(x_n)_n$ is a $K_{p,q}$-null sequence.

(b) There exists $(z_k)_k \in \ell_{p^*}^{u}(X)$ and $(\lambda_k)_k \in \ell_r$, such that for every $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that:

$$x_n \in \varepsilon q - conv(\lambda_k z_k)_k$$

for every $n \geq N$.

(c) There exists an $S \in K_{p,q}(\ell_{q^*}, X)$ such that for every $\varepsilon > 0$, there is an $N \in \mathbb{N}$ such that:

$$x_n \in \varepsilon S(B_{\ell_{q^*}})$$

for every $n \geq N$.

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Fractional Stochastic Integro-differential Equations with Non-instantaneous Impulsive

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Abstract
This paper proposes a class of Caputo fractional stochastic integro-differential equations (FSIDEs) with non-instantaneous impulsive and nonlocal conditions in a Hilbert space. The existence of mild solutions is investigated. The result is obtained by using stochastic analysis techniques, fixed point theory and generalized Grönwall inequality. An examples is given to illustrate the effectiveness of the proposed results.

Keywords: Fractional stochastic integro-differential equations, Non-instantaneous impulses, Nonlocal conditions, Fixed point theory.
2020 Mathematics Subject Classification: 60H10, 37H30, 26A33.

1 Introduction

The purpose of this research is studies the existence and uniqueness of mild solutions for a class of fractional stochastic integro-differential equations with non-instantaneous impulses and using nonlocal conditions for the following:

$$\begin{cases} {}^{c}_{0}D^{q}_{t}y(t) &= Ay(t) + P\Big(t, G_1y(t)\Big) + \sigma\Big(t, G_2y(t)\Big)\frac{dB(t)}{dt}, \quad t \in \cup_{k=0}^{m}(s_k, t_{k+1}], \\ y(t) &= h_k(t, y(t_k^-)), \qquad t \in \cup_{k=0}^{m}(t_k, s_k], \\ y(0) + \gamma(y) &= y_0 \in \mathbb{H}, \end{cases} \tag{1.1}$$

where ${}^{c}_{0}D^{q}_{t}$ is the Caputo fractional derivative of order q, $q \in (\frac{1}{2}, 1)$, $A : D(A) \subset \mathbb{H} \to \mathbb{H}$ is the infinitesimal generator of a strongly continuous semigroup $\{\mathcal{T}(t)\}_{t\geq 0}$ of bounded linear operators on a separable Hilbert space $\mathbb{H}$. Let $J = [0, T]$, $t > 0$ is a constant, $P : J \times \mathbb{H} \to \mathbb{H}$, $\sigma : J \times \mathbb{H} \to \mathbb{H}$ and $\gamma : \mathcal{PC}\big(J, \mathbb{H}\big) \to \mathbb{H}$ are given functions satisfying certain assumptions $0 = t_0 = s_0 < t_1 < s_1 < t_2 \cdots < t_m < s_m < t_{m+1} = T$, $h_k : (t_k, s_k] \times \mathbb{H} \to \mathbb{H}$ are non-instantaneous impulsive functions for all $k = 0, 1, 2, \cdots, m$, the terms $G_iy(t)$ is given by $G_iy(t) = \int_0^t K_i(t, s)y(s)ds$ where $K_i \in \mathcal{C}\big(D, \mathbb{R}^+\big)$, for $i = 1, 2$ are the set of all positive continuous functions on $D := \Big\{(t, s) : 0 \leq s < t \leq T\Big\}$, and y_0 is an $\mathcal{F}_0$-measurable random variable with $\mathbb{E}\|y_0\| < \infty$.

2 Main results

In this section we prove the existence and uniqueness of the mild solution for a class of FSIDEs using different fixed point theorems. Consider the space

$$\mathcal{PC}([0,T],\mathbb{H}) = \Big\{ \mathcal{F}_t\text{-adapted } \mathbb{H}\text{-valued process } y(t) \text{ is continuous every where except for some } t \neq t_k \text{ at which } y(t_k^-) \text{ and } y(t_k^+) \text{ exist and } y(t_k^-) = y(t_k),\ k = 1,2,\cdots,m \text{ and } \sup_{0\le t\le T} \mathbb{E}\|y(t)\|^2 < \infty \Big\},$$

which is a Banach endowed with the norm

$$\|y\|_{\mathcal{PC}} = \left(\sup_{0\le t\le T} \mathbb{E}\|y(t)\|^2 \right)^{\frac{1}{2}}.$$

Definition 2.1. A stochastic process $y \in \mathcal{PC}\big([0,T],\mathbb{H}\big)$ is a mild solution of the system (1.1) if y satisfies the following stochastic integral:

$$y(t) = \begin{cases} U(t)\big(y_0 - \gamma(y)\big) + \displaystyle\int_0^t \big(t-s\big)^{q-1} V\big(t-s\big) P\Big(s, G_1 y(s)\Big) ds \\ \qquad + \displaystyle\int_0^t \big(t-s\big)^{q-1} V\big(t-s\big) \sigma\Big(s, G_2 y(s)\Big) dB(s) & t \in [0,t_1], \\ h_k(t, y(t_k^-)), & t \in \cup_{k=0}^m (t_k, s_k], \\ U(t-s_k) h_k(s_k, y(s_k)) + \displaystyle\int_{s_k}^t \big(t-s\big)^{q-1} V\big(t-s\big) P\Big(s, G_1 y(s)\Big) ds \\ \qquad + \displaystyle\int_{s_k}^t \big(t-s\big)^{q-1} V\big(t-s\big) \sigma\Big(s, G_2 y(s)\Big) dB(s), & t \in \cup_{k=0}^m (s_k, t_{k+1}] \end{cases} \tag{2.1}$$

where

$$U(t) := \int_0^\infty \sigma_q(r) T(t^q r) dr, \qquad V(t) := q \int_0^\infty r \sigma_q(r) T(t^q r) dr,$$

$$\sigma_q(r) := \frac{1}{q} r^{-1-\frac{1}{q}} \varrho_q\big(r^{-\frac{1}{q}}\big)$$

and

$$\varrho_q(r) := \frac{1}{\pi} \sum_{n=1}^{\infty} \big(-1\big)^{n-1} r^{-qn-1} \frac{\Gamma\big(qn+1\big)}{n!} \sin(n\pi q), \quad r \in (0,+\infty).$$

σ_q is a probability density function defined on $(0,+\infty)$.

Now, we will present a list of hypotheses that will be enforced in our main theorem.

(H_1) The function $P : J \times \mathbb{H} \to \mathbb{H}$ satisfies

(1) For each $t \in [0,T]$, $P(.,u)$ is measurable and for any $u \in \mathbb{H}$, $P(t,.)$ is continuous.

(2) There exist a continuous non-decreasing function $\psi_P : \mathbb{R}^+ \to \mathbb{R}^+$, for some positive number $x > 0$, there exists a constant $L_P > 0$, function $\varphi_P \in L^{\frac{1}{p_1}}(J, \mathbb{R}^+)$ and a such that

$$\mathbb{E}\|P(t,u)\|^2 \leq \varphi_P(t)\psi_P\big(\|u\|^2_{\mathcal{PC}}\big), \quad \liminf_{u\to+\infty} \frac{\psi_P(x)}{x} = L_P < +\infty.$$

(H_2) The function $\sigma : J \times \mathbb{H} \to \mathbb{H}$ satisfies

(1) For all $t \in [0,T]$, $\sigma(.,u)$ is measurable and for any $u \in \mathbb{H}$, $\sigma(t,.)$ is continuous.

(2) There exist a continuous non-decreasing function $\psi_\sigma : \mathbb{R}^+ \to \mathbb{R}^+$, for some positive number $x > 0$, there exists a constant $L_\sigma > 0$, function $\varphi_\sigma \in L^{\frac{1}{p}}(J, \mathbb{R}^+)$ and a such that

$$\mathbb{E}\|\sigma(t,u)\|^2 \leq \varphi_\sigma(t)\psi_g\big(\|u\|^2_{\mathcal{PC}}\big), \quad \liminf_{u\to+\infty} \frac{\psi_\sigma(x)}{x} = L_\sigma < +\infty.$$

(H_3) The nonlocal function $\gamma : \mathcal{PC}([0,T], \mathbb{H}) \to \mathbb{H}$ is continuous, there exists a constant $C_\gamma > 0$, satisfies the conditions

$$\mathbb{E}\|\gamma(y)\|^2 \leq C_\gamma, \quad \forall y \in \mathcal{PC}([0,T], \mathbb{H}).$$

(H_4) $h_k : (t_k, s_k] \times \mathbb{H} \to \mathbb{H}$ is continuous function, for $k = 1, 2, \cdots, m$, there exist constants $C_{h_k} \in (0, \frac{1}{2})$, satisfies the conditions:

$$\mathbb{E}\|h_k(t, y(t_k^-))\|^2 \leq C_{h_k}\|y\|^2_{\mathcal{PC}}, \quad \forall y \in \mathbb{H}, \ \ t \in (t_k, s_k].$$

The following should be noted:

$$N_P = \max_{k=0,1,\cdots m}\{\|\varphi_P\|\}_{L^{\frac{1}{p_1}}[s_k,t_{k+1}]}, \quad N_\sigma = \max_{k=0,1,\cdots m}\{\|\varphi_\sigma\|\}_{L^{\frac{1}{p}}[s_k,t_{k+1}]}.$$

Theorem 2.1. *Assume that the assumptions* (H_1)*-*(H_2)*and* (H_3)*-*(H_4)*are satisfied. Then the system 1.1 has a mild solution on* $[0,T]$*, provided that*

$$C := 3M^2C_{h_k} + \frac{3M^2L_PT^{(a+1)(1-p_1)}N_P}{\Gamma^2(q)\,(a+1)^{(1-p_1)}} + \frac{3M^2Tr\big(\mathcal{Q}\big)L_\sigma T^{(b+1)(1-p)}N_\sigma}{\Gamma^2(q)\,(b+1)^{(1-p)}} < 1. \tag{2.2}$$

And

$$M^2C_\gamma + \big(M^2+1\big)C_{h_k} < 1.$$

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Aerosol Particles Coalescence Modelisation and Simulation along with Monte Carlo Probabilistic Model

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Abstract
The present paper is devoted to the analysis of a numerical approach for solving the Smoluchovski's equation for atmospheric aerosols coagulation process enhanced by the Van der Waals forces and monitored by the Brownian motion. We focus on the study of the number concentration of coagulated particles, which decreases with the growth of the particle size. We explore a method for solving Smoluchowski's coagulation equation using the Monte Carlo probabilistic method. Furthermore, we prove the convergence of the MC numerical scheme in the case of the coagulation equation.

Keywords: Atmospheric aerosols; Monte Carlo probabilistic method; Smoluchowski PDE equation.

1 Introduction

Atmospheric aerosols are tiny solid or liquid droplet particles suspended in atmosphere and composed of dirt, clay or air pollutants. Furthermore, depending on their magnitude, atmospheric aerosols have a significant impact on human life in a variety of ways [1]. In reality, it impacts the biosphere's global temperature and personal health. Hence, its health impact has been the topic of increasing scientific inquiry. One of the main concerns in aerosol physics is the process of coalescence which can occur due to various micro-physical transformations [2, 3]. In this context, we employ the Monte Carlo probabilistic method in order to analyze aerosol dynamics. The objective of this work is to study the coagulation process greatly influenced by Van der Waals forces and monitored through Brownian motion. Our focus is tracking the changes in the mass concentration of particles

which aligns with current regulations. Furthermore, we carry out numerous numerical simulations to evaluate their accuracy in measuring mass concentrations.

The beginning of the classical understanding of aggregation stem from the work of Smoluchowski [4] which has the unsteady-state form of:

$$\frac{\partial c}{\partial t}(i,t) = \frac{1}{2}\sum_{j=1}^{i-1} K(i-j,j)\; c(i-j,t)\; c(j,t) - \sum_{j=1}^{\infty} K(i,j)\; c(i,t)\; c(j,t), i,\; j \in \mathbb{N}^*, t \geq 0, \tag{1.1}$$

$$K(i,j) = 1 \text{ and } c(i,0) = c_i(0) = \begin{cases} 1 & \text{if } i = 1, \\ 0 & \text{else} \end{cases}.$$

System (1.1) is deduced for a set of particles or any other whole mass at time t, where $c(i,t)$ denotes the particle size distribution number of size i at time t usually defined by a continuous distribution and $c(i,t)\; di$ stands for the particle number concentration whose single particle mass lies between i and $i + di$. Their probability of fusion is $K(i,j)$, which is the coagulation Kernel proportional to the density of these two particles.

2 Main results

Numerical simulations were performed on computers with the following processor characteristics: Core i7-3632QM max turbo frequency 3.2 GHz. In the present study, the performance of the numerical simulations is evaluated by employing the analytical method ability assessing criterion proposed for solving the SE due to Brownian coagulation. Therefore, we consider that the investigated analytical method and the reference numerical method show the same precision in solving SE if the absolute relative errors are less than 0.050

The following table shows that given a particles number N, the lower the number of time steps Δt, the greater is the error in moments of order 0.

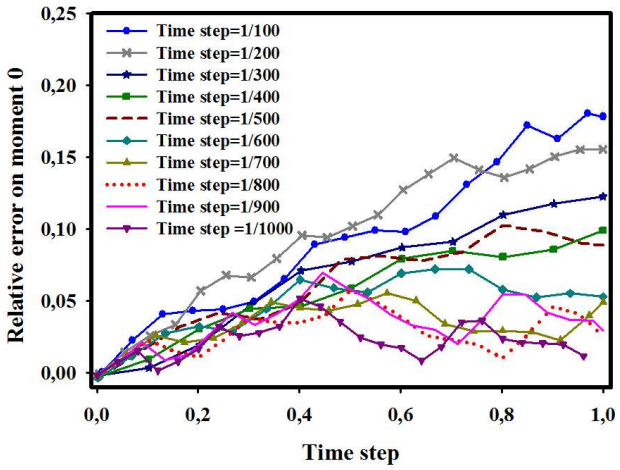

Figure 1: Relative error on zeroth moments corresponding to time steps varying between $\Delta T=T/100$ and $\Delta T=T/1000$ for N= 1400000.

Time step	**Relative error on moment 0**
$\Delta T=T/100$	0.178150
$\Delta T=T/200$	0.155440
$\Delta T=T/300$	0.122600
$\Delta T=T/400$	0.099193
$\Delta T=T/500$	0.088499
$\Delta T=T/600$	0.052801
$\Delta T=T/700$	0.049041
$\Delta T=T/800$	0.025677
$\Delta T=T/900$	0.029447
$\Delta T=T/1000$	-0.003129

Figure 2: Relative error on zeroth moments corresponding to time steps varying between $\Delta T=T/100$ and $\Delta T=T/1000$ for N= 1400000.

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Fractional Stochastic Integro-differential Equations with Non-instantaneous Impulsive

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The purpose of this research is studies the existence and uniqueness of mild solutions for a class of fractional stochastic integro-differential equations with non-instantaneous impulses and using nonlocal conditions for the following:

$$\begin{cases} {}^{c}_{0}D^{q}_{t}y(t) &= Ay(t) + P\Big(t, G_1y(t)\Big) + \sigma\Big(t, G_2y(t)\Big)\frac{dB(t)}{dt}, \quad t \in \cup_{k=0}^{m}(s_k, t_{k+1}], \\ y(t) &= h_k(t, y(t_k^-)), \qquad t \in \cup_{k=0}^{m}(t_k, s_k], \\ y(0) + \gamma(y) &= y_0 \in \mathbb{H}, \end{cases} \tag{1.1}$$

where ${}^{c}_{0}D^{q}_{t}$ is the Caputo fractional derivative of order q, $q \in (\frac{1}{2}, 1)$, $A : D(A) \subset \mathbb{H} \to \mathbb{H}$ is the infinitesimal generator of a strongly continuous semigroup $\{\mathcal{T}(t)\}_{t\geq 0}$ of bounded linear operators on a separable Hilbert space $\mathbb{H}$. Let $J = [0, T]$, $t > 0$ is a constant, $P : J \times \mathbb{H} \to \mathbb{H}$, $\sigma : J \times \mathbb{H} \to \mathbb{H}$ and $\gamma : \mathcal{PC}\big(J, \mathbb{H}\big) \to \mathbb{H}$ are given functions satisfying certain assumptions $0 = t_0 = s_0 < t_1 < s_1 < t_2 \cdots < t_m < s_m < t_{m+1} = T$, $h_k : (t_k, s_k] \times \mathbb{H} \to \mathbb{H}$ are non-instantaneous impulsive functions for all $k = 0, 1, 2, \cdots, m$, the terms $G_iy(t)$ is given by $G_iy(t) = \int_0^t K_i(t, s)y(s)ds$ where $K_i \in \mathcal{C}\big(D, \mathbb{R}^+\big)$, for $i = 1, 2$ are the set of all positive continuous functions on $D := \Big\{(t, s) : 0 \leq s < t \leq T\Big\}$, and y_0 is an $\mathcal{F}_0$-measurable random variable with $\mathbb{E}\|y_0\| < \infty$.

2 Main results

In this section we prove the existence and uniqueness of the mild solution for a class of FSIDEs using different fixed point theorems. Consider the space

$$\mathcal{PC}([0,T],\mathbb{H}) = \Big\{ \mathcal{F}_t\text{-adapted } \mathbb{H}\text{-valued process } y(t) \text{ is continuous every where except for some } t \neq t_k \text{ at which } y(t_k^-) \text{ and } y(t_k^+) \text{ exist and } y(t_k^-) = y(t_k),\ k = 1,2,\cdots,m \text{ and } \sup_{0\le t\le T} \mathbb{E}\|y(t)\|^2 < \infty \Big\},$$

which is a Banach endowed with the norm

$$\|y\|_{\mathcal{PC}} = \left(\sup_{0\le t\le T} \mathbb{E}\|y(t)\|^2 \right)^{\frac{1}{2}}.$$

Definition 2.1. A stochastic process $y \in \mathcal{PC}\big([0,T],\mathbb{H}\big)$ is a mild solution of the system (1.1) if y satisfies the following stochastic integral:

$$y(t) = \begin{cases} U(t)\big(y_0 - \gamma(y)\big) + \displaystyle\int_0^t \big(t-s\big)^{q-1} V\big(t-s\big) P\Big(s, G_1 y(s)\Big) ds \\ \qquad + \displaystyle\int_0^t \big(t-s\big)^{q-1} V\big(t-s\big) \sigma\Big(s, G_2 y(s)\Big) dB(s) & t \in [0,t_1], \\ h_k(t, y(t_k^-)), & t \in \cup_{k=0}^m (t_k, s_k], \\ U(t-s_k) h_k(s_k, y(s_k)) + \displaystyle\int_{s_k}^t \big(t-s\big)^{q-1} V\big(t-s\big) P\Big(s, G_1 y(s)\Big) ds \\ \qquad + \displaystyle\int_{s_k}^t \big(t-s\big)^{q-1} V\big(t-s\big) \sigma\Big(s, G_2 y(s)\Big) dB(s), & t \in \cup_{k=0}^m (s_k, t_{k+1}] \end{cases} \tag{2.1}$$

where

$$U(t) := \int_0^\infty \sigma_q(r) T(t^q r) dr, \qquad V(t) := q \int_0^\infty r \sigma_q(r) T(t^q r) dr,$$

$$\sigma_q(r) := \frac{1}{q} r^{-1-\frac{1}{q}} \varrho_q\big(r^{-\frac{1}{q}}\big)$$

and

$$\varrho_q(r) := \frac{1}{\pi} \sum_{n=1}^{\infty} \big(-1\big)^{n-1} r^{-qn-1} \frac{\Gamma\big(qn+1\big)}{n!} \sin(n\pi q), \quad r \in (0,+\infty).$$

σ_q is a probability density function defined on $(0,+\infty)$.

Now, we will present a list of hypotheses that will be enforced in our main theorem.

(H_1) The function $P : J \times \mathbb{H} \to \mathbb{H}$ satisfies

(1) For each $t \in [0,T]$, $P(.,u)$ is measurable and for any $u \in \mathbb{H}$, $P(t,.)$ is continuous.

(2) There exist a continuous non-decreasing function $\psi_P : \mathbb{R}^+ \to \mathbb{R}^+$, for some positive number $x > 0$, there exists a constant $L_P > 0$, function $\varphi_P \in L^{\frac{1}{p_1}}(J, \mathbb{R}^+)$ and a such that

$$\mathbb{E}\|P(t,u)\|^2 \leq \varphi_P(t)\psi_P\big(\|u\|^2_{\mathcal{PC}}\big), \quad \liminf_{u\to+\infty} \frac{\psi_P(x)}{x} = L_P < +\infty.$$

(H_2) The function $\sigma : J \times \mathbb{H} \to \mathbb{H}$ satisfies

(1) For all $t \in [0,T]$, $\sigma(.,u)$ is measurable and for any $u \in \mathbb{H}$, $\sigma(t,.)$ is continuous.

(2) There exist a continuous non-decreasing function $\psi_\sigma : \mathbb{R}^+ \to \mathbb{R}^+$, for some positive number $x > 0$, there exists a constant $L_\sigma > 0$, function $\varphi_\sigma \in L^{\frac{1}{p}}(J, \mathbb{R}^+)$ and a such that

$$\mathbb{E}\|\sigma(t,u)\|^2 \leq \varphi_\sigma(t)\psi_g\big(\|u\|^2_{\mathcal{PC}}\big), \quad \liminf_{u\to+\infty} \frac{\psi_\sigma(x)}{x} = L_\sigma < +\infty.$$

(H_3) The nonlocal function $\gamma : \mathcal{PC}([0,T], \mathbb{H}) \to \mathbb{H}$ is continuous, there exists a constant $C_\gamma > 0$, satisfies the conditions

$$\mathbb{E}\|\gamma(y)\|^2 \leq C_\gamma, \quad \forall y \in \mathcal{PC}([0,T], \mathbb{H}).$$

(H_4) $h_k : (t_k, s_k] \times \mathbb{H} \to \mathbb{H}$ is continuous function, for $k = 1, 2, \cdots, m$, there exist constants $C_{h_k} \in (0, \frac{1}{2})$, satisfies the conditions:

$$\mathbb{E}\|h_k(t, y(t_k^-))\|^2 \leq C_{h_k}\|y\|^2_{\mathcal{PC}}, \quad \forall y \in \mathbb{H}, \ \ t \in (t_k, s_k].$$

The following should be noted:

$$N_P = \max_{k=0,1,\cdots m}\{\|\varphi_P\|\}_{L^{\frac{1}{p_1}}[s_k,t_{k+1}]}, \quad N_\sigma = \max_{k=0,1,\cdots m}\{\|\varphi_\sigma\|\}_{L^{\frac{1}{p}}[s_k,t_{k+1}]}.$$

Theorem 2.1. *Assume that the assumptions* (H_1)*-*(H_2)*and* (H_3)*-*(H_4)*are satisfied. Then the system 1.1 has a mild solution on* $[0,T]$*, provided that*

$$C := 3M^2C_{h_k} + \frac{3M^2L_PT^{(a+1)(1-p_1)}N_P}{\Gamma^2(q)\,(a+1)^{(1-p_1)}} + \frac{3M^2Tr\big(\mathcal{Q}\big)L_\sigma T^{(b+1)(1-p)}N_\sigma}{\Gamma^2(q)\,(b+1)^{(1-p)}} < 1. \tag{2.2}$$

And

$$M^2C_\gamma + \big(M^2+1\big)C_{h_k} < 1.$$

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Quasistatic evolution of damage in thermo-electro-elastic-viscoplastic materials with normal compliance

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Abstract
We have studied contact problem in a quasistatic process with boundary conditions, for which we couple a thermal effect and damage and electrical effect. We assume that the contact is with normal compliance in a form with a gap function and is associated to a slip rate-dependent version of Coulomb's law of dry friction. We give the variational formulation, then the existence and the uniqueness of the weak solution. The proof is based on arguments of time-dependent variational inequalities, parabolic inequalities, and fixed point.

Keywords: Electro-viscoplastic, Quasistatic, Damage, temperature ,differential equations, fixed point.
2020 Mathematics Subject Classification:49J40,47H10, 74M10,74C10, 74M15.

1 Introduction

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2 Main results

3 Problem statement

Consider a material body situated within a bounded domain $\Omega \subset \mathbb{R}^d (d = 2, 3)$, featuring a well-defined boundary surface Γ. This surface is divided into three measurable parts, namely Γ_1, Γ_2 and Γ_3, with the condition $meas\Gamma_1 > 0$. The unit outward normal vector on Γ is denoted as ν. The body is fixed within a structure on Γ_1. Surface tractions of density f_2 and voluminal forces of density f_0, both varying slowly with respect to time, act on Γ_2 and Ω respectively. Additionally, the body is subject to electric charges with volume density q_0 and surface electric charges. To describe the latter, we introduce another partition of the boundary into three parts: Γ_a, Γ_b and Γ_3 with $meas(\Gamma_a) > 0$. Frictional contact with a conductive foundation occurs on Γ_3, where the electric potential vanishes on Γ_a, and a prescribed surface electric charge density q_2 is applied on Γ_b. At each time instant, Γ_3

is further divided into two parts: one where the body and foundation are in contact, and the other where they are separated. The boundary of the contact part represents a free boundary, determined by the solution of the problem. For generality, we assume the existence of a gap, denoted as $g = g(x)$, between Γ_3 and the foundation in the reference configuration. This gap is measured along the outer normal ν. It is crucial to note that the assumption $meas(\Gamma_1) > 0$ is essential in quasistatic problems. Without this assumption, the problem becomes noncoercive mathematically, and many estimates and results outlined below may not hold. This aligns with the physical scenario where, if $\Gamma_1 = 0$, the body is not restrained and can move freely in space as a rigid body.
The classical formulation of the mechanical problem involving an electro-elastic-viscoplastic body with damage and thermal effects can be articulated as follows.

Problem P

Find a displacement field $\boldsymbol{u} : \Omega \times (0,T) \to \mathbb{R}^d$, a stress field $\boldsymbol{\sigma} : \Omega \times (0,T) \to \mathbb{S}^d$, an electric potential field $\varphi : \Omega \times (0,T) \to \mathbb{R}$, a temperature field $\theta : \Omega \times (0,T) \to \mathbb{R}$, an electric displacement field $\boldsymbol{D} : \Omega \times (0,T) \to \mathbb{R}^d$, and a damage field $\alpha : \Omega \times (0,T) \to \mathbb{R}$ such that

$$\begin{aligned}\boldsymbol{\sigma}(t) =& \mathcal{A}\varepsilon(\dot{\boldsymbol{u}}(t)) + \mathcal{B}\varepsilon(\boldsymbol{u}(t)) - \mathcal{E}^* E(\varphi(t)) \\ &+ \int_0^t \mathcal{G}(\boldsymbol{\sigma}(s) - \mathcal{A}(\varepsilon(\dot{\boldsymbol{u}}(s)) + \mathcal{E}^* E(\varphi(t)), \varepsilon(\boldsymbol{u}(s)), \theta(s), \alpha(s))ds \\ &\text{in } \Omega, \text{ a.e } .t \in (0,T),\end{aligned} \tag{3.1}$$

$$\boldsymbol{D} = \mathcal{E}\varepsilon(\boldsymbol{u}) + \boldsymbol{B}\nabla(\varphi) \qquad \text{in } \Omega \times (0,T), \tag{3.2}$$

$$\operatorname{Div} \boldsymbol{\sigma} + f_0 = 0 \qquad \text{in } \Omega \times (0,T), \tag{3.3}$$

$$\operatorname{div} \boldsymbol{D} - q_0 = 0 \qquad \text{in } \Omega \times (0,T), \tag{3.4}$$

$$\begin{aligned}\dot{\theta} - k_0 \Delta\theta = \Theta\left(\boldsymbol{\sigma}(t) - \mathcal{A}(\varepsilon(\dot{\boldsymbol{u}}(t)) + \mathcal{E}^* E(\varphi(t)), \varepsilon\left(\boldsymbol{u}\right), \theta, \alpha\right) + \rho \\ \text{in } \Omega \times (0,T),\end{aligned} \tag{3.5}$$

$$\begin{aligned}\dot{\alpha} - k_1 \Delta\alpha + \partial\varphi_K(\alpha) \ni S\left(\boldsymbol{\sigma}(t) - \mathcal{A}(\varepsilon(\dot{\boldsymbol{u}}(t)) + \mathcal{E}^* E(\varphi(t)), \varepsilon(\boldsymbol{u}), \theta, \alpha\right) \\ \text{in } \Omega \times (0,T),\end{aligned} \tag{3.6}$$

$$\boldsymbol{u} = \boldsymbol{0} \qquad \text{on } \Gamma_1 \times (0,T), \tag{3.7}$$

$$\boldsymbol{\sigma\nu} = f_2 \qquad \text{on } \Gamma_2 \times (0,T), \tag{3.8}$$

$$-\sigma_\nu = p\left(u_\nu - g\right) \quad \text{on } \Gamma_3 \times (0,T), \tag{3.9}$$

$$\left.\begin{aligned}&\|\boldsymbol{\sigma}_\tau\| \le \mu\left(\|\dot{\boldsymbol{u}}_\tau\|\right)|\sigma_\nu| \\ &-\boldsymbol{\sigma}_\tau = \mu\left(\|\dot{\boldsymbol{u}}_\tau\|\right)|\sigma_\nu| \frac{\dot{\boldsymbol{u}}_\tau}{\|\boldsymbol{u}_\tau\|} \quad \text{if} \quad \dot{\boldsymbol{u}}_\tau \neq \boldsymbol{0}\end{aligned}\right\} \quad \text{on } \Gamma_3 \times (0,T), \tag{3.10}$$

$$\frac{\partial\alpha}{\partial\nu} = 0 \qquad \text{on } \Gamma \times (0,T), \tag{3.11}$$

$$\varphi = 0 \qquad \text{on } \Gamma_a \times (0,T), \tag{3.12}$$

$$\boldsymbol{D}.\nu = q_2 \qquad \text{on } \Gamma_b \times (0,T), \tag{3.13}$$

$$\boldsymbol{D} \cdot \boldsymbol{\nu} = \psi\left(u_\nu - g\right)\phi_l\left(\varphi - \varphi_0\right) \quad \text{on } \Gamma_3 \times (0,T), \tag{3.14}$$

$$k_0 \frac{\partial\theta}{\partial\nu} + k_2\theta = 0 \quad \text{on } \Gamma \times (0,T), \tag{3.15}$$

$$\boldsymbol{u}(0) = \boldsymbol{u}_0, \quad \theta(0) = \theta_0, \quad \alpha(0) = \alpha_0, \qquad \text{in } \Omega. \tag{3.16}$$

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Limit Cycles Bifurcating from a Degenerate center

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Abstract
This talk deals with the maximum number of limit cycles bifurcating from the degenerate center

$$\dot{x} = -y(3x^2 + y^2),\ \dot{y} = x(x^2 - y^2),$$

when we perturb it inside the class of all quintic homogeneous polynomial differential systems. By using averaging theory of second order, we show that at most five limit cycles produced from the periodic orbits surrounding the degenerate center under the quintic perturbation. In addition, this upper bound can be reached.

Keywords: Averaging theory, Limit cycles, Polynomial differential systems, Degenerate center.
2020 Mathematics Subject Classification: 34C29, 34C25, 47H11.

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The existence and multiplicity of non-trivial solutions for a class of superlinear fractional Schrödinger-Poisson systems

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Abstract
The main aim of this work is to establish the existence and multiplicity of non-trivial solutions to a new class of fractional Schrödinger-Poisson systems of the following form

$$\begin{cases} -div^{\sigma}(\nabla^{\sigma}u) + V(x)u + K(x)\phi u = g(x,u) & \text{in } \mathbb{R}^3, \\ -div^{\beta}(\nabla^{\beta}\phi) = K(x)u^2 & \text{in } \mathbb{R}^3, \end{cases} \tag{0.1}$$

where $\sigma, \beta \in (0,1)$, $4\sigma + 2\beta > 3$, and $div^{\sigma}(\nabla^{\sigma})$ is the distributional Riesz fractional derivative. First, we introduce the latter operator and investigate their natural functional space. Then, we pose the given problem in that space. By using variational methods, based on the symmetric mountain pass theorem under certain assumptions imposed on g, K and V, we investigate the existence of infinitely many non-trivial solutions.

Keywords: Fractional Schrödinger-Poisson systems, Bessel potential spaces, non-trivial solutions, Riesz fractional derivatives.
2020 Mathematics Subject Classification: 35J60, 35R11, 35A15.

1 Introduction

Over the past few years, the search for a good concept to the nonlocal operator has led many researchers to develop a variety of different definitions of fractional derivatives operators. Particularly, the distributional Riesz fractional derivatives, these operators have achieved an immense attention, and many excellent results have emerged. On the other hand, the pioneering work of Shieh and Spector of 2015 [2] leads to a new class of fractional PDEs and new problems in the calculus of variations based on distributional Riesz fractional gradient, they proved that the fractional Laplacian coincides with the composition of fractional divergence and fractional gradient andshowed that the fractional gradient is an interesting object for the study of fractional partial differential equations problems. Furthermore, this kind of operator has an impressive applications in the theory of electromagnetic fields, multidimensional processes, and in fractal media.

In last years, non local systems involving fractional Laplacian operators have received the attention of several scholars, mainly the following fractional Schrödinger-Poisson system

$$\begin{cases} (-\Delta)^{\sigma}u + V(x)u + K(x)\phi u = g(x,u) & \text{in } \mathbb{R}^3, \\ (-\Delta)^{\beta}\phi = K(x)u^2 & \text{in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

which used to study the solitary waves solutions for the fractional Schrödinger equations. Recently, there have been many papers devoted to the study on the existence of solutions to (1.1) by means of variational tools and critical point theory under different assumptions on V, K and g. We call attention to the work of He and Lei [1], where it was proved the multiplicity of non-trivial solutions for (1.1) by exploiting the symmetric mountain pass theorem. We notice that when $g(x,u) = \lambda g(x,u)$, we [4] proved the existence of infinitely many solutions by using the fountain theorem.
In this viewpoint, this work aims to improve the previous results related to the (1.1) as well as to improve the hypotheses required and to extend the result in [1] to our case considering the presence of distributional Riesz fractional derivative instead of fractional Laplacian.
The starting concept of this research for the development of a general theory for fractional PDEs involving the distributional Riesz fractional derivative, is the distributional Riesz fractional gradient ∇^{σ}which can be written in finit integral form for sufficiently smooth function u (see [3]) by

$$\nabla^{\sigma}u(x) = C_{d,\sigma}\int_{\mathbb{R}^d}[u(x)-u(y)]\frac{x-y}{|x-y|^{d+\sigma+1}}dy,$$

It was observed in [2] that for $u \in C_0^{\infty}(\mathbb{R}^d)$, the composition of div^{σ} and ∇^{σ} it coincides with the well known fractional Laplacian $(-\Delta)^{\sigma}u$ as follows:

$$\begin{aligned} (-\Delta)^{\sigma}u &= \frac{1}{2}C_{d,\sigma}^2\int_{\mathbb{R}^d}\frac{u(x+y)+u(x-y)-2u(x)}{|y|^{d+2\sigma}}dy \\ &= -\sum_{j=1}^{d}\frac{\partial^{\sigma}}{\partial x_j^{\sigma}}\frac{\partial^{\sigma}}{\partial x_j^{\sigma}}u \\ &= -div^{\sigma}(\nabla^{\sigma}u), \end{aligned} \tag{1.2}$$

Next, we require the following assumptions on g, K and V where $G(x,u) = \int_0^u g(x,t)dt$.

(g_1) : $g \in C(\mathbb{R}^3\times\mathbb{R};\mathbb{R})$ $\forall (x,u)\in\mathbb{R}^3\times\mathbb{R}$, and there exist constants $C_1>0$, and $p\in\,]2;2_{\sigma}^*[$ such that

$$|g(x,u)| \le C_1(|u|+|u|^{p-1}),$$

where $2_{\sigma}^* = \frac{6}{3-2\sigma}$ is the fractional critical Sobolev exponent.
(g_2) : $g(x,-u) = -g(x,u)$, $x\in\mathbb{R}^3$, $u\in\mathbb{R}$.
(g_3) : $\lim\limits_{|u|\to\infty}\frac{G(x,u)}{|u|^4} = \infty$ unifomly in $x\in\mathbb{R}^3$ and $G(x,u)\ge 0$ $\forall (x,u)\in\mathbb{R}^3\times\mathbb{R}$.
(g_4) : There exist $\lambda>0$ and $L>0$ such that

$$4G(x,u) \le ug(x,u) + \lambda|u|^2 \quad \text{for a.e. } x\in\mathbb{R}^3 \text{ and } \forall\,|u|\ge L.$$

(K) : $K(x) \in L^{\frac{6}{4\sigma+2\beta-3}}(\mathbb{R}^3) \cup L^{\infty}(\mathbb{R}^3)$, $K(x) \geq 0$ $\forall x \in \mathbb{R}^3$.
(V): $V(x) \in C(\mathbb{R}^3, \mathbb{R})$, $\inf_{x\in\mathbb{R}^3} V(x) > 0$, and for any $V_0 > 0$
meas $\{x \in \mathbb{R}^3 \; V(x) \leq V_0\} < +\infty$.

2 Main results

The main result of this paper is as follows:

Theorem 2.1. *Assume that system* (**??**) *satisfies* (g_1)-(g_4), (K) *and* (V). *Then,* (**??**) *has infinitely many distributional solutions.*

To prove our main result, we need the following symmetric mountain-pass theorem [**?**].

Theorem 2.2. *Assume that* $E = Y_k \oplus Z_k$ *be a Banach space where* Y *is finite dimensional, let* $\Phi \in C^1(E, \mathbb{R})$ *be even and* $\Phi(0) = 0$, *satisfies the* $(C)_c$ *condition, if*
(i) there exist constants $\rho, \delta > 0$ *satisfying* $\Phi\big|_{\partial B_\rho \bigcap Z} \geq \delta$, *where* B_ρ *is the open ball in* E *of radius* ρ *about* 0 *and* ∂B_ρ *is its boundary,*
(ii) for any finite dimensional subspace $\tilde{E} \subset E$, *there is* $C = C(\tilde{E}) > 0$ *such that* $\Phi(u) \leq 0$ *on* $\tilde{E} \setminus B_C$,
then, Φ *has an unbounded sequence of critical points.*

Lemma 2.1. Φ *satisfies* $(C)_c$ *condition for every* $c \in \mathbb{R}$ *on* E, *if* (g_1)-(g_4) *and* (V) *hold.*

Lemma 2.2. *Suppose that* (g_1),(g_3) *and* (V) *are satisfied. Then, for any finite dimensional subspace* $\tilde{E} \subset E$, *we have*

$$\Phi(u) \to -\infty, \qquad \|u\| \to \infty, \quad u \in \tilde{E}.$$

Corollary 2.1. *Assume that* (g_1),(g_3) *and* (V) *are satisfied. Then, there is a constant* $C = C(\tilde{E}) > 0$ *for every finite dimensional subspace* $\tilde{E} \subset E$ *such that*

$$\Phi(u) \leq 0 \quad \textit{for all} \;\; u \in \tilde{E} \quad \textit{with} \; \|u\| \geq C.$$

Lemma 2.3. *If* (g_1) *and* (V) *hold. Then, there exist constants* $\rho, \delta > 0$ *satisfying* $\Phi\big|_{\partial B_\rho \bigcap Z_m} \geq \delta > 0$.

Proof of Theorem 2.1 Let $Y = Y_m$ and $Z = Z_m$. Due to (g_2), and based on Lemma 2.1, Lemma 2.3 and Corollary 2.1, the energy functional related to the system (0.1) satisfies all conditions of Theorem 2.2. Consequently, the system (0.1) possesses infinitely many distributional solutions.

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Maximum number of limit cycles of a second order differential system

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Abstract
In this paper, we study the limit cycles of a perturbed differential system in $\mathbb{R}^2$, given by

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= -x - \epsilon(1+\sin^n(\theta)\cos^m(\theta))H(x,y), \end{cases}$$

where ϵ is a small parameter, m and n are non-negative integers, $\tan(\theta) = y/x$, and $H(x,y)$ is a real polynomial of degree $l \geq 1$. Using Averaging theory of first order we provide an upper bound for the maximum number of limit cycles. Also, we provide some examples to confirm and illustrate our results.

Keywords: Limit cycles, Averaging theory, Polynomial differential systems.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

The limit cycle is one of the most used notions in the qualitative theory of planar differential systems recently with the arising of its maximum number determination problem. Poincaré initiated the study of limit cycles cite6 in 1881 where he defined this notion as a periodic orbit isolated in the set of all periodic orbits of a differential system. He also defined the notion of a center of a real planar differential system which is an isolated critical point in the neighborhood of periodic orbits. In this paper, we will produce limit cycles by perturbing the periodic orbits of a center using Averaging theory, which is a powerful tool for studying the existence, number, and behavior of limit cycles. Many results concern the maximum number of limit cycles bifurcating from the linear center of planar polynomial differential systems using averaging theory (see for instance [1]).

The Mathieu differential equation ([2])

$$\ddot{x} + b(1+\cos t)x = 0, \tag{1.1}$$

where b is a real constant, is the simplest mathematical model of an excited system depending on a parameter, it describes the dynamics of a system with harmonic parametric excitation and a nonlinear

term corresponding to a restoring force. In [3], T. Chen & J. Llibre studied the limit cycles of a kind of generalization of the equation (1.1). They considered the second-order differential system

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= -x - \epsilon(1+\cos^m(\theta))Q(x,y), \end{cases}$$

where ϵ sufficiently small, m is an arbitrary non negative integer, $Q(x,y)$ is a polynomial of degree $n \geq 1$ and $\theta = \arctan(y/x)$. They studied the maximum number of limit cycles that can bifurcate from the linear center of the previous system with $\epsilon = 0$. They obtained an upper bound for this system with respect to the parity of m and n.
In this work, using the averaging theory of first order, we study the maximum number of limit cycles of the following differential system

$$\begin{cases} \dot{x} &= y, \\ \dot{y} &= -x - \epsilon(1+\sin^n(\theta)\cos^m(\theta))H(x,y), \end{cases} \tag{1.2}$$

where ϵ sufficiently small, m and n are arbitrary non negative integers, $H(x,y)$ is a polynomial of degree $l \geq 1$ and $\theta = \arctan(y/x)$.

2 Main results

Theorem 2.1. *Using the averaging theory of first order, the maximum number of limit cycles of the differential system (1.2) bifurcating from the periodic solutions of the linear center $\dot{x} = y$, $\dot{y} = -x$ is as follows*

1. *If l is* ***even:***

   (1.a) *we have at most $l-1$ limit cycles, if m is even and n is odd or m is odd and n is even.*

   (1.b) *we have at most $(l-2)/2$ limit cycles, if m is even and n is even or m is odd and n is odd.*

2. *If l is* ***odd:***

   (2.a) *we have at most l limit cycles, if m is even and n is odd or m is odd and n is even.*

   (2.b) *we have at most $(l-1)/2$ limit cycles, if m is even and n is even or m is odd and n is odd.*

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multivalued contractions and coincidence points

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Abstract
In the search for new fixed point results . We present novel coincidence point findings in complete metric spaces by integrating the concepts of former works in weak contractions for multivalued and single valued maps . .

Keywords: Metric space, Generalized contraction, Simulation function, Fixed point, Complete space, multivalued.
2020 Mathematics Subject Classification: Primary

1 Introduction

Fixed point Theory One of the outstanding sub-fields of nonlinear functional analysis , this important theory only truly bloomed in 1922 when Steven Banach proved the existence and uniqueness of the fixed point under the conditions that he proposed in the his theory . Ever since then , Banach's method has attracted many great authors to follow in his footsteps . among these authors many have came up with alternative concepts for counteractive mappings in the set of a complete metric space . This study explores new coincidence point findings in complete metric spaces by integrating concepts from weak contractions for both single-valued and multivalued maps. By building on previous works, we aim to uncover fresh insights into the existence and properties of these points,

2 Main results

Let (X, d) be a metric space, $f : X \to X$ be a self-mapping and $T : X \to CB(X)$, assuming that there exists $\zeta \in \mathcal{Z}_G$ and $\lambda \in]0, +\infty[$ such that :

$$\zeta(H(Tx, Ty), \lambda d(fx, fy)) \geq 0$$

for all $x, y \in X$ with $fx \neq fy$. Suppose also that for all $x \in X$ we have $Tx \subseteq fX$. Also assume that at least one of the following conditions hold (i) $(f(X), d)$ is complete ; (ii) (X, d) is complete. Then f and T have a point of coincidence .

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26-27 Jun 2024 Constantine (Algeria).

Stabilization of fractional order Liu chaotic system using a sliding mode control strategy

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Abstract—

In this paper a fractional sliding mode controller (FOSMC) is proposed to control the chaotic fractional-order Liu's system. Sliding mode control is a robust controller and it's objective is to allow the system states to move to the sliding surface and remain on it. The stability analysis for the proposed control is studied based on the Lyapunov stability theorem.

The fractional-order Liu system is defined as follows [1] ,

(1)

With the parameter values given as,

To propose a sliding mode control scheme, we define the sliding surface as [2-3]

(2)

And the switching control action can be defined as follows,

(3)

Simulation results show the ability of the proposed controller to improve the system performance (see Figure 1).

Keywords: Fractional-order Liu chaotic system, fractional-order control, Sliding mode control, Stability.

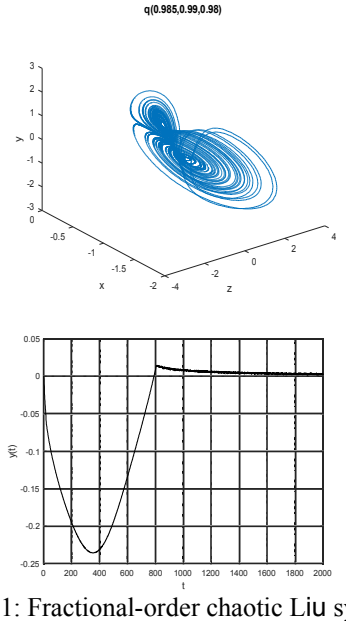

Figure 1: Fractional-order chaotic Liu system :
(a) Without control. (b) with states sliding mode control.

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Title

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Abstract
In this study, we examine the Odd Lindley Exponentiated Exponential distribution's dependability features and the estimation of the unknown parameters. With full data and a gamma prior distribution, several loss functions are used to construct the Bayes estimators and associated hazards. The computation of all the results was done through a simulation research. The integrated mean squared error and Pitman's proximity criterion were used to evaluate the effectiveness of the maximum likelihood and Bayesian estimators. Finally, we analyse a real-world data set to demonstrate our methods.

Keywords: Reliability, Estimation, Bayesian analysis, Simulations, Monte Carlo Methods.
2020 Mathematics Subject Classification: Primary 62E99,60E05.

1 Introduction

The Odd Lindley Exponentiated Exponential model, which was first introduced by Goual et al. (2019) [7], is the basis for this work. In that study, the validity of the OLEE was examined through the use of a modified goodness-of-fit test, and its applications to censor and full data were examined. It was discovered that the suggested lifetime model outperforms the exponentialexponential, Moment exponential, Log Burr Hatke exponential, and the Lindley exponential model with two parameters that are odd. In order to model failure time data, the new lifespan model is a good substitute for these models. This model is the result of numerous studies, including those by Korkmaz and Yousof (2017) [9] who examined the one-parameter odd Lindley distribution and Gupta and Kundu (2001) [5] who examined the Exponentiated Exponential Family: an Alternative to Gamma and Weibull Distributions. The probability density function (PDF) (1), cumulative distribution function (CDF) (2) reliability function (RF) (3) and hazard rate function (HRF) (4) are respectively given by

$$f_{(\alpha,\theta,\lambda)}(x) = \frac{\alpha^2\theta\lambda\exp(-\lambda x)(1-\exp(-\lambda x))^{\theta-1}}{(1+\alpha)\left(1-(1-\exp(-\lambda x))^{\theta}\right)^3} exp\left(-\alpha\frac{(1-\exp(-\lambda x))^{\theta}}{1-(1-\exp(-\lambda x))^{\theta}}\right), x \geq 0 \quad (1.1)$$

$$F_{(\alpha,\theta,\lambda)}(x) = 1 - \frac{\alpha + \left(1-(1-\exp(-\lambda x))^{\theta}\right)}{(1+\alpha)\left(1-(1-\exp(-\lambda x))^{\theta}\right)} exp\left(-\alpha \frac{(1-\exp(-\lambda x))^{\theta}}{1-(1-\exp(-\lambda x))^{\theta}}\right), x \geq 0 \quad (1.2)$$

$$R_{(\alpha,\lambda,\theta)}(x) = \frac{\alpha + \left(1-(1-\exp(-\lambda x))^{\theta}\right)}{(1+\alpha)\left(1-(1-\exp(-\lambda x))^{\theta}\right)} exp\left(-\alpha \frac{(1-\exp(-\lambda x))^{\theta}}{1-(1-\exp(-\lambda x))^{\theta}}\right), x \geq 0 \quad (1.3)$$

$$h_{(\alpha,\lambda,\theta)}(x) = \frac{\alpha^2 \theta \lambda \exp(-\lambda x)(1-\exp(-\lambda x))^{\theta-1}}{\left(1-(1-\exp(-\lambda x))^{\theta}\right)^2 \left(\alpha+1-(1-\exp(-\lambda x))^{\theta}\right)} \quad (1.4)$$

$x \geq 0$
Where $\alpha > 0$, $\lambda > 0$, $\theta > 0$.

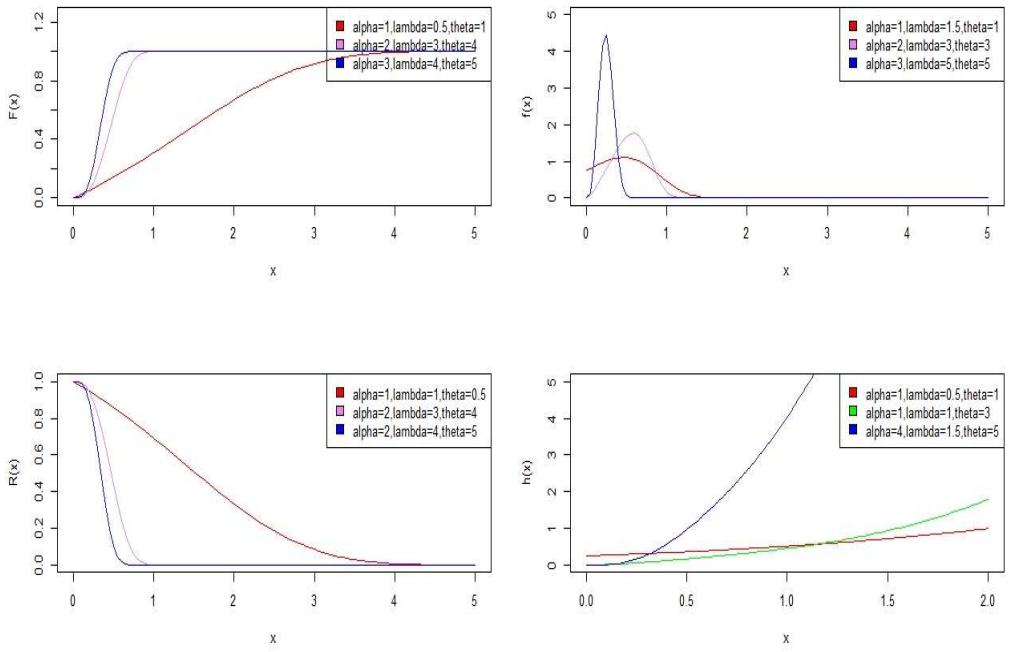

Figure 1: The CDF, PDF, SF and HRF of OLEE distribution.

2 Maximum Likelihood Estimation

Let $(x_1, ..., x_n)$ a random sample of size n from the OLEE(α,λ,θ) distribution, the likelihood function is given by

$$L\left(\underline{x}, \alpha, \theta, \lambda\right) = \frac{\alpha^{2n}\theta^n\lambda^n}{(1+\alpha)^n} \exp\left(-\lambda\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \frac{\alpha\left(1-\exp\left(-\lambda x_i\right)\right)^\theta}{1-\left(1-\exp\left(-\lambda x_i\right)\right)^\theta}\right) \prod_{i=1}^{n} \frac{(1-\exp(-\lambda x_i))^{\theta-1}}{\left(1-(1-\exp(-\lambda x_i))^\theta\right)^3} \quad (2.1)$$

For $\alpha > 0$, $\lambda > 0$, $\theta > 0$.
The log-likelihood function

$$\ln(L\left(\underline{x}, \alpha, \theta, \lambda\right)) = n\ln\left(\frac{\alpha^2\theta\lambda}{1+\alpha}\right) + \sum_{i=1}^{n}\left(-\lambda x_i - \alpha\left(\frac{\phi_\lambda(x_i)^\theta}{1-\phi_\lambda(x_i)^\theta}\right) + (\theta-1)\ln\left(\phi_\lambda(x_i)\right) - 3\ln\left(1-\phi_\lambda(x_i)^\theta\right)\right) \quad (2.2)$$

Where $\phi_\lambda(x) = 1-\exp\left(-\lambda x\right)$
The estimators of the parameters are obtained by solving the following non-linear system:

$$\begin{cases} -\sum_{i=1}^{n}\frac{\phi_\lambda(x_i)^\theta}{1-\phi_\lambda(x_i)^\theta} - \frac{n}{\alpha+1} + \frac{2n}{\alpha} = 0 \\ \frac{n}{\theta} + \sum_{i=1}^{n}\ln\left(\phi_\lambda(x_i)\right) + \sum_{i=1}^{n}\frac{(3-\alpha)\ln(\phi_\lambda(x_i))(\phi_\lambda(x_i)^\theta)}{1-\phi_\lambda(x_i)^\theta} - \sum_{i=1}^{n}\frac{\alpha\ln(\phi_\lambda(x_i))(\phi_\lambda(x_i))^{2\theta}}{\left(1-\phi_\lambda(x_i)^\theta\right)^2} = 0 \\ \frac{n}{\lambda} - \sum_{i=1}^{n}x_i + \sum_{i=1}^{n}\frac{(\theta-1)(\phi_\lambda(x_i))'}{\phi_\lambda(x_i)} \\ +\sum_{i=1}^{n}\frac{(3-\alpha)\theta(\phi_\lambda(x_i))'(\phi_\lambda(x_i))^{\theta-1}}{1-\phi_\lambda(x_i)^\theta} - \sum_{i=1}^{n}\frac{\alpha\theta(\phi_\lambda(x_i))'(\phi_\lambda(x_i))^{2\theta-1}}{\left(1-\phi_\lambda(x_i)^\theta\right)^2} = 0 \end{cases} \quad (2.3)$$

Here, there is no analytical solution. So, we use numerical methods to obtain the corresponding approximate maximum likelihood estimates[12].
Then, the reliability characteristics are:

$$\hat{R}_{\left(\hat{\alpha},\hat{\lambda},\hat{\theta}\right)}\left(x\right)_{MLE} = \frac{\hat{\alpha}+\left(1-\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}}\right)}{(1+\hat{\alpha})\left(1-\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}}\right)} \exp\left(-\hat{\alpha}\frac{\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}}}{1-\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}}}\right), x \geq 0 \quad (2.4)$$

And

$$\hat{h}_{\left(\hat{\alpha},\hat{\lambda},\hat{\theta}\right)}(x)_{MLE} = \frac{\hat{\alpha}^2\hat{\theta}\hat{\lambda}\exp\left(-\hat{\lambda}x\right)\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}-1}}{\left(1-\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}}\right)^2\left(\hat{\alpha}+(1-\left(\phi_{\hat{\lambda}}(x)\right)^{\hat{\theta}})\right)}, x \geq 0 \quad (2.5)$$

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Modeling for the dynamics of a current-actuated piezoelectric beam and a control point of view.

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Abstract
The talk of this conference will be on how to model the dynamics of a piezoelectric beam when it is current actuated, what is the best actuation for these beams, how to use Variational approach, Hamilton's principle, surface electrical conituity conditions, electroc-static, quasistatic and fully dynamic approach with the requared assymptions to give the equivalent mathematical model and a control point of view for some works.

Keywords: Piezoelectric beams, Modeling, Current acutation, control
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

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Qualitative analysis and traveling wave solutions for Klein-Gordon equation

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Abstract
We study the bounded traveling wave solutions for the Klein-Gordon equation by employing the theory of planar dynamical systems. Our investigation focuses on determining the conditions under which these solutions exist, quantifying their number, and elucidating their general shapes. Specifically, we categorize the solutions into three distinct types: pulse waves, periodic waves, and cusp waves. By analyzing these waveforms, we aim to contribute to a deeper understanding of the Klein-Gordon equation and its applications in various physical contexts. This comprehensive analysis provides valuable insights into the behavior of nonlinear wave phenomena.

Keywords: Qualitative analysis, Solitary wave solution, Periodic wave, Cusp wave.
2020 Mathematics Subject Classification: Primary 35C07, 34C25, 35Q51.

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Cosmological models with linearly varying deceleration parameter

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Abstract
We consider the spatially homogeneous and anisotropic Bianchi-I spacetime with variable cosmic "constants" $\Lambda(t)$. The exact solutions of Einstein's field equations were obtained using the law for the deceleration parameter that changes linearly with time. It was found that the solutions are consistent with recent observations of Type Ia supernovae.

Keywords: Anisotropic, Cosmological Models, deceleration parameter, Bianchi space-time.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

Through observations of Type Ia Supernova (SNIa), it was found that the current universe is expanding at an accelerated rate in 1998, and this was confirmed by numerous independent observational data such as SNIa sample (Union2), baryonic acoustic oscillation (BAO) and cosmic microwave background (CMB) radiation together. Today, the focus of research is on how acceleration changes through the study of the deceleration parameter.
In this work we propose a linearly variable deceleration coefficient (LVDP). This law was first proposed by Akarsu and Dereli (2012). LVDP is used to obtain accelerated cosmological solutions. The spatially homogeneous and anisotropic Bianchi-I space-time is described by the line element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \tag{1.1}$$

where A(t), B(t) and C(t) are potential functions of cosmic time t. The Einstein field equations can be written as follows,

$$G_{\mu\nu} - (6\lambda^2 u_\mu u_\nu + (3 - 6\alpha)\,\lambda^2 g_{\mu\nu}) = T_{\mu\nu}, \tag{1.2}$$

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu} = \rho_m u_\mu u_\nu$ is the energy-momentum tensor, λ is real, α is a real and $g^{\mu\nu}u_\mu u_\nu = -1$. The Hubble parameter in anisotropic models can be defined by the average scale factor $V = (ABC)^{\frac{1}{3}}$ where we write,

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3), \tag{1.3}$$

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$, and $H_3 = \dot{C}/C$ are the directional Hubble's parameters in the directions of x, y and z respectively. We have assumed $8\pi\mathrm{G} = \mathrm{c} = 1$ in proper unit. The field equation (1.2) for the line element (1.1) takes the form

$$\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C}\right) = \rho_m + 3(1+2\alpha)\lambda^2, \tag{1.4}$$

$$\left(\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C}\right) = -3(1-2\alpha)\lambda^2, \tag{1.5}$$

$$\left(\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C}\right) = -3(1-2\alpha)\lambda^2, \tag{1.6}$$

$$\left(\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B}\right) = -3(1-2\alpha)\lambda^2. \tag{1.7}$$

The energy conservation equation

$$T^{\mu}_{\nu;\mu} = 0, \tag{1.8}$$

leads to

$$\dot{\rho_m} + 3\rho_m H = 0, \qquad \dot{\lambda} + \frac{3\lambda}{(1+2\alpha)}H = 0. \tag{1.9}$$

where an overhead prime denote derivative with respect to time t only, ρ_m is the energy density of the matter. To solve these equations, a special law was proposed for the deceleration parameter with a negative slope. It is used to obtain accurate cosmological models, in the context of dark energy. The linearly varying deceleration parameter (q) is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -kt + m - 1 \tag{1.10}$$

Solving this equation for $k > 0$, $n > 1$, we obtain the law of variation for average scale factor a as

$$a(t) = C_1 \exp\left\{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n} - 1\right)\right\}. \tag{1.11}$$

where C_1 is constant of integration. The geometry of the space-time (1.2) takes the form

$$ds^2 = -dt^2 + C_1^2\gamma^2\left(\Delta_1^2 \exp\left\{\delta X_1 \gamma^{-(3+n)}\right\}dx^2 + \Delta_2^2 \exp\left\{\delta X_2 \gamma^{-(3+n)}\right\}dy^2 + \Delta_3^2 \exp\left\{\delta X_3 \gamma^{-(3+n)}\right\}dz^2\right), \tag{1.12}$$

and

$$\gamma = \exp\left\{\frac{2}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)\right\},$$
$$\delta = -\frac{4n^2}{C_3^3 k(3+n)}\,{}_2F_1\left(2;1+\frac{3}{n};2+\frac{3}{n};-e^{\frac{2}{n}\tanh^{-1}\left(1-\frac{kt}{n}\right)}\right).$$

where ${}_2F_1$ is hyper geometric function. The other cosmological parameters of the model have the following expressions

$$H = -\frac{2}{(kt^2-2nt)}. \tag{1.13}$$
$$\theta = -\frac{6}{(kt^2-2nt)}, \tag{1.14}$$
$$A_p = \frac{X_1^2+X_2^2+X_3^2}{12C_1^6}(kt)^2\exp\left\{-\left(\frac{12}{n}+2\right)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right\}, \tag{1.15}$$
$$\sigma^2 = \frac{\{(X_1-X_2)^2+(X_2-X_3)^2+(X_3-X_1)^2\}}{6C_1^6(2nt-kt^2)^2}(kt)^2\exp\left\{-\left(\frac{12}{n}-4\right)\tanh^{-1}\left(\frac{kt}{n}-1\right)\right\}. \tag{1.16}$$

where H is the generalized mean Hubble parameter, θ is the expansion scalar, (σ^2) is the shear scalar , and A_p is the anisotropy parameter. The energy density is read as

$$\rho_m = \rho_0 C_1^{-3}\exp\left\{-\frac{6}{n}\tanh^{-1}\left(\frac{kt}{n}-1\right)\right\} \tag{1.17}$$

where ρ_0 is constant of integration.
we find Lambda terme in general relativity as follows

$$\Lambda = -6\lambda_0^2 C_1^{-\frac{6}{1+2\alpha}}\exp\left\{-\frac{12}{n(1+2\alpha)}\tanh^{-1}\left(\frac{kt}{n}-1\right)\right\},$$

where λ_0 is constant of integration.

2 Main results

• By using the linearly varying deceleration parameter, the Universe would exhibit expansion as: (a) $q<-1$ Super exponential expansion, (b) $-1\le q<0$ Exponential expansion (for $q=-1$, known as de-Sitter expansion), (c) $q=0$ Expansion with constant rate, (d) $-1<q<1$ Accelerating power expansion, (e) $q>0$ Decelerating expansion.
• The model of the universe starts with big bang and ends with Big Rip.
• ρ_m is very high at initial time and it decreases sharply with passage of time and ultimately approaches to zero at t$\to\infty$.

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Dirac equation on a curved surface

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Abstract
The objective of this study is twofold. First, we will show by concrete examples that the non-appearance the geometric potential in the non-relativistic limit is essentially due to the inclusion of spin connection terms in the Dirac equation. These terms give a scalar repulsive terms which contribute to cancel the geometric potential. Second, we deduce the energy spectrum for relativistic particle confined on a cylindrical and a spherical surfaces using the thin-layer approach.

Keywords: Dirac quantification, Geometric potential, Thin-layer quantization.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

Even in flat Euclidean space it might be useful to use curvilinear coordinates. However, in a curved space we have no other choice, because Cartesian coordinates can only exist locally, that is, in an infinitesimal neighborhood. In this section we will see how to write the Dirac equation in a curved space. In reality, what we want to write is the covariant Dirac equation for an arbitrary metric. Since this equation is written for an arbitrary metric, we can also use it to write the Dirac equation on a curved surface. Let's start by writing the Dirac equation in flat space-time for a particle of mass M and spin $1/2$ in cartesian coordinates x^a, $a = 0, 1, 2, 3$, as follows

$$\left(i\hbar\gamma^a\partial_a - Mc\right)\psi = 0, \tag{1.1}$$

where γ^a are constant matrices relative to Minkowski metric $\eta_{ab} = diag\,(1, -1, -1, -1)$. We can extend the Dirac equation from flat space-time to curved space-time using the tetrad formalism. These tetrads $e^a_{\ \mu}$ and its inverses $e_a{}^{\mu}$ allow us to transform between a local inertial frame and a global frame. Indeed, in this formalism the metric $h_{\mu\nu}$ of a curved space-time is linked to the Minkowski metric by the relation

$$h_{\mu\nu} = e^a_{\ \mu} e^b_{\ \nu} \eta_{ab} \tag{1.2}$$

Since η_{ab} is a constant matrix. Where γ^a are constant matrices relative to Minkowski metric $\eta_{ab} = diag\,(1,-1,-1,-1)$ Indeed, in this formalism the metric $h_{\mu\nu}$ of a curved space-time is linked to the Minkowski metric by the relation

$$h_{\mu\nu} = e^{a}_{\ \mu} e^{b}_{\ \nu} \eta_{ab} \tag{1.3}$$

Since η_{ab} is a constant matrix, these tetrads necessarily depend on space-time coordinates. In addition, these tetrads verify the orthogonality relations

$$e_{\mu}^{\ a} e_{a}^{\ \nu} = \delta_{\mu}^{\ \nu} \quad , \quad e^{a}_{\ \mu} e_{b}^{\ \mu} = \delta^{a}_{\ b}$$

Dirac's equation takes the following form in curved space-time

$$\left(\frac{1}{\sqrt{-h}} D_\mu \left(\sqrt{-h} h^{\mu\nu} D_\nu \right) + \frac{\mathcal{R}}{4} + \frac{M^2 c^2}{\hbar^2} \right) \psi = 0$$

We consider a two-dimensional surface (S) with parametric equations $\mathbf{r} = \mathbf{r}\,(q^1, q^2)$. In a three-dimensional Euclidean space, the very close neighborhood of this surface is described by the vector

$$\mathbf{R} = \mathbf{r}\left(q^1, q^2\right) + q^3 \mathbf{n}\left(q^1, q^2\right), \tag{1.4}$$

th e Dirac equation is written as
$\left[-\frac{\hbar^2}{2M\sqrt{G}} \partial_i \sqrt{G} G^{ij} \partial_j - \frac{\hbar^2}{M} G^{ij} \Gamma_i \partial_j - \frac{\hbar^2}{2M\sqrt{G}} \partial_i \left(\sqrt{G} G^{ij} \Gamma_j \right) - \frac{\hbar^2}{2M} G^{ij} \Gamma_i \Gamma_j + \frac{\hbar^2}{8M} \mathcal{R} \right] \psi = E_{NR} \psi$
apply the confining process

$$\psi = f^{-\frac{1}{2}} \chi$$

$$\mathcal{R} = 2\kappa$$

2 Main results

The general configuration of the confinement $V(q)$ requires a difficult Dirac configuration in the decoupler. It's for this raison that we have adopted a simple form that is effective as a pair of infinity inserts as a minimal coupling. This chooses our aid to i) decoupler the Dirac coupling. ii) Separate the surface area from the normal part. iii) Obtenir la limite non relativiste.

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Geometric of Acoustics by Jordan-Moore-Gibson-Thompson Equations

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Abstract
This paper examines the existence and uniqueness of solutions under the traveling wave forms for a free boundary Cauchy problem of fractional Jordan-Moore-Gibson-Thompson equations of nonlinear acoustics, which describe sound propagation in thermo-viscous elastic terms. It does so by applying the properties of Banach's fixed point theorem.

Keywords: Traveling wave, acoustics, fractional derivative, existence and uniqueness.
2020 Mathematics Subject Classification: Primary 35R11, 35A01, 34A08, 35C06.

1 Introduction

Several mathematical models are used to describe nonlinear acoustics phenomena. For example, In this work, we shall give a fractional model of nonlinear acoustics that is named the space-fractional Jordan-Moore-Gibson-Thompson (JMGT) equation. This equation results from modeling high-frequency ultra sound waves, and is written as follows:

$$\begin{cases} \tau\psi_{ttt}+\mu\psi_{tt}-\kappa^2\partial_x^{\alpha}\psi-\eta\partial_x^{\alpha}\psi_t=\varphi\left(x,t,\psi,\psi_t,\psi_{tt},\psi_{xx},(\psi_t)_{xx}\right), & (x,t)\in\Omega, \\ \psi\left(x,0\right)=\psi_0\left(x\right),\ \psi_t\left(x,0\right)=\psi_1\left(x\right),\ \psi_{tt}\left(x,0\right)=\psi_2\left(x\right), & \psi_0,\psi_1,\psi_2\in\mathbb{C}, \\ \psi_x\left(\kappa t,t\right)=0,\ \psi_{xx}\left(\kappa t,t\right)=0, & \kappa>0, \end{cases} \tag{1.1}$$

where $\tau,\mu,\kappa,\eta\in\mathbb{R}_+^*$ and $\varphi:\Omega\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\to\mathbb{C}$ is a nonlinear function.
The major goal of this work is to determine the existence and uniqueness of the fractional-order's partial differential equation (1.1), under the traveling wave form

$$\psi\left(x,t\right)=\exp\left(-\frac{\kappa^2}{\eta}t\right)u\left(x-\kappa t\right),\ \text{with}\ \kappa,\eta\in\mathbb{R}_+^*. \tag{1.2}$$

The basic profile u is not known in advance and is to be identified.

For the forthcoming analysis, we impose the following assumptions:
$(A1)$ φ is a continuous function that is invariant by the change of scale (1.2). It gives us

$$\varphi\left(x,t,\psi,\psi_t,\psi_{tt},\psi_{xx},\left(\psi_t\right)_{xx}\right)=\exp\left(-\frac{\kappa^2}{\eta}t\right)\left(\eta\kappa f\left(\xi,u\left(\xi\right),u'\left(\xi\right),u''\left(\xi\right)\right)-\kappa^3\tau u'''\left(\xi\right)\right), \tag{1.3}$$

where $\xi=x-\kappa t$ and $f:[0,\ell]\times\mathbb{C}\times\mathbb{C}\times\mathbb{C}\to\mathbb{C}$ is a continuous function.
$(A2)$ There exist three positive constants $\beta,\gamma,\lambda>0$ so that the function f given by (1.3) satisfies

$$\left|f\left(\xi,u,v,w\right)-f\left(\xi,\bar{u},\bar{v},\bar{w}\right)\right|\leq\beta\left|u-\bar{u}\right|+\gamma\left|v-\bar{v}\right|+\lambda\left|w-\bar{w}\right|,\ \text{for }\beta,\gamma,\lambda>0,$$

for any $u,v,w,\bar{u},\bar{v},\bar{w}\in\mathbb{C}$.
Now, we give the principal theorems of this work.

Theorem 1.1. *Assume that the assumptions* $(A1)$, $(A2)$ *hold. We give*

$$\tau\kappa^5+\beta\eta^4-\mu\eta\kappa^3\neq0\ \textit{and}\ \omega\in(0,1)$$

If we put

$$\ell<\left(\frac{\eta^4\Gamma\left(\alpha+2\right)\left(1-\omega\right)}{\left|\tau\kappa^5+\beta\eta^4-\mu\eta\kappa^3\right|}\right)^{\frac{1}{\alpha+1}}, \tag{1.4}$$

then the Cauchy problem (1.1) admits a unique solution in the traveling wave form (1.2) on Ω.

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GSPN MODEL FOR A CHALLENGING RELIABILITY MODEL

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Abstract

The aim of this paper is to study the performance of a $K-out-of-M+W+C:G$ retrial system with a unreliable repairer through a Generalized Stochastic Petri Net ($GSPN$). The repairer can fail in two ways: while it is repairing a component (*activebreakdown*) or while it is idle (*passivebreakdown*). The system is equipped with M primary operating components, W warm standby components, and C cold standby components. The lifetimes of primary components and warm standby components are assumed to be exponentially distributed random variables. The failure times of the repairer and the components are all assumed to be exponentially distributed. When a component fails, it is either repaired immediately if the repairer is available and free, or it enters a retrial orbit if the repairer is busy or under repair. We use the $GSPN$ model to derive a Continuous Time Markov Chain ($CTMC$), which allows us to calculate the system's main stationary probabilities and some performance reliability measures..

Keywords: Generalized stochastic Petri net, $K-out-of-n$ system, reliability, active and passive breakdowns

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Modelisation and phenomenology of vector-like fermions within left-right symmetric model

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Abstract
In this work we built a model based essentially on the gauge structure of the left-right symmetric model. We introduce a new kind of exotic particles called vector-like fermions. Those particles are well motivated in literature. We exploited the numurical tools of high enegy physics such as $MadGraph5_aMC@NLO$ to make a simulation for pairs production of those particles. Thus a phenomenological study has been done and we found that we are able to produce this kind of particles at the LHC.

Keywords: left-right symmetric model, vector-like fermions, $MadGraph5_aMC@NLO$, phenomenology.

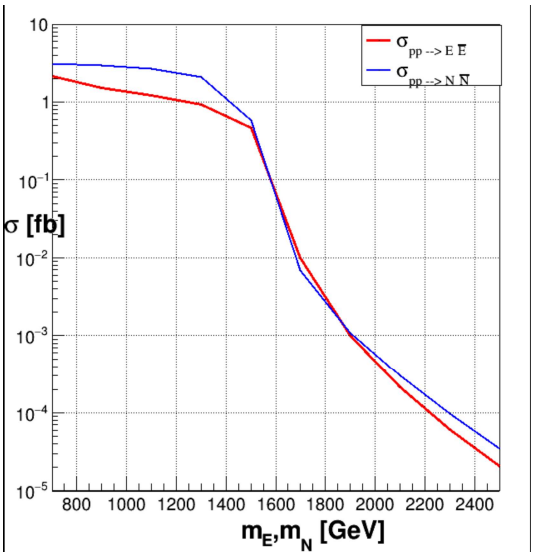
1 Introduction

Standard model (SM) of particle physics is an extremely successful theory from theoretical and experimental perspectives. Even though, it still incomplete theory where it doesn't give an answer for several problems, such as parity violation in weak interactions, existence and smallness of neutrino masses, quadratic divergence in Higgs boson mass and dark matter, which summons the necessity of existence of physics beyond the SM.
Many extensions have been introduced to solve one or some of the mentioned problems, and one of the most important extensions which have been considered since a long time is the left-right symmetric model (LRSM) [1, 2, 3, 4, 5]. This model is belonging to the category with extension in gauge symmetry, it gives a probable solution for neutrinos mass problem and explain their smallness naturally through see-saw mechanism [6]. Moreover, it provide an explanation for the parity violation at weak scale, where the parity is expected to be conserved at TeV scale under this framework. Another category of extensions is based on adding new matter content to SM, and one kind of the most attractive exotice particles is the hypothetical vector-like fermions (VLF). Mixing of the VLF with the SM fermions hase been studied compehensevly [7, 8, 9, 10]. Existence of the new vector-like top quark(T) would explain the quadratic divergence in Higgs boson mass [11]. Otherelse, VLF has been exploited to explain the B meson anomalies ($R(\{D, D^*\})$ and $R_{K(K^*)}$) [12]. However, the last measurements of LHCb collaboration shows an agreement with the SM predictions for $R_{K(K^*)}$ [13].

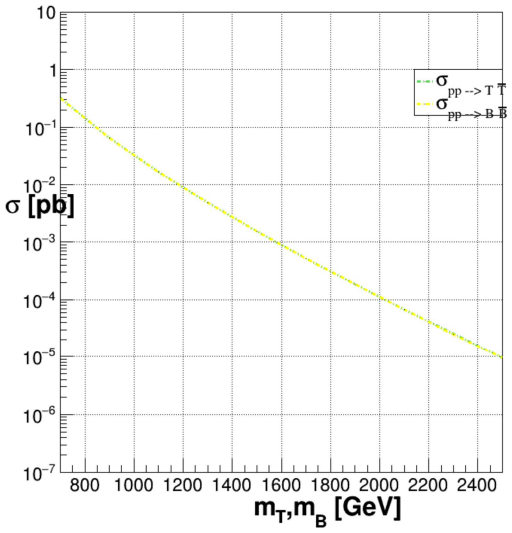
Therefore, it is reasonable to ask whether a major of these problems can accommodated under a unified framework.
We give at this work an alternative model for the LRSM. We extend the gauge group structure by an extra gauge factor $SU(2)_V$ includes one generation of VLF. Since the model is left-right symmetric, the parity violation is explained where it has been considered that it's preserved at TeV scale. Parity violation after breaking the symmetry leads to an asymmetry in the mixing of the vector-like fermion doublets to the SM chiral fermions where this feature appears in the works [9, 10] without explanation.
The theoretical model has been implemented using `FeynRules` [14] package to extract the numerical `UFO` format for the model. The last format was proposed for introducing a new way to pass model information to high-energy physics software such as `MadGraph5_aMC@NLO`. A phenomenological study using `MadGraph5_aMC@NLO` to simulate the pairs production the new VLF particles in our model has been done.
Since the gauge structure of the model allows the existence of new heavy gauge and scalar bosons. This is leads to riche pairs production channels. Vector-like quarks (VLQ) pairs production are dominated by the QCD contribution through their gauge couplings to gluons. Thus, it's mostly favored than the pairs production of the vector-like leptons (VLL) which are mediated by the higgs boson, Z_μ gauge boson, the new heavy neutral gauge and neutral scalar (psudo-scalar) bosons. However, the research for such heavy states is relatively accessible at Large Hadron collider (LHC).

2 Main results

In this section we give briefly the results of the simulated VLF production at the LHC. We simulated the collision of two protons at the initial state where we get in final state pairs for each kind of the VLF at center of mass of energy $\sqrt{s} = 13\ TeV$. The total number of final states is four, where we have two VLQ (up state denoted by T and down state denoted by B) and two VLL (charged state denoted by E and neutral state denoted by N). The total cross sections are presented in Fig. (1a) for VLQ and Fig.(1b) for VLL production. As we have mentioned already in the introduction, the cross sections of pairs production of VLQ are mostly 1000 times bigger than VLL production since the mediators are the gluons which are massless particles. Moreover, we would notice that the cross sections drops when we increase the mass of the VLF at the final state since we need more energy to produce highly massive particles. However, we could say at good level of confidence that all particles could be produced at the LHC.



(a) Vector-like leptons



(b) Vector-like quarks

Figure 1: Vector-like fermions pairs production cross sections at $\sqrt{s} = 13\ TeV$.

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Formal Modeling and Analysis of Concurrent Distributed Systems Using UML Activity Diagrams

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Abstract
Concurrent distributed systems play a crucial role in many modern applications, ranging from web services to sensor networks and cloud computing platforms. These systems are characterized by the simultaneous execution of processes across multiple interconnected nodes, which provides advantages in terms of performance, scalability, and resilience. However, the design and analysis of such systems present significant challenges due to their inherent complexity and the need to effectively coordinate concurrent processes.
Model-Driven Engineering (MDE) offers a methodological approach to developing complex systems by using abstract models to represent different facets of a software system. The Unified Modeling Language (UML) is central to this approach as it provides a comprehensive set of diagrams for modeling both the static and dynamic aspects of systems. By utilizing UML, we gain significant interoperability and extensive support from various modeling tools, facilitating integration and communication throughout the different phases of development.
In this context, UML activity diagrams are particularly useful for modeling and analyzing concurrent behaviors and control flows in distributed systems. Activity diagrams provide a visual and structured representation of parallel processes, synchronization points, and interactions between system components.
However, despite UML being a semi-formal language, it has certain limitations regarding the rigorous structural and behavioral verification of models. To address these limitations, it is essential to integrate formal verification techniques into the modeling process. In this work, we propose an approach to represent activity diagrams in a formally analyzable form, utilizing simple graphs. This representation enables the application of formal structural analysis techniques, ensuring the coherence and reliability of models for concurrent distributed systems.

Keywords: Concurrent Distributed Systems, Model-Driven Engineering, UML, UML Activity Diagrams, Simple Graphs, formal Structural Analysis.

1 Introduction

Concurrent distributed systems are essential in many modern applications, including sensor networks and cloud computing platforms [1]. These systems involve the simultaneous execution of processes across multiple interconnected nodes, offering significant advantages in performance, scalability, and resilience [2]. Despite these benefits, designing and analyzing such systems is challenging due to their inherent complexity and the need for effective coordination of concurrent processes [3].
Model-Driven Engineering (MDE) provides a methodological approach to developing complex systems by using abstract models to represent different facets of a software system [4,5]. The Unified Modeling Language (UML)[6] is central to this approach, offering a comprehensive set of diagrams for modeling both static and dynamic aspects of systems. UML facilitates significant interoperability and provides extensive support from various modeling tools, aiding integration and communication throughout the different phases of development.
In particular, UML activity diagrams [7] are highly effective for modeling and analyzing concurrent behaviors and control flows in distributed systems. These diagrams provide a visual and structured representation of parallel processes, synchronization points, and interactions between system components. Despite UML being a semi-formal language, it has certain limitations in terms of rigorous structural and behavioral verification of models. To overcome these limitations, it is essential to integrate formal verification techniques into the modeling process.
This paper proposes an approach to representing activity diagrams in a formally analyzable form using simple graphs. This representation allows the application of formal structural analysis techniques, ensuring the coherence and reliability of models for concurrent distributed systems. Specifically, we aim to leverage the formal analysis tool GROOVE (Graph-based Object-Oriented Verification), which provides powerful capabilities for model checking and validation through graph transformations. By translating activity diagrams into simple graphs, we can benefit from GROOVE's ability to perform exhaustive state space exploration, ensuring the correctness and robustness of the modeled systems.
The remainder of this paper is organized as follows: Section 2 presents the methodology for modeling and analyzing concurrent distributed systems based on UML activity diagrams and GROOVE. Section 3 provides a detailed case study to illustrate the practical application of the proposed approach. Finally, Section 4 concludes the paper and discusses potential future work

2 Methodology for Modeling and Analyzing Concurrent Distributed Systems Using UML Activity Diagrams and GROOVE

2.1 Modeling Concurrent Distributed Systems with UML Activity Diagrams

UML activity diagrams are a powerful tool for modeling concurrent distributed systems. These diagrams provide a clear and structured way to represent parallel processes, synchronization points, and interactions between system components. The advantages of using activity diagrams include:

- Visualization: They offer a visual representation of complex workflows and processes, making it easier to understand and communicate system behaviors.
- Concurrency Representation: They naturally represent concurrent behaviors, allowing for the modeling of parallel activities and their interactions.
- Interoperability: Being part of UML, they integrate seamlessly with other UML diagrams and tools, supporting a comprehensive modeling approach.
- Structured Analysis: They facilitate the identification of synchronization points, bottlenecks, and potential issues in the system.

Activity diagrams consist of various components, including:

- Actions: Representing executable steps within the process.
- Control Nodes: Including initial nodes, final nodes, decision nodes, merge nodes, fork nodes, and join nodes.

Table 1 below lists the different types of nodes with their graphical representations:

Type	*Description*	*Graphical Representation*
Activity Node	Represents an action or operation of the system	
Initial Node	Starting point of the activity flow	
Final Node	Endpoint of the activity flow	
Decision Node	Point where the flow divides into multiple paths based on a condition	
Merge Node	Combines multiple alternative flows into a single flow	
Fork Node	Separates an input flow into multiple concurrent output flows	
Join Node	Synchronizes multiple parallel flows into a single flow	

Table 1: Types of Nodes in UML Activity Diagrams

The different nodes in the diagram are connected by arcs called transitions, representing the sequencing of activities. Each transition can have a name and a guard condition.
Figure 1 presents an example of an activity diagram modeling the behavior of an Automobile Accident Alert Management Service. The UML activity diagram describes an autonomous service reacting to an alert following a car accident. It begins by calling the driver to check if assistance is needed. If the alert is confirmed, the accident location and other specific parameters are transmitted to the appropriate service, such as an ambulance or a towing service. This scenario comes from the European project SENSORIA.

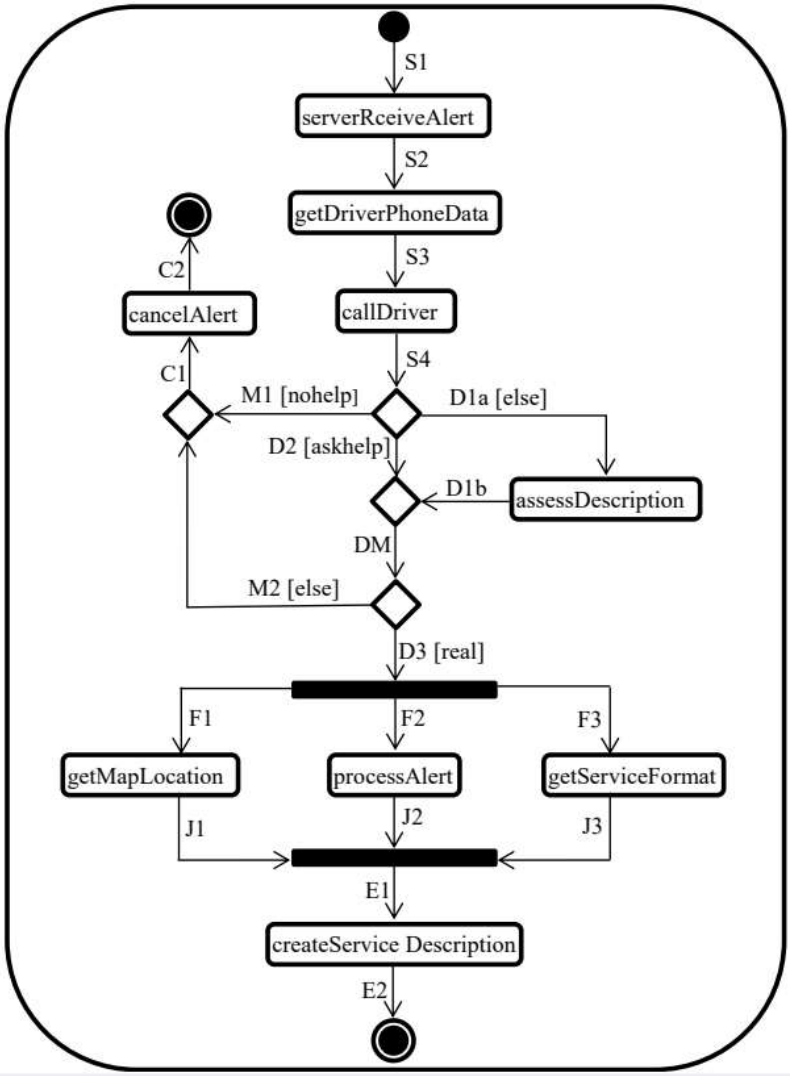

Figure 1: Activity Diagram Example

2.2 Using GROOVE for Formal Analysis

To use GROOVE for analyzing activity diagrams, we proposed a meta-model for activity diagrams. The meta-model defines the different components of these diagrams and ensures the conformity of models with the principles of activity diagrams. The representation of this meta-model in GROOVE is shown in Figure 2.

Based on the meta-model, activity diagrams are represented in the GROOVE tool as simple graphs. The idea is simple: each transition in the activity diagram is represented by a vertex in the simple graph, with incoming and outgoing vertices representing the different entering and exiting nodes of the activity diagram. A basic example of representing an activity diagram as a simple graph is shown in Figure 3.

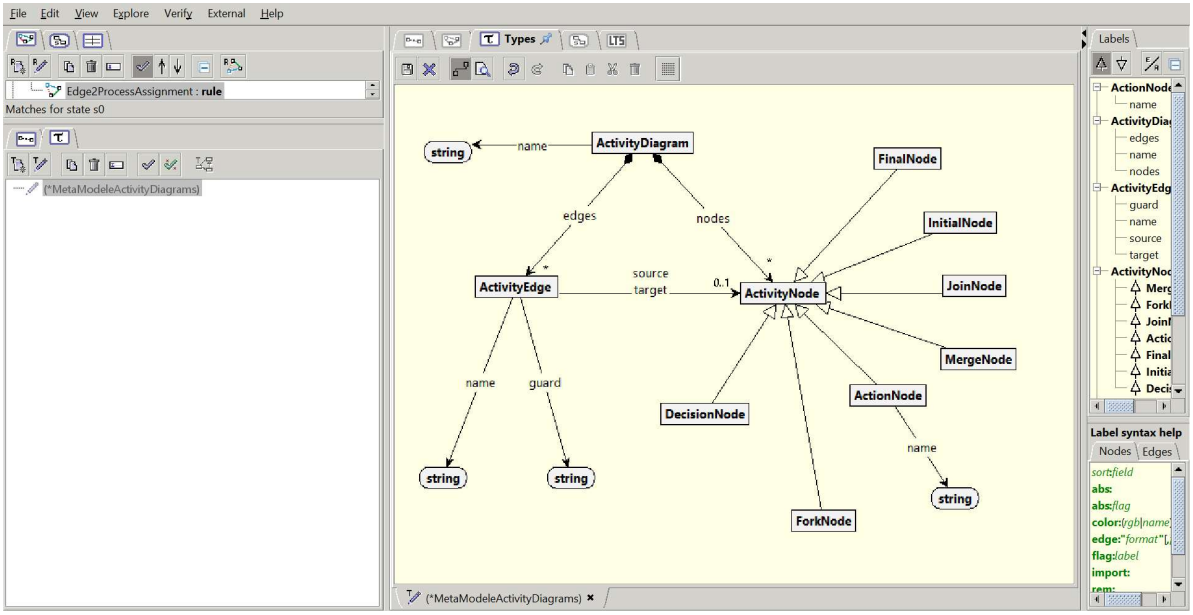

Figure 2: Activity Diagram Meta-Model

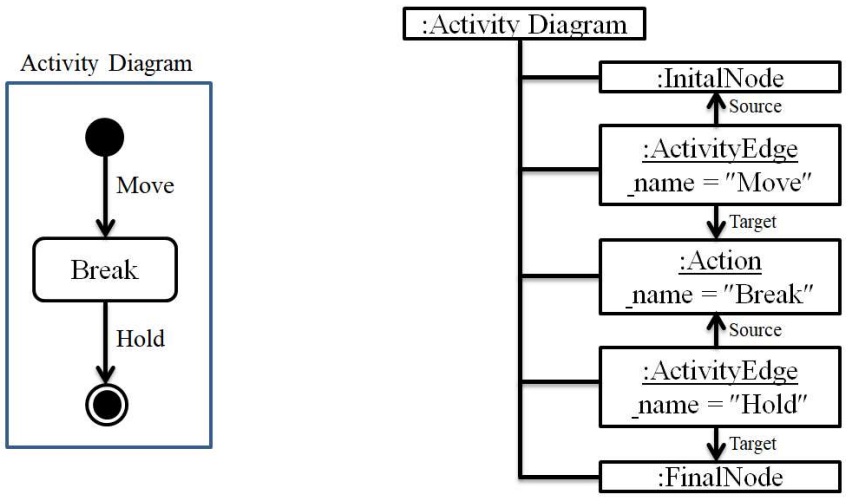

Figure 3: Activity Diagram as Simple Graph

3 Case Study

In this section, we take the activity diagram illustrated in Figure 1 and demonstrate its representation using a simple graph in GROOVE. Figure 4 shows the corresponding graph representation.

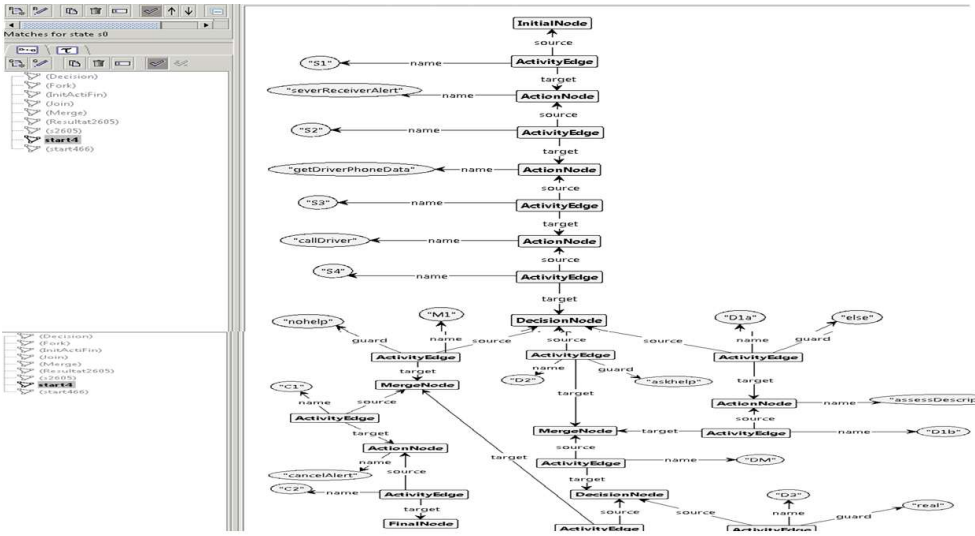

Figure 4: Automobile Accident Alert Management Service as Simple Graph in Groove

Once the activity diagram is specified in GROOVE, we can use the tool to analyze and verify the structure of our model formally. GROOVE provides powerful capabilities for ensuring the correctness of models, which is crucial for verifying the behavior of concurrent distributed systems.
Our choice to use GROOVE is primarily motivated by its verification tool, Check LTL Property, which allows us to verify properties that describe the static semantics of the model. Linear Temporal Logic (LTL) is used to express these properties, providing a robust framework for specifying and checking temporal aspects of the model.
The verification process ensures that the model conforms syntactically to its meta-model. By doing so, we can identify and rectify any discrepancies or errors in the model, ensuring that it accurately represents the intended system behavior. This formal analysis step is essential for validating the design of concurrent distributed systems, as it helps to detect issues early in the development process, reducing the risk of errors in the final implementation.
In this section, we will verify two properties: **EdgeUniqueName** and **Deadlock DJ**.
EdgeUniqueName: This property verifies if the source model satisfies the specification that no two edges can have the same name. The EdgeUniqueName property is presented in Figure 5.

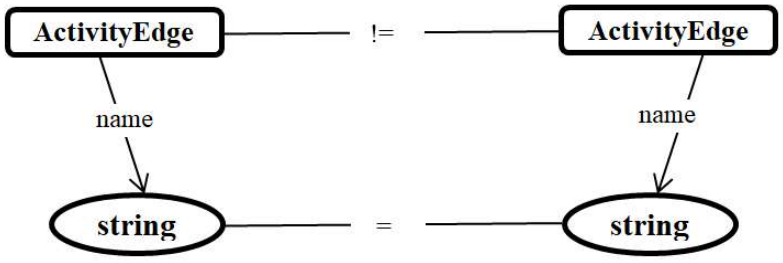

Figure 5: The EdgeUniqueName Property Representation

The logical formula G!(EdgeUniqueName) expresses that two edges can never have the same name throughout the entire execution. The verification of this formula using Check LTL shows that it is satisfied in the source model. Using the temporal operator G, which signifies that the property holds throughout the entire execution, followed by the negation (!) of the EdgeUniqueName property, we find that this property is satisfied. The result of the simulation is illustrated in Figure 6.

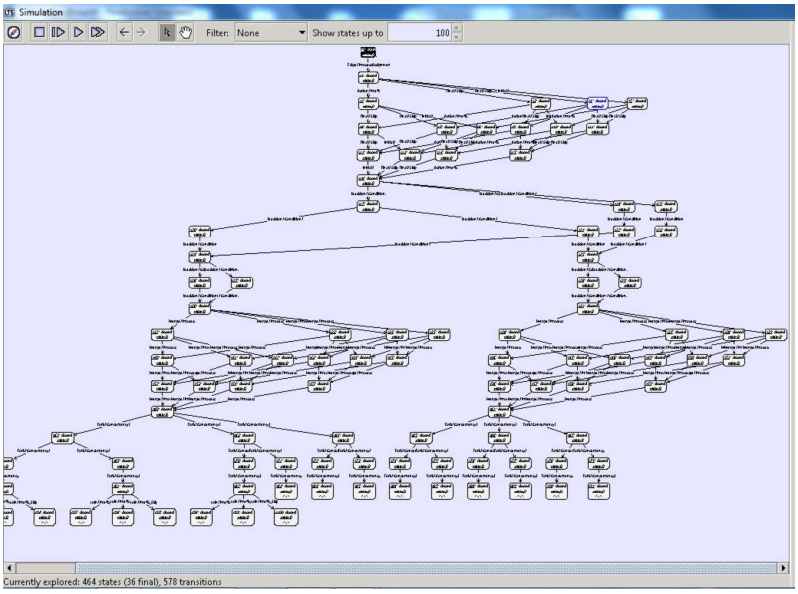

Figure 6: LTS Simulation of G!(EdgeUniqueName)

Deadlock DJ: This is a structural deadlock property at the level of the UML activity diagram, indicating that two different outgoing edges from a decision node can be joined at a join node. This situation causes a deadlock because the join node must wait for both edges to execute, but only one edge will reach the join node due to the preceding decision. The Deadlock DJ property is presented in Figure 7.

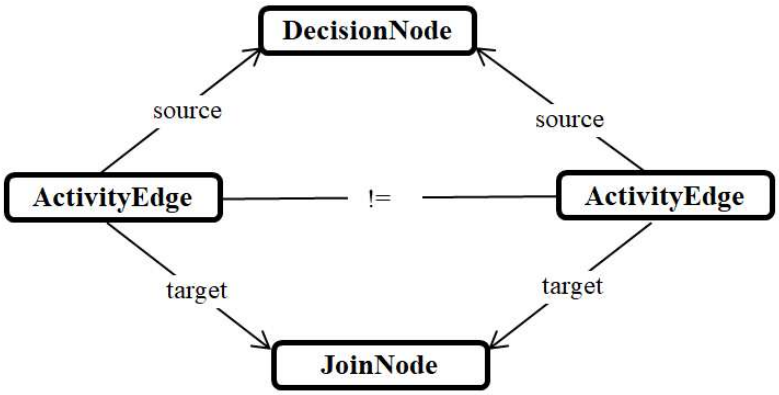

Figure 7: The Deadlock DJ Property Representation

The logical formula G!(Deadlock DJ) expresses that two different outgoing edges from a decision node can never be joined at a join node throughout the entire execution. The verification of this formula using Check LTL shows that it is satisfied in the source model. The result of the simulation is illustrated in Figure 8.

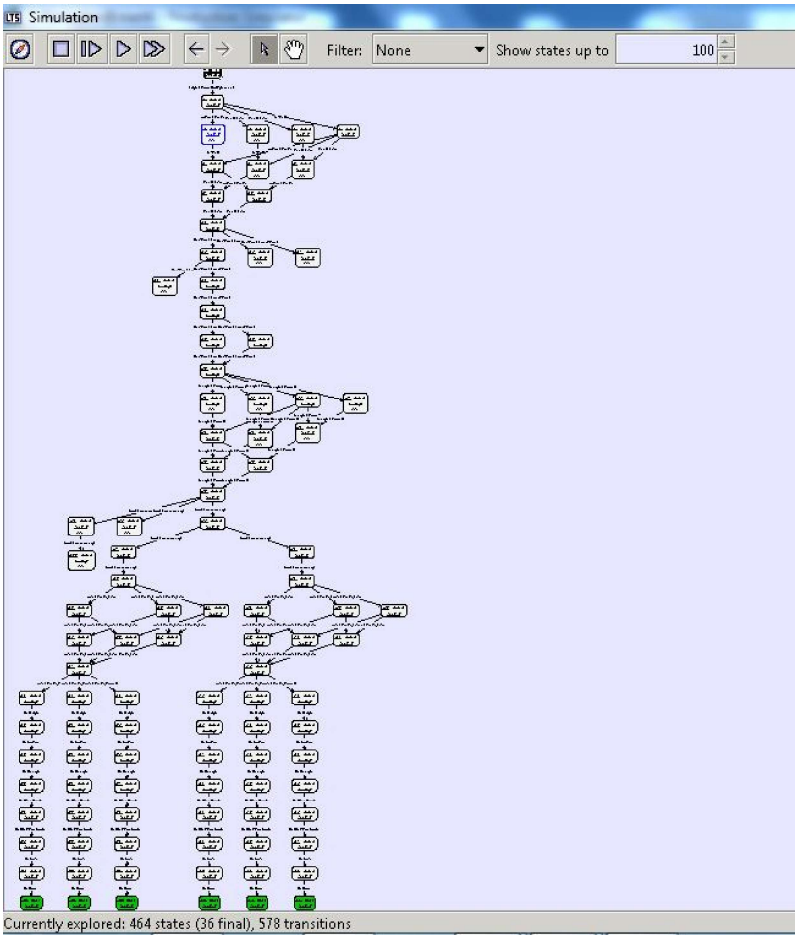

Figure 8: LTS Simulation of G!(Deadlock DJ)

4 Conclusion

In this paper, we presented a methodology for modeling and analyzing concurrent distributed systems using UML activity diagrams and the GROOVE tool. UML activity diagrams provide a clear and structured way to represent concurrent processes and interactions within a system. By transforming these diagrams into simple graphs, GROOVE allows for formal verification of properties like EdgeUniqueName and Deadlock DJ, ensuring model correctness and reliability.
Our case study demonstrated the practical application of this approach, verifying key properties of an Automobile Accident Alert Management Service model. The successful verification highlights the effectiveness of integrating UML activity diagrams with formal analysis tools like GROOVE.

This methodology enhances the accuracy and reliability of system models and facilitates early detection of design flaws, contributing to the development of robust concurrent distributed systems. Future work can extend this approach to other UML diagrams and more complex verification scenarios.

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James' Theorem and Characterization

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Abstract
In this work, we have dealt with reflexive spaces, James' theorem, and its linear and multilinear versions in Banach spaces, providing multilinear and polynomial characterizations at the end.

Keywords: Linear operator, multilinear operator, reflexive space, tensor product, James' theorem, homogeneous polynomial
2020 Mathematics Subject Classification: Primary 47A35, 60Fxx, 60G10.

1 Introduction

We are interested in this work in the characterization theorems of James in their multilinear and polynomial versions. In the linear case, James provided a characterization of a reflexive space using their linear forms. In other words, a space X is reflexive if and only if every linear form of X^* attains its norm. In our work, for the multilinear version, we consider the multilinear forms that are written as products of linear forms, as well as the case of polynomials. Let X be a normed space over the field $\mathbb{K}$. We define the canonical embedding, denoted by J_X, of X into its bidual X^{**} by :

$$J_X(x) : X^* \longrightarrow \mathbb{K}$$

with $J_X(x)(x^*) = x^*(x)$ The application J_X is always isometric. Indeed, let $x \in X$ then:

$$\begin{aligned} \|J_X(x)\| &= \sup_{x^* \in \mathcal{B}_{x^*}} |J_X(x)(x^*)| \\ &= \sup_{x^* \in \mathcal{B}_{x^*}} |x^*(x)| \\ &= \|x\| \end{aligned}$$

The operator J_X is not necessarily surjective. If J_X is surjective, we present the following definition.
Let X be a Banach space. The space X is reflexive if its canonical embedding J_X is surjective.

2 Main results

Definition 2.1. Let X be a Banach space. The linear form x^* of X^* attains its norm if there exists x_0 in B_X such that

$$\|x^*\| = \sup_{x \in B_X} |x^*(x)| = |x^*(x_0)|.$$

The following theorem is a characterization of James' reflexivity.

Theorem 2.1. *A Banach space X is reflexive if and only if every linear functional in X^* attains its norm.*

Let $X_1, \ldots, X_m$ be Banach spaces. Let $\varphi \in \mathcal{L}(X_2, \ldots, X_m)$ be a multilinear form. We say that the norm of φ is attained if there exist $x_1^0 \in B_{X_1}, \ldots, x_m^0 \in B_{X_m}$ such that

$$\begin{aligned} \|\varphi\| &= \sup_{x_j \in B_{X_j}, 1 \leq j \leq m} |\varphi(x_1, \ldots, x_m)| \\ &= \left|\varphi(x_1^0, \ldots, x_m^0)\right|. \end{aligned}$$

The following theorem generalizes James' theorem to several Banach spaces.

Theorem 2.2. *Let $X_1, \ldots, X_m$ be Banach spaces. The following two assertions are equivalent.*

(a) The spaces $X_1, \ldots, X_m$ are reflexive.

(b) Every multilinear form in $\mathcal{L}(X_1, \ldots, X_m)$ of the form $\varphi = x_1^* \otimes \ldots \otimes x_m^*$ attains its norm.

Definition 2.2. Let X be a Banach space. Let $x^* \in X^*$, we define the following homogeneous polynomial of degree m

$$P_{x^*}(x) = \underbrace{x^*(x) \cdots x^*(x)}_{m \text{ times}} = x^*(x)^m.$$

In this case, we have

$$\|P_{x^*}\| = \sup_{x \in B_X} |P_{x^*}(x)| = \sup_{x \in B_X} |x^*(x)^m| = \|x^*\|^m.$$

The symmetric multilinear form associated with this polynomial is given by

$$T = \underbrace{x^* \otimes \cdots \otimes x^*}_{m \text{ times}}.$$

Definition 2.3. [3] Let $P \in \mathcal{P}(^m X)$ be a polynomial from X to $\mathbb{K}$. This polynomial is said to attain its norm if there exists $x^0 \in B_X$ such that

$$\|P\| = \sup_{x \in B_X} |P(x)| = \left|P(x^0)\right|.$$

Here is the first polynomial version of James' Theorem.

Theorem 2.3. *Let X be a Banach space. The following two assertions are equivalent.*

1. *The space X is reflexive.*
2. *Every polynomial in $P(^m X)$ of the form P_{x^*} attains its norm.*

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Encryption Scheme Based on General Linear Group over Groupring

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Abstract

We propose a new public key cryptosystem based on the difficulty of the conjugacy search problem (CSP) in general linear group over group ring, the prime advantage of using $GL(n, GR)$ is that the multiplication of matrices is very efficient, also this group is resistant to attacks such as eigenvalue and determinant attacks.

Keywords: Group ring, General Linear Group, Public key cryptography.
2020 Mathematics Subject Classification: 94A60, 20G40, 15B33.

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Dosimetry using Monte Carlo simulation

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Abstract

The computation of particle transport in complex geometries requires efficient and unbiased methods to taking account the competing interactions, material variations and accurate tracking. Monte Carlo techniques address these challenges by using a variety of methods including mean free path and cross sections. Cross sections are used to determine the probability of a particle interacting with matter as it travels through a material, making it possible to track particle interactions and determine the probability of each interaction type, providing an accurate representation of the physical processes involved. When a particle interacts, a portion of its energy is transferred to the material, which depends on the specific interaction type, properties and energy of the particle and the material. This information is crucial for predicting the attenuation and energy deposition of particles. These values can then be converted into terms of dosimetric parameters (S-values).
Several simulation tools utilize the Monte Carlo techniques to realize these calculations, offering pre-defined physics models for different particle types and materials. One of these tools is GATE code (Geant4 Application for Tomographic Emission), which plays a crucial role in transporting radiation and calculating the delivered energy to living matter or biological environment. In this context, GATE Monte Carlo packages have been used to evaluate the S-values in an anthropomorphic voxel-based phantom. The obtained S-values results depend on the organ mass and the type of irradiation.

Keywords: Monte Carlo, Cross section, Voxelized phantom, S-values, GATE code.

1 Introduction

It has become frequent to use the radiation transport simulation by Monte Carlo codes as an important method in calculating doses using computational phantoms, to treat problems in radiation protection and radiotherapy. It is based on the object orientated programming language C++ and has a large variety of applications. Several program packages are currently available, however updated development codes are necessary to perform calculations. One such valuable tool is the GATE Monte Carlo code. GATE, allows the user to set up a geometry to act as the object receiving the radiation. A particle source radiating set particles can then be created in this geometry. These particles will then interact with materials created in the geometry and thus interact according to the set physics in that geometry. Information such as energy deposited in the material can be stored and presented, and can then be convert into dosimetric parameters (S-values).

2 Main results

In this work, the radiation energy and the organ mass have an important effect on the S-values for both beta and gamma emissions during self-absorption. Moreover, in the case of cross-irradiation, the distance between the source and the target organs influences the S-values. This confirms the consistent of MC simulation in estimating dosimetric parameters for different emission types.

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Lewis-Riesenfeld invariant method and particle creation in time dependent electric field

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Abstract
We have studied the evolution of the number of particles created in a time dependant electric field by the use of Lewis-Riesenfeld method. We have obtained the density of the particles created which is written as a function of the solution to the Ermakov-Pinny equation. We have considered an exactly solvable example.

Keywords: Particle creation, Lewis-Riesenfeld invariant method, Electric field.
2020 Mathematics Subject Classification: Primary 81Q20; 81Q05; 34A34.

1 Introduction

Quantum field theory in the presence of an external time-dependent electric field $E(t)$ is usually associated with an unstable vacuum and leads to the phenomenon of particle creation. This phenomenon which is known as the Schwinger effect can be explained as follows: When the quantum state of the matter field corresponds to the vacuum state at time t_i, then at any subsequent time, the quantum state will be a superposition of the ground state and excited states of the system. This is generally interpreted as particles created from vacuum under the influence of $E(t)$ [1].
Recently the Lewis-Riesenfeld invariant method, initially developed to study the time-dependent harmonic oscillator [2], is adopted to the case of scalar fields in an external time-dependent electric field $E(t)$ and used to calculate the density of the particles created [3, 4, 5, 6].
The aim of this work is to adopt the Lewis-Riesenfeld invariant method to calculate the number of particles created in an external time-dependent electric field $E(t)$.

2 Particle creation and invariant method

According to the Lewis-Riesenfeld invariant method in QFT, the expression of the time-dependent number of scalar particles created from the invariant vacuum $|0_i\rangle$ for each mode k is given by

$$N_{\mathrm{k}}(t) = |\beta_{\mathrm{k}}|^2 = \frac{\rho^2}{4\Omega_{\mathrm{k}}}\left[\left(\frac{1}{\rho^2} - \Omega_{\mathrm{k}}\right)^2 + \left(\frac{\dot{\Omega}_{\mathrm{k}}}{2\Omega_{\mathrm{k}}} + \frac{\rho'}{\rho}\right)^2\right]. \tag{2.1}$$

The above formula is written in terms of the function $\rho(\eta)$, which is a solution to the nonlinear Ermakov-Pinny differential equation

$$\left[\frac{\partial^2}{\partial\eta^2}+\Omega_k^2(\eta)\right]\rho(\eta)=\frac{1}{\rho^3(\eta)}, \tag{2.2}$$

and takes the form

$$\rho(\eta)=\sqrt{\left|\frac{1}{W}\right|\frac{1}{\sinh 2\alpha}}\left|e^{-\alpha}\rho_1+e^{\alpha}\rho_2\right|, \tag{2.3}$$

where $\rho_1(\eta)$ and $\rho_2(\eta)$ are solutions of the linear equation

$$\left[\frac{\partial^2}{\partial\eta^2}+\Omega_k^2(\eta)\right]\phi_{\mathrm{k}}(\eta)=0, \tag{2.4}$$

α is a real constant and W is the Wronskian of $\rho_1(\eta)$ and $\rho_2(\eta)$, $W=\rho_1\dot{\rho}_2-\rho_2\dot{\rho}_1$.

3 Application

We consider a constant and homogeneous electric field described by the gauge $A_\mu=(0,0,0,Et)$. In this case, the equation (2.4) becomes

$$\left[\frac{d^2}{d\tau^2}-\frac{\tau^2}{4}+\frac{1}{2}+p\right]\phi_{\mathrm{k}}(\tau)=0 \tag{3.1}$$

where

$$\tau=\sqrt{2ieE}\left(t+\frac{k_z}{eE}\right)\ ,\ p=-\frac{1}{2}-i\frac{k_\perp^2+m^2}{2eE}. \tag{3.2}$$

The solutions of the equation (3.1) are given by

$$\rho_1=\left(\frac{2}{eE}\right)^{\frac{1}{4}}e^{-\frac{\pi\lambda}{8}}D_p((1+i)\tau) \tag{3.3}$$

$$\rho_2=\left(\frac{2}{eE}\right)^{\frac{1}{4}}e^{-\frac{\pi\lambda}{8}}D_{-p-1}((1-i)\tau), \tag{3.4}$$

and thus the solution to the Ermakov-Pinny equation is written as

$$\rho_0=\left(\frac{2}{eE}\right)^{\frac{1}{4}}e^{-\frac{\pi\lambda}{8}}\left|D_p(-(1+i)z)\right|. \tag{3.5}$$

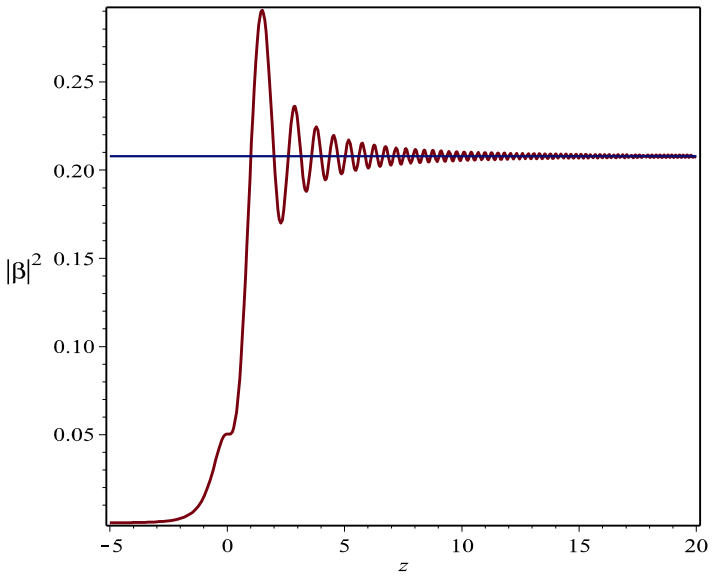

Figure 1: This figure shows the number of particles created $N_k(z)$. The straight lines represent the Schwinger limit, $n(k) = \exp\left(-\pi\tilde{\lambda}\right)$. We can clearly see that $N_k(z)$ is oscillatory in the neighborhood of 0 but in the limit $z \to \infty$, the oscillations will be damped and $N_k(z)$ becomes well defined.

4 Conclusion

In this work we were able to show the importance of the invariant method to study the creation of particles in the presence of a time-dependent external field where the density of the particles created is written as a function of the solution of the Ermakov equation.

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Global Optimization Method of Multivariate hölderian Functions Using secant Minorants

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Abstract
This paper is devoted to solving the multi-dimensional global optimization problems where the objective function f satisfies a Hölder condition over hyper-rectangle of $\mathbb{R}^n$. The generalization of Piyavskii's algorithm to the multivariate case requires finding the intersection of many non-linear hyper-surfaces. In this paper, we propose an algorithm which is composed of two steps. The first one is to transform the multivariate function f into a single variable function $F(x)$ using the α-dense curves and the second one is to apply the extended version of Piyavskii's algorithm to $F(x)$. For minimizing $F(x)$, we construct a sequence of lower bounding piecewise secant functions. A convergence result is proved and the numerical experiments on some test functions are encouraging.

Keywords: Global optimization, hölderian multivariate function, α-dense curves, Piyavskii's algorithm.
2020 Mathematics Subject Classification: Primary 93-03, 93A30, 93B40, 93C35.

1 Introduction

Let us consider the box constrained global optimization problem:

$$\begin{cases} \min\limits_{t\in\mathbf{P}=\prod\limits_{i=1}^{n}[a_i,b_i]} f(t) \end{cases} \tag{1.1}$$

where f is a real continuous multiextrimal function defined on the hyper-rectangle $\mathbf{P}$ and satisfies the following condition:

$$|f(t)-f(t')| \leq h\,\|t-t'\|^m, \forall t,t' \in \mathbf{P} \tag{1.2}$$

with the two parameters $h>0$ and $(m<1)$, where $\|.\|$ stands for the Euclidean norm.
Global optimization is of interest in many complex industrial applications. But it can also be applied to a variety of other multidimensional problems such as the resolution of systems of nonlinear functional equations involving objective functions, which are only continuous and do not possess strong mathematical proprieties such as convexity or differentiability, and which should be optimized. The kind of problem (1.1) arises in several applications, for instance, the simple plant

location problem under a uniform delivered price policy, see Hanjoul et al. [1], infinite horizon optimization problems, see Kiatsupaibul et al. [2], etc. The local irregularity of the objective function, particularly when the value of m is small, is what causes the problem to be complex to solve in this case. When applied to higher dimensions, the traditional multidimensional global optimization methods present significant challenges. For this, there are numerous methods for reducing a multidimensional global optimization problem to one or more optimization problems with a smaller dimension, especially with one dimension. Finding the global minima of multivariate functions with a lot of local minima has proven to be quite effective with the Alienor method coupled with some covering one-dimensional methods. The concept is to densify the hyper-rectangle $\mathbf{P}$ as accurately as we need, using pretty regular so-called α-dense curves [4] , and then approach the objective function f with n variables defined on the hyper-rectangle $\mathbf{P}$, by a function F with a single variable x on a real interval A of $\mathbb{R}$. This allows the multidimensional optimization problem to be reduced to a one-dimensional optimization problem, which can then be solved using one-dimensional methods that are well-known for their effectiveness and performance. This coupling has proved to be efficient for solving diverse non-Lipschitz global optimization problems.

2 Main Results

In this paper, a method for solving non-convex multivariate global optimization problems is presented where the objective function is Hölderian over hyper-rectangle of $\mathbb{R}^n$ with the knowledge a priori of its parameters h and m

The concept relies on using a new α-dense curve for reducing the size of the space $\mathbb{R}^n$ to 1. When minimizing a non-convex function F of single variable , the general principle behind most deterministic global optimization methods is to relax the original non-convex problem in order to make the relaxed problem convex by utilizing an underestimator of the objective function. Here we apply the one-dimensional version of Piyavskii's algorithm based on constructing secant minorant functions [3]. This method is simple and easy to implement on any multivariate non-Lipschitz function even if $1/m$ is not an integer. We suggested a series of numerical applications, followed by a comparative study of two mixed algorithms applied to the proposed problem. We see that the mixed proposed method offer interesting prospects for reducing the computation time and the number of evaluations. Finally, we want to elaborate on these investigations in cases where the constant h is a priori unknown.

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Comparative analysis of the $ASQL$ and the AST of Matched queues with matching batch pair (m, n).

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Abstract
This paper describes a practical application of the matrix-analytic method to deal with matched queue as approximations for matched system with matching batch pair (m, n). We consider two types of customers (m A-customers and n B-customers) arrive according to two independent Poisson processes. Inter arrival times follow an exponential distribution with parameters λ_1 and λ_2 (for each type, respectively) according to a First-Come-First-Match discipline. We assume that the impatient time of an A-customer (respectively, a B-customer) is exponentially distributed with impatient rate θ_1 (respectively, θ_2) for θ_1, $\theta_2 > 0$. Once m A-customers and n B-customers are matched as a group, the $(m + n)$ customers immediately leave the system. We assume that all the random variables defined above are mutually independent.
Numerical analysis and graphical representations were used to compare the average stationary queue lengths ($ASQL$) and the average sojourn times (AST) for both type (the A-customers and he B-customers) under the system stability.

Keywords: Matched queue; Impatient customer; QBD process; Matrix-analytic method; Average Stationary Queue Lengths ($ASQL$); Average Sojourn Times (AST).
2020 Mathematics Subject Classification: 60K25, 60K30, 68M20.

1 Introduction

The present study deals an interesting bilateral stochastic matching problem: A matched queue with matching batch pair (m, n) and two types of impatient customers. Which are widely encountered in many practical areas, for example, sharing economy, ride sharing platform, bilateral market, organ transplantation, taxi services, and assembly systems.
We show that HEDADJI et al. (2023) in [3] used the matrix-analytic method as a tool to obtain an approximation of the limiting probabilities for a single-server Markovian queueing model with several parameters. The matched queue with matching batch pair (m, n) can be expressed as a novel level-dependent quasi-birth-and-death (QBD) process with bidirectional infinite levels. Based on this, we can provide an effective matrix-analytic method to deal with this matched queue and compute the average stationary queue lengths ($ASQL$), the average sojourn times (AST) under the system stability, as it is shown in the theoretical results of Heng-Li Liu et al. (2024) [4].

A physical illustration of the matched queue in illustrated in Figure 1.

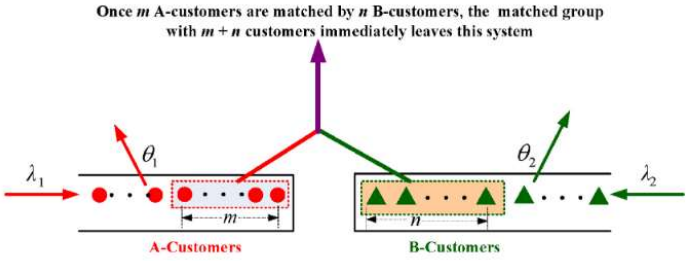


Figure 1: Matched queues with matching batch pair (m, n). [4]

2 Main results

We denote by $Q^{(1)}$ and $Q^{(2)}$ the stationary queue lengths of the A-customers and B-customers, respectively.
We consider that the average stationary queue length of the A-customers is given by $E[Q^{(1)}]$ and the average stationary queue length of the B-customers is given by $E[Q^{(2)}]$.
Let W represent the sojourn time of any arriving A-customer and W' represent the sojourn time of any arriving B-customer. Where

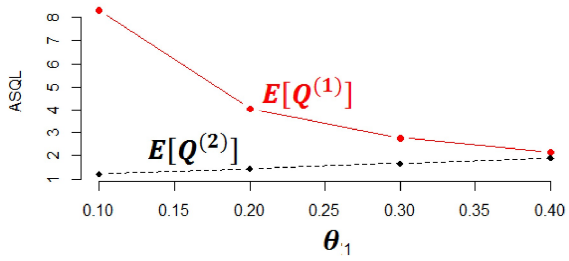
$$E[Q^{(1)}] = \sum_{k=1}^{\infty}\sum_{i=0}^{m-1}(km+i)\sum_{j=0}^{n-1}\pi_{km+i,j} + \sum_{\infty<l\leq 0}\sum_{i=0}^{m-1} i\sum_{j=0}^{n-1}\pi_{i,(-l)n+j};$$

$$E[Q^{(2)}] = \sum_{k=0}^{\infty}\sum_{j=0}^{n-1} j\sum_{i=0}^{m-1}\pi_{km+i,j} + \sum_{\infty<l\leq 0}\sum_{j=0}^{n-1}[(-l)n+j]\sum_{i=0}^{m-1}\pi_{i,(-l)n+j};$$

$$E[W] = \frac{E[Q^{(1)}]}{\lambda_1}$$

and

$$E[W'] = \frac{E[Q^{(2)}]}{\lambda_2}.$$



θ_1	$E[Q^{(1)}]$	$E[Q^{(2)}]$
0.1	8.3049	1.2168
0.2	4.0546	1.4239
0.3	2.7659	1.6639
0.4	2.1454	1.8924

Figure 2: The effect of θ_1 on: $E[Q^{(1)}]$ and $E[Q^{(2)}]$, for $\theta_2 = 0.1$.

θ_2	$E[Q^{(1)}]$	$E[Q^{(2)}]$
0.1	2.1454	1.8924
0.2	2.3002	1.6143
0.3	2.4294	1.4581
0.4	2.5467	1.3491

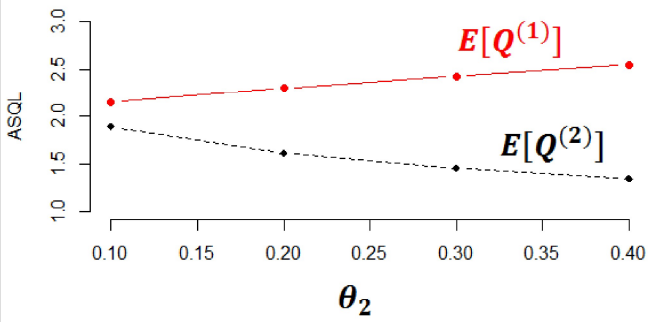
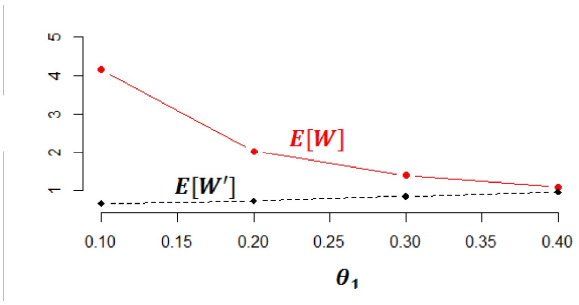


Figure 3: The effect of θ_2 on: $E[Q^{(1)}]$ and $E[Q^{(2)}]$, for $\theta_1 = 0.4$.



θ_1	$E[W]$	$E[W']$
0.1	4.1525	0.6084
0.2	2.0273	0.7119
0.3	1.3830	0.8320
0.4	1.0727	0.9462

Figure 4: The effect of θ_1 on: $E[W]$ and $E[W']$, for $\theta_2 = 0.1$.

θ_2	$E[W]$	$E[W']$
0.1	1.0727	0.9462
0.2	1.1501	0.8071
0.3	1.2147	0.7290
0.4	1.2734	0.6745

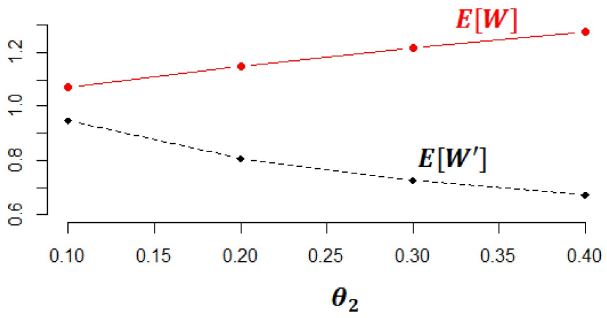


Figure 5: The effect of θ_2 on: $E[W]$ and $E[W']$, for $\theta_1 = 0.4$.

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Chaotic Optimization Algorithm Using New Two-Dimension Map And Its Application

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Abstract

Chaos optimization algorithm is an effective global optimization with low computation complexity and fast search speed.

The ergodicity of chaos is a new and interesting property for optimization, of which the main idea is to search according to a series of successive fields, first search in the total field, then in a smaller one, and so on.

The most famous phenomena of chaos are described by logistic equation. But the distribution of chaostic sequences produced by logistic map is non-uniform leading to the slow constringency. The rational fraction is uniform map with which we replace logistic map to accelerate the rate of convergence.

Keywords: 2-D rational chaotic map; new chaotic attractor; Chaos optimization; Global optimization.

2020MathematicsSubjectClassification:

1 Introduction

A discrete-time dynamical system is given by a map $T: X \to X$ from a space X into itself, we are interested in the asymptotic behavior of sequences $\big(x(n)\big)$ defined by $x(n+1) = T\big(x(n)\big)$ depending on the initial condition $x(0)$ interest in dynamical systems exploded in the 1960s-70s when it was shown in the following items: (a) very simple dynamical systems can have an extremely complex "chaotic" behavior, which appears to be "random",(b) such "chaotic" behavior can paradoxically be "stable", (c) The behavior of some dynamical systems is so "chaotic" and "random" that it is best studied statistically. One of these models is the Lozi map[1,2]. Moreover, it is possible to change the form of the Lozi map for obtaining others chaotic attractors [1,3-6].

2 Mainresults

The conclusion is confirmed by simulation.Therefore it is of great importance to applying rational fraction in global optimization.

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« Inégalités intégrales multidimensionnelles de type Gronwall-Bellman et Applications. »

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Local Linear Estimation of Regression Functions for iid Data and Mixing Processes

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Abstract
Local linear regression has many interesting statistical applications. This work is devoted to the study of local linear regression for a real data. We start by the iid case then we developpe the theory of local linear fitting for dependent data.

Keywords: Local linear estimation, regression function, mixing processes.

1 Introduction

Local polynomial regression was studied by Stone 1977, Cleveland 1979, Tsybakov 1986, Fan Gijbels 1992 and Fan 1993, it has many interesting statistical applications, in particular for the estimation of statistical functions. This work is devoted to the study of local linear regression in the real case. For each data point x, we fit a low order polynomial (line) through the data in the neighborhood of x, the result will be a smooth function on the data support. Local polynomial fitting has many exciting statistical properties which where established under i.i.d. setting. However, the need for nonlinear time series modeling, constructing predictive intervals, understanding divergence of nonlinear time series requires the development of the theory of local linear fitting for dependent data. We study the problem of estimating conditional mean function and its derivative via a local linear fit. The functions include conditional moments, conditional distribution as well as conditional density functions. Joint asymptoti normality for derivative estimation is established for both strongly mixing and ρ-mixing processes.

2 Main results

2.1 The iid case

Let $(X_1, Y_1), ..., (X_n, Y_n)$, pairs of identically distributed independent random variables. The goal is to estimate $m(x) = E(Y|X = x)$ and its derivative $m'(x)$

$$E(m(x)|X) = e_1^t S^{-1} \varsigma \frac{m^{(2)}(x)}{2!} h^2 [1 + o(1)],$$

$$E(m'(x)|X) = e_2^t S^{-1} \varsigma \frac{m^{(2)}(x)}{2!} h [1 + o(1)]$$

$$Var(\hat{m}|X) = \frac{\sigma^2(x)}{nh^3 f(x)} e_2^t S^{-1} S^* S^{-1} e_2 [1 + o(1)],$$

2.2 Dependent data

Suppose thet we observe a stationary sequence $(X_1, Y_1), ..., (X_n, Y_n)$

$$\sqrt{nh}\left(\hat{m}(x) - m(x) - \frac{1}{2}m''(x)B_0h^2\right) \longrightarrow^L N\left(0, \frac{V_0\sigma^2(x)}{f(x)}\right)$$
$$\sqrt{nh^3}\left(\hat{m}'(x) - m'(x) - \frac{1}{2}m''(x)B_1h\right) \longrightarrow^L N\left(0, \frac{V_1\sigma^2(x)}{f(x)}\right).$$

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A New Presentation of Some LCD Linear Codes over the ring $\mathfrak{R}$

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Abstract
Linear codes play a crucial role in improving the reliability of information exchange over noisy channels and are extensively used in modern communication technologies. The study of coding theory, especially codes over rings, has become a prominent area of research. This work aims to construct Simplex and MacDonald linear codes that serve as LCD (Linear Complementary Dual) codes over the ring $\mathfrak{R} = \mathbb{F}_9 + w_1\mathbb{F}_9 + w_2\mathbb{F}_9 + w_3\mathbb{F}_9 + w_4\mathbb{F}_9 + w_5\mathbb{F}_9$, where $w_i^2 = (2 + 2\varrho)w_i$ and $w_iw_j = w_jw_i = 0$ for $1 \leq i \neq j \leq 5$. Initially, we will present the algebraic structures of simplex and MacDonald linear codes over this finite ring, followed by a discussion on the specific properties of these codes. Additionally, we will define some families of linear codes over the finite field $\mathbb{F}_9$, which are Gray images.

Keywords: :Simplex linear codes, Gray images, LCD code, MacDonald linear codes.
2020 Mathematics Subject Classification: 94B05, 94B15, 94B35.

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Construction of DNA Secret Sharing Scheme Using Binary Linear Codes

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Abstract
This research aims to create a DNA secret-sharing scheme tailored for a general access structure, which is vital in the domain of cryptography. This scheme incorporates two fundamental DNA computing techniques specific DNA mixing and sequencing. By leveraging these simple yet effective methods, we enhance the security and efficiency of cryptographic protocols.

Keywords: DNA Secret Sharing, Cryptography, Binary String, Participant.
2020 Mathematics Subject Classification: Primary 11TXX, 11T71, 14G50.

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On the GARCH model with random coefficients

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Abstract
In the usual GARCH model introduced by Bolerslev (1986), the coefficients have a degenerate distribution, and it is thus constant over realizations. In this Note we introduce the GARCH model, involving coefficients that are independent random variables and may vary over realizations.
Conditions for the existence of a stationary solution and conditions ensuring the existence of higher order moments are obtained.

Keywords: RC-ARCH, RC-GARCH, stationarity, Heteroscedasticity, volatility.
2020 Mathematics Subject Classification: .

1 Introduction

Recently, there has been growing interest in using nonlinear time series models in finance and economics. Inference for nonlinear time series had been studied in (Thavaneswaran 1988) using estimating function theory. A nonlinear model had been proposed in [B. Abraham 1991] and using nonlinear state space formulation, filtering and smoothing had been studied (see [1] for more details). Many financial series, such as returns on stocks and foreign exchange rates, exhibit leptokurtosis and time-varying volatility. These two features have been the subject of extensive studies ever since Nicholls and Qninn (1982) and Engle (1982,1991) reported them. Random coefficient autoregressive (RCA) models, the autoregressive conditional heteroscedastic (ARCH) model, and its generalization, the GARCH model, provide a convenient framework to study time-varying volatility in financial markets. Financial time series models for intra-day trading are typical example of random coefficient GARCH models.

2 Main results

1- **Existence of stationary solutions and higher moments for *RC-GARCH(p,q),***

2- Existence of higher order moments,

3- Covariance structure of RC-GARCH(p,q).

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On Two Parameters Rani Distribution and Its Applications

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Abstract

This paper introduces a new continuous two parameters lifetime distribution called the two parameters Rani distribution (TPRD). Some statistical properties of the proposed distribution are derived including the moments, moment generating function, the coefficients of variation, skewness and kurtosis. Also, the distributions of order statistics and the reliability analysis are presented. The distribution parameters are estimated using the maximum likelihood estimation. The Rényi entropy is derived and the stochastic ordering as well as the mean deviations about the mean and the median are investigated..

Keywords: Rani distribution; Reliability analysis; Rényi entropy; Stochastic ordering, Mean and median deviations

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Neutron stars in scalar torsion gravity

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Abstract
In the presence of a non-minimally coupled scalar field ϕ characterized by a symmetron-like potential, we have studied the neutron stars (NS) configuration in teleparallel framework, this latter can be identified as a gauge theory for the translation group in which the gravitational interaction is not geometrized but it rather incarnates itself in torsion. We numerically solve the modified hydrostatic equations for both the interior and the exterior regions using two different realistic equations of state then we extract the mass-radius relations. Our study has been carried out to investigate the compatibility of the obtained results with the observational constraints imposed by the GW170817 event and leads to a maximum mass of $2.37M_{\odot}$ for BSk21 which aligns closely with the recent discovery of the most massive NS ever observed namely, PSRJ0952-0607.

Keywords: neutron stars, teleparallel gravity, scalar tensor theories.
2020 Mathematics Subject Classification: 83F05, 83C10, 83C15.

1 Introduction

Neutron stars (NS) are considered as ideal astrophysical laboratories to test modified theories of gravity in strong field regimes. Recently, the structure and the proprieties of NS have been investigated within the framework of scalar tensor theories, which include scalar fields as an additional degree of freedom alongside the metric tensor [1].
In this context, the aim of this manuscript revolves around the exploration of NS solutions within a specific subclass of teleparallel scalar-tensor theories $F(T, \phi, X)$ or namely scalar torsion gravity [2]. The teleparallel theory of gravity is equivalent to Einstein's general relativity, where the geometry is described by the Weitzenböck connection and thus the gravitational field is inherent in torsion [3] instead of curvature. We promote the teleparallel action to include scalar fields that couple non-minimally with torsion scalar with a coupling parameter ξ. Our investigation delves into the consequences of incorporating a symmetron-like potential $V(\phi) = -\mu^2\phi^2/2 + \lambda\phi^4/4$ on a spherically symmetric and static background. By deriving the structure equations, we plot the mass-radius relations using two different equations of state, and compare these outcomes against the available observational data from gravitational wave events.

2 Scalar torsion gravity

Usually, the dynamical variables in teleparallel gravity are given by a tetrad fields $e^a = e^a{}_\mu dx^\mu$ and a flat spin connection $\omega^a{}_b = \omega^a{}_{b\mu} dx^\mu$. In this framework, gravity is described using the Weitzenböck connection, which lacks curvature and satisfies the metricity condition. Therefore, the field strenght in teleparallel gravity is inherent in torsion tensor $T^\rho{}_{\mu\nu}$. Using the tetrads and the spin connections one can define the torsion tensor as

$$T^\rho{}_{\mu\nu} = -2\Gamma^\rho{}_{[\mu\nu]} = e_a{}^\rho \left(\partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^a{}_{b\mu} e^b{}_\nu - \omega^a{}_{b\nu} e^b{}_\mu \right) . \tag{2.1}$$

The torsion scalar T is defined as a combination of the torsion tensor as follows

$$T = \frac{1}{4} T^\rho{}_{\mu\nu} T_\rho{}^{\mu\nu} + \frac{1}{2} T^\rho{}_{\mu\nu} T^{\nu\mu}{}_\rho - T^\lambda{}_{\lambda\mu} T_\rho{}^{\rho\mu} , \tag{2.2}$$

the teleparallel action is constructed from the above torsion scalar, which differs from the Ricci scalar R of a general relativity by a boundary term implying the equivalence between the two theories at the level of the field equations. Based on curvature-scalar theories, one can also generalize the theory by allowing scalar fields ϕ to non-minimally coupled to torsion throughout the following general action

$$\mathcal{S} = \int d^4x e \left[\frac{\kappa}{2} (1 + \frac{\xi}{\kappa} \phi^2) T - X + V(\phi) + \mathcal{L}_m \right] , \tag{2.3}$$

where the scalar field is accompanied with its kinetic term $X = -(1/2)\, g^{\mu\nu} \phi_{,\mu} \phi_{,\nu}$ and $\mathcal{L}_m$ refers to the matter Lagrangian. The tetrad field equations are given by

$$G_{\alpha\beta} F_T + \frac{1}{2} g_{\alpha\beta} (F - F_T T) + S_{\beta\alpha}{}^\rho \overset{\circ}{\nabla}_\rho F_T + \frac{1}{2} F_X \phi_{,\alpha} \phi_{,\beta} = \Theta_{\alpha\beta} , \tag{2.4}$$

where $G_{\alpha\beta}$ is the Einstein tensor and $S_{\beta\alpha}{}^\rho$ is the superpotential tensor. On the other hand, the scalar field equation are given by

$$g^{\mu\nu} \overset{\circ}{\nabla}_\mu (F_X \phi_{,\nu}) + F_\phi = 0 , \tag{2.5}$$

where $F_A \equiv \partial F / \partial A$ such that $A = T, X, \phi$.

3 Structure equations

Solving the field equations for a spherically symmetric and static background

$$ds^2 = -f(r) dt^2 + h(r) dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) , \tag{3.1}$$

and assuming that the stellar matter is modeled by a perfect fluid, one can obtain the structure equations

$$\frac{f'}{f} = \frac{h-1}{r} + \frac{2r^2 h (P - V) + r^2 \phi'^2}{2r (\kappa + \xi \phi^2)} , \tag{3.2}$$

$$\frac{h'}{h} = \frac{1-h}{r} + \frac{2r^2 h (\rho + V) - 8\xi \left(\sqrt{h} - 1 \right) r \phi \phi' + r^2 \phi'^2}{2r (\kappa + \xi \phi^2)} , \tag{3.3}$$

$$\phi'' = \left[\frac{r^2 h (\rho - P + 2V)}{2r (\kappa + \xi \phi^2)} - \frac{1+h}{r} \right] \phi' + \xi h T \phi + h V_\phi , \tag{3.4}$$

$$P' = -(\rho + P) \left(\frac{h-1}{2r} + \frac{2rh (P - V) + r \phi'^2}{4 (\kappa + \xi \phi^2)} \right) . \tag{3.5}$$

The equations (3.2)-(3.5) together with an equation of state collectively provide a description of the internal structure of the star and they represent the modified TOV equations that account for the hydrostatic equilibrium within a static and spherically symmetric background. In order to numerically solve them and generate the mass-radius diagram, we must derive the appropriate boundary conditions for regions both inside and outside of the star. In this work, we will focus mainly on exploring the effects of the symmetron-like potential on the NS stellar configurations whose explicit form $V(\phi) = -\mu^2\phi^2/2 + \lambda\phi^4/4$.

4 Mass-Radius Graphs

We find that the mass-radius relation deviates significantly from the GR counterpart. Additionally, the results of Fig. (1) agree with the constraints imposed by GW170817 [4]. A NS with mass $1.6M_\odot$ must have a radius greater than 10.68 km, and the maximum mass must be greater than 9.6 km. The EoSs SLy and BSk21 can support NSs with masses exceeding $2M_\odot$. Remarkably, the case of BSk21 yields to a maximum mass of $2.37M_\odot$ for $\xi = 0.25$, which situates within the scope of the measured mass $2.35 \pm 0.17M_\odot$ of the PSR J0952-0607, the heaviest and fastest known NS [5].

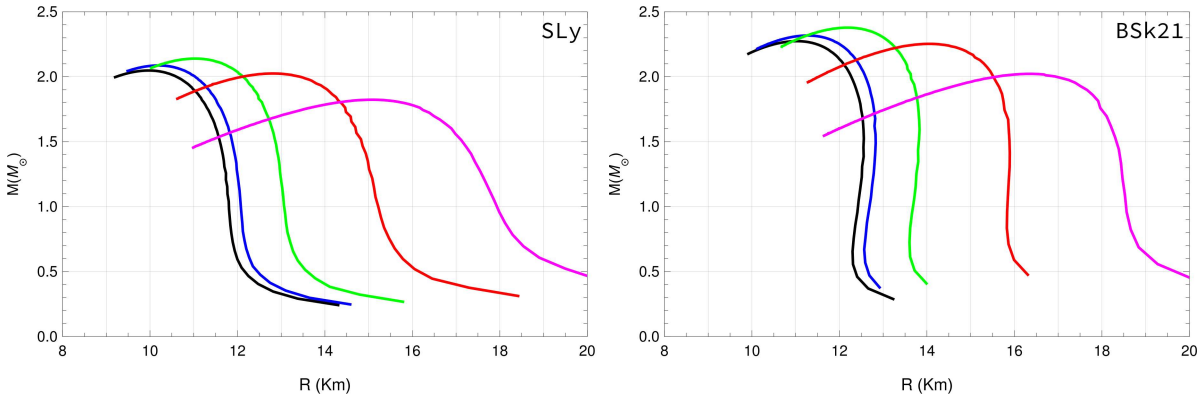

Figure 1: The Mass-radius relation for the symmetron-like potential with $\mu = 15.04 \times 10^{-8} r_0^{-1}$ and $\lambda = 2.512 \times 10^{-14} \rho_0^{-1} r_0^{-2}$ for two realistic EoSs and different coupling parameters ξ. For the GR solutions $\xi = 0$ (black line), $\xi = 0.05$ (Blue), $\xi = 0.25$ (Green), $\xi = 0.75$ (Red) and $\xi = 1.5$ (Magenta).

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Construction a novel generating functions for the products of Mersenne-Lucas numbers with some numbers and polynomials

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Abstract
In this paper, we derive some new generating functions for the products of $k-$Mersenne and Mersenne-Lucas numbers with $k-$Fibonacci numbers, $k-$Pell numbers and $k-$Jacobsthal numbers. Additionally, we give some new generating functions of the products of these numbers with Chebyshev polynomials.

Keywords: Symmetric functions, $k-$Mersenne numbers, Mersenne-Lucas numbers, Generating functions, $k-$Fibonacci numbers, $k-$Pell numbers.
2020 Mathematics Subject Classification: Primary 05E05, Secondary 11B39.

1 Introduction

Over many years, many researchers have studied well-known number sequences such as Fibonacci numbers, Pell numbers, Mersenne and Mersenne Lucas numbers. The Mersenne sequence $\{M_n\}_{n\in\mathbb{N}}$ is defined recursively by [3]

$$M_n = 3M_{n-1} - 2M_{n-2}, \quad n \geq 2, \tag{1.1}$$

with initial conditions $M_0 = 0$, and $M_1 = 1$. In [3] the authors defined the $k-$Mersenne numbers $\{M_{k,n}\}_{n\in\mathbb{N}}$ as a generalization of the Mersenne numbers by the recurrence relation

$$M_{k,n} = 3kM_{k,n-1} - 2M_{k,n-2}, \quad n \geq 2, \tag{1.2}$$

with initial conditions $M_{k,0} = 0$ and $M_{k,1} = 1$. The characteristic equation corresponding to the recurrence relation (1.4) is

$$r^2 - 3kr + 2, \tag{1.3}$$

whose roots are

$$r_1 = \frac{3k + \sqrt{9k^2 - 8}}{2}, \qquad r_2 = \frac{3k - \sqrt{9k^2 - 8}}{2}.$$

The Mersenne Lucas numbers $\{m_n\}_{n\in\mathbb{N}}$ is defined recursively by

$$m_n = 3m_{n-1} - 2m_{n-2}, \quad n \geq 2, \tag{1.4}$$

with initial conditions $m_0 = 2$ and $m_1 = 3$.

Definition 1.1. [3] Let n be positive integer and $A = \{a_1, a_2\}$ are set of given variables, then the n-th symmetric function $S_n(a_1 + a_2)$ is defined by

$$S_n(A) = S_n(a_1 + a_2) = \frac{a_1^{n+1} - a_2^{n+1}}{a_1 - a2}.$$

Definition 1.2. [3] The symmetrizing operator $\delta^k_{b_1b_2}\delta^k_{a_1a_2}$ is defined by

$$\delta^k_{b_1b_2}\delta^k_{a_1a_2}f(a_1,b_1) = \frac{a_1^k b_1^k f(a_1b_1) - a_1^k b_2^k f(a_1b_2) - a_2^k b_1^k f(a_2b_1) + a_2^k b_2^k f(a_2b_2)}{(a_1-a_2)(b_1-b_2)}, \quad k \in \mathbb{N}.$$

2 Main results

In this section, we give some new generating functions for the products of Mersenne-Lucas numbers with some well-know numbers and polynomials.

Theorem 2.1. *[3] Let A, B and C be three alphabets, respectively, $\{a_1, a_2\}$, $\{b_1, b_2\}$ and $\{c_1, c_2\}$ then we have*

$$\begin{aligned}
&\sum_{n=0}^{\infty} S_n(A)S_{n+k-1}(B)S_{n+k-1}(C)z^n = \\
&\frac{b_1^k b_2^k \left(\begin{array}{l} \left(\sum\limits_{n=0}^{\infty} S_n(-A)b_2^n c_1^n z^n\right)\left(\sum\limits_{n=0}^{\infty} S_n(-A)b_1^n c_1^n z^n\right)\sum\limits_{n=0}^{\infty} S_n(-A)S_{n-k-1}(B)C_2^{n+k}z^n \\ -\left(\sum\limits_{n=0}^{\infty} S_n(-A)b_2^n c_2^n z^n\right)\left(\sum\limits_{n=0}^{\infty} S_n(-A)b_1^n c_2^n z^n\right)\sum\limits_{n=0}^{\infty} S_n(-A)S_{n-k-1}(B)C_1^{n+k}z^n \end{array}\right)}{(c_1-c_2)\prod\limits_{a\in A}(1-ab_1c_1z)\prod\limits_{a\in A}(1-ab_2c_1z)\prod\limits_{a\in A}(1-ab_1c_2z)\prod\limits_{a\in A}(1-ab_2c_2z)},
\end{aligned} \tag{2.1}$$

for all $k \in \mathbb{N}_0$.

Based on the previous theorem, we obtain the following results.

Theorem 2.2. *For $n \in \mathbb{N}$, the new generating function of the product of Mersenne-Lucas numbers with $k-$Fibonacci numbers is given by*

$$\sum_{n=0}^{\infty} m_n F_{k,n}^2 z^n = \frac{2 - 20z^6 + 12k^2z^5 + (16k^2+33)z^4 - 3k^2z^3 + (-10k^2-15)z^2 - 3k^2z}{D_{m_nF^2_{k,n}}}, \tag{2.2}$$

with

$$\begin{aligned}
D_{m_nF^2_{k,n}} &= 1 - 3k^2z - (9k^2 + (k^2+2)(-2k^2+5))z^2 - 3k^2(-4k^2-1)z^3 \\
&\quad + (26k^4 + 32k^2 + 33)z^4 - 6k^2(-4k^2-1)z^5 - (36k^2 + (4(k^2+2))(-2k^2+5))z^6 \\
&\quad - 24k^2z^7 + 16z^8.
\end{aligned}$$

Theorem 2.3. *For $n \in \mathbb{N}$, the new generating function for the product of Mersenne-Lucas numbers with k-Pell numbers is given by*

$$\sum_{n=0}^{\infty} m_n P_{k,n}^2 z^n = \frac{-24k^6z^7 + 64k^4z^6 + (42k^4 + 96k^3)z^5 + (-21k^2 - 48k)z^3 - 16z^2 + 3z}{D_{m_n P_{k,n}^2}}, \tag{2.3}$$

with

$$\begin{aligned} D_{m_n P_{k,n}^2} &= 1 - 12z - (36k + (2k+4)(5k-8))z^2 - (12(-k^2 - 16k))z^3 + (128k^2 \\ &+ 105k^4 - 2k^2(36k^2 - 64k - 144))z^4 - 24k^2(-k^2 - 16k)z^5 - (144k^5 + 4k^4(2k+4)(5k-8))z^6 \\ &- 96k^6z^7 + 16k^8z^8. \end{aligned}$$

Theorem 2.4. *For $n \in \mathbb{N}$, the new generating function for the product of Mersenne-Lucas numbers with k-Jacobsthal numbers is given by*

$$\sum_{n=0}^{\infty} m_n J_{k,n}^2 z^n = \frac{-24z^7 + 16k^2z^6 + (24k^2 + 42)z^5 + (-12k^2 - 21)z^3 - 4k^2z^2 + 3z}{D_{m_n J_{k,n}^2}}, \tag{2.4}$$

with

$$\begin{aligned} D_{m_n J_{k,n}^2} &= 1 - 3k^2z - (9k^2 + (k^2+2)(-2k^2+5))z^2 - 3k^2(-4k^2 - 1)z^3 + (26k^4 + 32k^2 + 33)z^4 \\ &- 6k^2(-4k^2 - 1)z^5 - (36k^2 + (4(k^2+2))(-2k^2+5))z^6 - 24k^2z^7 + 16z^8. \end{aligned}$$

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A mixed algorithm for smooth global optimization

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Abstract
This paper presents a covering algorithm for solving bound-constrained global minimization problems with a differentiable cost function. In the proposed algorithm, we suggest to explore the feasible domain using a one-dimensional global search algorithm through a number of parametric curves that are relatively spread and simultaneously scan the search space. To accelerate the corresponding algorithm, we incorporate a multivariate quasi-Newton local search algorithm to spot the lowest regions. The proposed algorithm converges in a finite number of iterations to an ε-approximation of the global minimum. The performance of the algorithm is demonstrated through numerical experiments on some typical test functions.

Keywords: Global optimization, Alienor dimensionality reduction technique, $\alpha-dense$ curve, One-dimensional global search algorithm, Limited Memory BFGS-B algorithm
2020 Mathematics Subject Classification: 90C26, 90C30

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Identification of a bilinear models

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Abstract
In this paper, We deal with the frequency domain method, i.e., using the Wiener-Itô spectral representation, to describe some probabilistic properties of different representations of BL models, as this method was shown more informative. So, we give a necessary and sufficient condition for the regular second order stationarity. It is shown that the spectrum characterizes only the linear part of these representations, therefore it is necessary to use the third order properties, i.e., the bispectrum for the identification.

Keywords: BL models, Transfer function, Stationarity, Spectrum.
2020 Mathematics Subject Classification: Primary 60G65, 60G10.

1 Introduction

We consider the simple bilinear model $(Y_t)_{t\in\mathbb{Z}}$:

$$Y_t = \beta Y_{t-l} e_{t-k} + e_t \tag{1.1}$$

where the observation Y_t is a real stochastic process defined on some probability space $(\Omega, \mathcal{A}, P)$, for a real constant β and the errors $(e_t)_{t\in\mathbb{Z}}$ are distributed symmetrically around mean zero and satisfied $E(e_t^i) = \begin{cases} \mu_i \text{ if } i \text{ is even} \\ 0 \text{ if } i \text{ is odd} \end{cases}$. Model (1.1) is called diagonal if $l = k$, superdiagonal if $l > k$ and subdiagonal if $l < k$. It was introduced in 1978 by Granger and Andersen, and a comprehensive survey of general bilinear process was made by Subba Rao and Gabr (1984). From the literature it appears that the subdiagonal model has never been investigated seriously. The superdiagonal models have been investigated in detail because of the simplicity of their expansions; from a physical point of view, it seems more interesting to investigate the subdiagonal class of models.
The purpose of this paper is to: a) overview the analysis of the simple BL models in the frequency domain, b) to provide some results probabilistic for the different representations of BL process in terms of multiple Wiener-Itô integrals: Conditions for the second order stationarity of a differents representation with the Hermite polynomial of second order, and c) to show that all the information for the identification of the model is given either by the spectrum and the bispectrum or by the spectrum and the cross spectrum between the process and its square.
An outline of the paper can be given as follows. Having defined the Winer's models and its spectral representation in section 2. Section 3 deals with the representations of bilinear models. we shall consider three particular bilinear models and we deduce the transfer functions, which play a fundamental role in the identification and estimation of the model. The main probabilistic properties are presented.

2 The bilinear models

In this section, we investigate different representations of bilinear models:
One of the simplest bilinear models is given by

$$Y_t = \beta Y_{t-k} e_{t-k} + e_t, k \geq 1, \tag{2.1}$$

we have $E(Y_t) = \beta\mu_2$ and the covariance of Y_t is then given by $cov(Y_t, Y_{t-j}) = \begin{cases} \beta^2\mu_2^2 \text{ if } j = k \\ 0 \text{ if } j \neq k \end{cases}$.

Theorem 2.1. *A necessary and sufficient condition for the existence of regular second order stationary solution for diagonal model (2.1) is that*

$$\sum_{r=0}^{\infty} (r)! \left\| sym\left\{\mathbf{f}_r\right\}\right\|^2 < \infty \tag{3.2}$$

where $f_r\left(\lambda_{(r)}\right)$ are the transfer functionsare of (3.1).

For the superdiagonal model

$$Y_t = \beta Y_{t-l} e_{t-k} + e_t, l > k, \tag{3.3}$$

where e_t a Gaussian white noise with mean zero and variance μ_2, we have $E(Y_t) = 0$ and $E(Y_t^2) = \frac{\mu_2}{1-\beta\mu_2}$. It is easy to see that Y_t is an uncorrelated non Gaussian process. We have

Lemma 2.1. *Assume that the model (3.3) has regular second order stationary solution, then the transfer functions of this solution are given by the symmetrization of the following functions defined recursively by*

$$\begin{cases} f_0 = 0, \text{ if } r = 0 \\ f_1(\lambda_1) = 1, \text{ if } r = 1 \\ f_2\left(\lambda_{(2)}\right) = \beta e^{-i.(l.\lambda_1 + k.\lambda_2)} f_1(\lambda_1), \text{ if } r = 2 \\ f_r\left(\lambda_{(r)}\right) = \beta e^{-i\left(l.\sum_{j=1}^{r-1}\lambda_j + k.\lambda_r\right)} f_{(r-1)}\left(\lambda_{(r-1)}\right), \text{ if } r > 2 \end{cases} \tag{3.4}$$

Corollary 2.1. *Under the conditions of Lemma 2.1 the necessary and sufficient condition for the existence of a regular second order stationary solution of the model (3.3) is*

$$\mu_2\beta^2 < 1.$$

For the subdiagonal model

$$Y_t = \beta Y_{t-k} e_{t-k-1} + e_t, k > 0, \tag{3.5}$$

we have $E(Y_t) = 0$ and $E(Y_t^2) = \begin{cases} \mu_2 + \lambda\mu_2 + \beta^2\lambda\mu_4/1 - \lambda \text{ if } k = 1 \\ \mu_2/1 - \lambda \text{ if } k \geq 2 \end{cases}$ were $\lambda = \beta^2\mu_2$. Assumed that e_t a Gaussian white noise with mean zero and variance 1, it is easy to see that (3.5) is equivalent to the system of equations

Theorem 2.2. *A necessary and sufficient condition for the existence of regular second order stationary solution for model (3.5) is that*

$$\sum_{r=0}^{\infty} (r)! \left\| sym\left\{\mathbf{f}_r\right\}\right\|^2 < \infty \tag{3.6}$$

where $f_r\left(\lambda_{(r)}\right)$ are the transfer functions of (3.5).

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Non-relativistic Quantum Mechanics in the Framework of the Snyder-de Sitter Algebra.

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Abstract
In this study, we apply the supersymmetric approach in Quantum Mechanics (SUSYQM) to analyze the one-dimensional harmonic oscillator (OH) within the Snyder-de Sitter model. We determine i particular the energy spectrum of the OH both in the conventional case and within the Snyder-de Sitter space (SdS), using the mentioned method. This approach allows us to explore the implications of combining SUSYQM with the SdS model and provides insights into the modifications of the harmonic oscillator energy spectrum under these conditions.

Keywords: Snyder-de Sitter model (SdS), Supersymmetry in Quantum Mechanics (SUSYQM), One-dimensional harmonic oscillator (OH), Energy spectrum.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

In theoretical physics, the Snyder-de Sitter (SdS) model is highly significant[1]. This extension of the Snyder model introduces a constant curvature to spacetime, resulting in a non-commutative spacetime that remains invariant under the de Sitter group and incorporates two additional fundamental scales beyond the speed of light[2]. Our objective is to examine the energy spectrum of a one-dimensional harmonic oscillator within the framework of the Snyder-de Sitter algebra using the quantum supersymmetry approach (SUSYQM).

2 Main results

In the literature, the energy spectrum of a one-dimensional harmonic oscillator in Snyder-de Sitter (SdS) space has been found using the hypergeometric method [2]. Here, we utilize supersymmetry in quantum mechanics (SUSYQM) to determine the energy spectrum by deriving the stationary Schrödinger equation for the harmonic oscillator in SdS space. We then transform this equation into a Schrödinger-type equation using a specific representation. By proposing a superpotential that satisfies the Riccati equation, we can apply the shape invariance method. Consequently, we derive the energy spectrum of the harmonic oscillator within the framework of deformed Heisenberg algebra, by:

$$E_n = (n+\frac{1}{2})\omega_0\sqrt{1+\frac{(\omega_0^2\beta^2+\alpha^2)^2}{4\omega_0^2}}+(n^2+n+\frac{1}{2})\frac{(\omega_0^2\beta^2+\alpha^2)}{2}$$

This study confirmed the effectiveness of the SUSYMQ approach in precisely determining the energy spectrum of a one-dimensional harmonic oscillator in SdS space, with potential applications to other physical systems.

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Limit cycles for a class of differential systems in dimension 3

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Abstract We shall study the maximum number of limit cycles which can bifurcate from a zero–Hopf equilibrium point of a polynomial differential system in $\mathbb{R}^3$ with cubic homogeneous non-linearities using the averaging theory of the fourth-order. More precisely, we consider the differential systems of the form

$$\begin{aligned}
\dot{x} &= (a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + a_4\varepsilon^4)x - (b + b_1\varepsilon + b_2\varepsilon^2 + b_3\varepsilon^3 + b_4\varepsilon^4)y + \sum_{j=0}^{2} \varepsilon^j X_j(x,y,z),\\
\dot{y} &= (b + b_1\varepsilon + b_2\varepsilon^2 + b_3\varepsilon^3 + b_4\varepsilon^4)x + (a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + a_4\varepsilon^4)y + \sum_{j=0}^{2} \varepsilon^j Y_j(x,y,z),\\
\dot{z} &= (c_1\varepsilon + c_2\varepsilon^2 + c_3\varepsilon^3 + c_4\varepsilon^4)z + \sum_{j=0}^{2} \varepsilon^j Z_j(x,y,z),
\end{aligned}$$

where

$$\begin{aligned}
X_j(x,y,z) \;=\; & a_{j0}x^3 + a_{j1}x^2y + a_{j2}x^2z + a_{j3}xy^2 + a_{j4}xyz + \\
& a_{j5}xz^2 + a_{j6}y^3 + a_{j7}y^2z + a_{j8}yz^2 + a_{j9}z^3,
\end{aligned}$$

$Y_j(x,y,z)$ and $Z_j(x,y,z)$ have the same expression as $X_j(x,y,z)$ by replacing a_{ji} respectively by b_{ji} and c_{ji} for $j = \overline{0,2}$ and $i = \overline{0,9}$. The coefficients a_{ji}, b_{ji}, c_{ji}, a_1, a_2, a_3, a_4, b, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4 are real parameters with $b \neq 0$. When $\varepsilon = 0$ the system has eigenvalues $\pm bi$ and 0 at the origin, i.e. the origin is a zero–Hopf equilibrium point.

Keywords: Zero–hopf bifurcation; Limit cycles; Averaging theory.
2020 Mathematics Subject Classification: Primary 37J70; Secondary 37L15.

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Projective Synchronization of the Fractional-Order Lorenz Hyperchaotic System

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Abstract
This work investigates the projective synchronization of a four dimensional fractional-order Lorenz hyperchaotic system. First, we analyzethe system's dynamic using phase portraits and lyapunov exponenets spectrum confirming its hyperchaotic nature. Then, we propose projective synchronization scheme using active feedback control method. The effectiveness of the proposed method is demonstrated through numerical simulations.

Keywords: fractional-order systems , Lyapunov exponents, Projective synchronization, Active feedback control.
2020 Mathematics Subject Classification: Primary 34C25, 37D45.

1 Introduction

Fractional calculus, with roots dating back to the 17th century, has emerged as a powerful tool for modeling complex systems across diverse interdisciplinary fields [1]. In 1979, Rössler introduced a groundbreaking concept – the first hyperchaotic system [2]. This system exhibits intriguing dynamic behavior characterized by multiple positive Lyapunov exponents. Since then, the exploration of fractional-order hyperchaotic systems has flourished, with notable examples including the fractional Chua system [3], the fractional Chen system [4], and the fractional Lü system [5]. Following the pioneering work of Pecora and Carroll on chaos synchronization [6], this field has garnered significant scientific interest due to its potential in various domains. This work delves into achieving projective synchronization of the four-dimensional fractional-order Lorenz system by employing an active feedback control method.

2 Main results

2.1 The Fractional-Order Lorenz Hyperchaotic System.

The Lorenz system is described by [6]

$$\begin{cases} \dot{x_1} = a(x_2 - x_1), \\ \dot{x_2} = -x_1x_3 + cx_1 - x_2, \\ \dot{x_3} = x_1x_2 - bx_3, \end{cases} \tag{2.1}$$

where $a; b; c$ are real constants. When $a = 10; b = 8/3; c = 28$, Lorenz system exhibits a chaotic behavior. [6].
Add a nonlinear controller x_4 to the system 2.1, then we obtain a new system defined as [2]

$$\begin{cases} \dot{x_1} = a(x_2 - x_1) + x_4, \\ \dot{x_2} = -x_1x_3 + cx_1 - x_2, \\ \dot{x_3} = x_1x_2 - bx_3, \\ \dot{x_4} = -x_2x_3 + rx_1, \end{cases} \tag{2.2}$$

where $x_1,\ x_2,\ x_3,\ x_4$ are the state variables, $a,\ b,\ c$ and r are real constants.
When $a = 10; b = 8/3; c = 28$, ans $r = -1$ system 2.2 exhibits a hyperchaotic attractor.
The four-dimensional fractional order Lorenz system can be described as[1]

$$\begin{cases} D^\alpha x_1 = a(x_2 - x_1) + x_4, \\ D^\alpha x_2 = -x_1x_3 + cx_1 - x_2, \\ D^\alpha x_3 = x_1x_2 - bx_3, \\ D^\alpha x_4 = -x_2x_3 + rx_1, \end{cases} \tag{2.3}$$

Where $\alpha \in (0, 1)$ is the fractional-order derivative, D^α is Caputo differential operator [7] defined by

$$D^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\xi)^{m-\alpha-1} x^{(n)}(\xi) d\xi \tag{2.4}$$

The proposed system has the following properties:
1. The system (2.3) is hyperchaotic when the parameter systems take the following values

$$(a,\ b,\ c,\ r) = (10,\ 8/3,\ 28,\ -1) \quad and \quad q = 0.98 \tag{2.5}$$

2. When the parameter systems are taken as (2.5), the Lyapunov exponents of the proposed system are obtained as:

$$L_1 = 0.237,\ L_2 = 0.062,\ L_3 = 0.024 \text{ and } L_4 = -1.673,$$

and the Kaplan-Yorke dimension of the system is calculated as:

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.193,$$

which means that the system is hyperchaotic.

2.2 Projective synchronization Analysis

Consider a class of master-slave hyperchaotic systems described by

$$D^{\alpha}x(t) = Ax(t) + f(x(t)), \tag{2.6}$$
$$D^{\alpha}y(t) = Ay(t) + g(y(t)) + u, \tag{2.7}$$

where $\alpha \in (0,1)$ is the fractional-order, x, $y \in \mathbb{R}^n$ are state variables of the master system and the slave system, respectively, $A \in \mathbb{R}^{n\times n}$, is the linear part of the systems, $f, g : \mathbb{R}^n \to \mathbb{R}^n$ are continuous non linear functions and $u = (u_1 u_2, ..., u_n)^T \in \mathbb{R}^n$ is a control input vector.
Our goalis to achieve projective synchronization between the drive system equation (2.6) and response system equation (2.7), for arbitrary initial conditions $x(0)$ and $y(0)$. Projective synchronization is achieved when the error, defined as

$$e(t) = y(t) - \lambda x(t), \tag{2.8}$$

converges to zero as time approaches infinity. Mathematically, our objective is to find a suitable control function u, such that the following condition holds

$$\lim_{t\to\infty} ||e(t)|| = 0, \tag{2.9}$$

Theorem 2.1. *The synchronization between the master system (2.6) and the slave system (2.7) can be achieved if the following conditions are satisfied*
(a) $U = g(y(t)) - \lambda f(x(t)) + \gamma(t)$, *where* $\gamma(t)$ *is a control vector.*
(b) $\gamma(t) = N.(e_1, e_2, e_3, e_4)^t$ *where* N *is a negative definite matrix.*

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Solving PDEs by Kansa's method with Coupled Radial Basis Functions using Chebyshev-Gauss-Lobatto collocation points.

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Abstract
Radial basis functions are an advanced technique in scientific computing fields for solving partial differential equations due to its high accuracy and stability. However, the selection of the optimal shape parameter has long been a challenging task. Coupled radial basis functions based on coupling one of the classical RBFs with the conical spline r^5. CRBFs are proposed and used to solve PDEs. In this work, our investigation focuses on solving PDEs by Kansa's method with CRBFs using various conical splines r^m, $(m = 5, 7, 9)$ as the shape parameter c ranges in a wide scope. In this study, we consider Chebyshev-Gauss-Lobatto collocation points to enhance the performance of the proposed method. Test example is given to show the efficiency of the CRBFs method for solving PDEs.

Keywords: Partial differential equations, Radial basis functions, Kansa's method, Shape parameter, Coupled radial basis functions, Chebyshev-Gauss-Lobato collocation points.
2020 Mathematics Subject Classification: Primary 31J05, 35J25, 35J60.

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The Dynamic Behavior of a New class of Kirchhoff Equations with variable exponent and High Order Strong Damping.

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Abstract
In this paper, we study the long time behavior of a New class of Kirchhoff Equations with variable exponent and High Order Strong dissipative terms. based on the appropriate assupmtions regarding the rigid and nonlinear terms of the Kirchhoff equation, we first evaluate the equation through a priori estimates in th spaces E_0 and E_k, and demonstrate the existence and uniquness of the solution using Galerkin's method.Subsequently, we derive the bounded absorbing set B_{0k} based on the a priori estimates.Furthermore, utilizing the Rellich-Kondrachov Copmact Embedding theorem, we prove that the solution semigroup $S(t)$ of the equation possesses a familly of global attractors A_k in the space E_k . finally, we establish that the solution semigroup $S(t)$ is Frechet differentiable in E_k by linearizing the equation.

Keywords: Kirchhoff equation ; Prior Estimate ; The Existeance and Uniquness of the solution ; Family of Global Attractor ; Dimension Estimattion.
2020 Mathematics Subject Classification: 35B45, 35A01, 37D45.

1 Introduction

In this paper, we mainly study the initial boundry value problem of the Kirchhoff equation with variable exponent and High Order strong damping :

$$\begin{aligned} &u_{tt} + M([\nabla^m u]_p^p)(-\Delta)^{2m}u + \beta(-\Delta)^{2m}u_t + |u|^p u = f(x), \\ &u(x,t) = \frac{\partial^j u}{\partial v^j} = 0, \;\; j = 1, 2, \ldots, 2m-1, \;\; x \in \partial\Omega, \\ &u(x,0) = u_0(x), u_t(x,0) = u_t(x), \;\; x \in \partial\Omega, \;\; t > 0, \end{aligned} \tag{1.1}$$

where $m > 1$, $p \geq 2$, $\Omega \subseteq \mathbb{R}^n (n \geq 1)$ is a bounded region with a smooth boundary $\partial\Omega$, $\mathcal{Q} = \Omega \times [0, \infty)$ stands for the cylinder in $\mathbb{R}^n_x \times \mathbb{R}_t$, the rigid term $M(s) \in C^1[0, \infty)$ is a general function, $\beta(-\Delta)^{2m}u_t (\beta > 0)$ is the strong dispative term, $|u|^p u$ is the nonlinear term and $\rho \geq -1$, $f(x)$ denots the external force.
Kirchhoff equation is a kind of important nonlinear wave equation, wich is widely used in engineering physics, especially provides a strong support for measuring bridge vibration.

2 Main results

Lemma 2.1. *Let $(u_0, u_1) \in E_0$ $f(x) \in L^2(\Omega)$, then the initial boundary value problem (1.1) has a global solution (u, v) that satisfies $u \in L^\infty(0, +\infty; H_0^{2m}(\Omega))$,*

$$v \in L^\infty(0, +\infty; H_0^{2m}(\Omega)) \cap L^2(0, T; H_0^{2m}(\Omega)), \text{and}$$

$$\|(u, v)\|_{E_0}^2 = \|\Delta^2 u\|^2 + \|v\|^2 \leq y_1(0)e^{\alpha_1 t} + \frac{C_1}{\alpha_1}, \tag{2.1}$$

$$\frac{\beta}{2}\int_0^T \|\Delta^m v\|^2 dt \leq C_1 T + y_1(0), \tag{2.2}$$

where $v = u_1 + \epsilon u$, $\alpha_1 =\leq \min\{\frac{\beta\lambda_1^{2m}}{2} - 2\epsilon - \epsilon^2, 2\epsilon, \frac{2\epsilon\mu - \beta\epsilon 2}{\mu}, \epsilon(\rho+2)\}$, $y_1(0) = \|v_0\|^2 + \epsilon^2\|u_0\|^2 + \mu\|\Delta^m u_0\|^2 + \frac{2}{\rho+2}\|u_0\|_{\rho+2}^{\rho+2}$.
So there exists a nonnegative real number R_1 and $t_1 = t_1(\Omega) > 0$ such that

$$\|(u, v)\|_{E_0}^2 = \|\Delta^m u\|^2 + \|v\|^2 \leq R_1^2 \qquad (t > t_1). \tag{2.3}$$

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Finding the Best Mix: Optimizing Renewable Resources in Hybrid Energy Systems

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Abstract
This paper explores hybrid renewable energy systems, which combine multiple renewable energy sources. These systems offer a sustainable solution for installing autonomous power systems. They reduce reliance on storage methods and minimize environmental impact.
The key challenge lies in optimizing the allocation of each renewable resource during design and operation. Given an estimate of the site's renewable energy potential, this work aims to determine the optimal combination of resources within a hybrid system to meet a specific region's energy demands. The problem can be formulated as an optimal allocation problem with limited resources, and the linear programming method is adopted to find the best solution. The paper concludes with practical examples demonstrating how this technique can be applied to optimize resource allocation for various regions.

Keywords: hybrid energy, renewable energy, optimal allocation of resources, linear programming.

1 Introduction

The hybrid renewable energy systems are systems that combine two or more sources of renewable (RE) energy. When integrated, they can overcome the limitations inherent to the use of a single energy source. The hybrid technique applied to renewable energy reduces the storage means because of the complementarily of different resources. This leads to a reduction in the cost per kWh. In recent years, many studies and applications have been conducted on hybrid energy to optimize the integration and allocation of available resources to meet the demands required by the consumers. In this context, there are generally two types of issues to consider. The first problem is, given the resource capacity already installed, to determine the mode of management to make the best use of the facility.

The second problem is, given an estimate of potential resources to determine the optimal distribution for each of these resources. To this end, several methods have been used, such as Linear Programming,

Dynamic Programming, Genetic Algorithms, etc... [1-4]. Simulation programs and optimization of hybrid systems have also been developed, are in particular HOMER, SOMES, RAPSIM, SOSIM [5-8]. In this study, we will focus on the second problem for which little work has been done [1]. In reference [1], we modeled the problem of resource allocation in terms of linear programming. The considered resources can be considered as a combination of the following energy according to the specificity of the regions: solar photovoltaic panel, solar thermal, wind turbines and biodigesters. A graphical user interface was developed to allow users to introduce the unit cost and the capacity of each energy so easy to automatically get the distribution that minimizes the cost function.

2 An experimental hybrid system

The purpose of this study is to combine renewable energy sources to meet the energy demand of a given location (home, farm, urban...). (Figure1) shows an experimental prototype house that uses hybrid energy sources: wind, solar, and biomass (Figure1). The picture (a) shows a solar photovoltaic system; the image (b) shows a photo-thermal system, the image (c) shows a wind turbine, image (d) shows an experimental bioreactor for the production of methane using wastewater and organic waste. The management system is used to adapt the energy available to meet the demand. As renewable energies are fluctuating, it is customary to add storage batteries as a reserve.

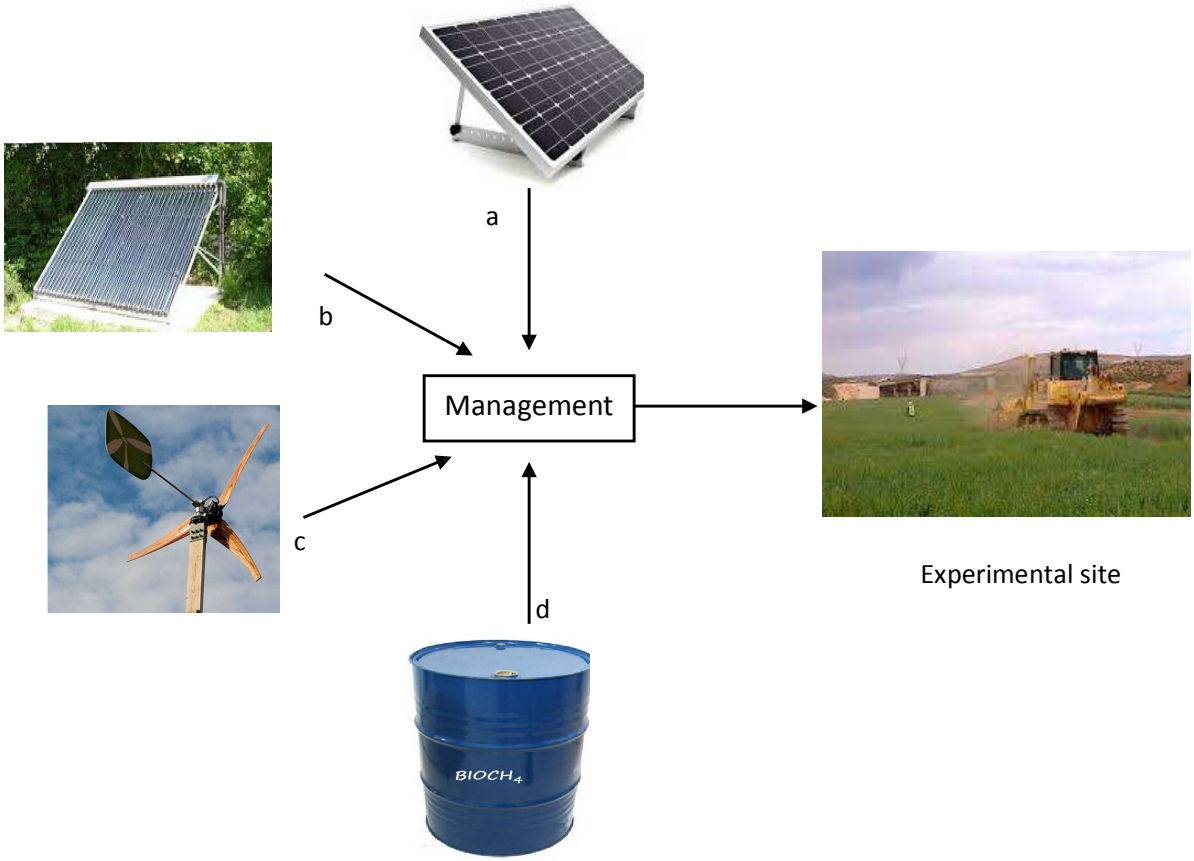

Figure. 1 Experimental hybrid renewable energy systems **(HRES)**

3 Modeling in terms of linear programming

Given the characteristics and the energy demand for a specific site, the problem under consideration is

to determine the capacity of renewable resources to be installed in order to meet the demand. Such an approach was also carried out by the reference [1] to solve a problem of rural electrification based on the integration and distribution of renewable resources in India. Hybrid problems can be reduced to those of optimal distribution of energy resources with the constraint of satisfying the demand taking into account the limitations of the available resources. We consider the following renewable energy sources: solar photovoltaic E_{ph} with the available limit S_{ph}, solar thermal E_{th} with the available limit S_{th}, the wind turbine E_{wi} with the available limit S_{wi} and biomass (biodigesters) E_{bi} with existing limit L_{bi}.We provide also the unit costs of each energy resource c_i as well as the demand D to be met. The general form may be that of a linear programming problem of minimizing a cost function Z_T under given constraints [1, 9].The problem can be expressed as follows

$$\left\{\begin{array}{l} \min Z_T = \sum C_i.E_i \\ \sum E_i = D \\ E_i \le L_i \\ E_i \ge 0 \end{array}\right\} \quad \text{(i = ph, th, wi, bi)} \tag{1}$$

The resolution of these problems can be done by techniques derived from the simplex or interior point methods. Many computer programs are available as (LINDO [9] LINSOLVE [10], Z-PL...). In our case, we used the LINSOLVE program to determine the optimal allocation of resources.

4 Applications

In this section, we illustrate our study by analyzing two examples of hybrid resources by considering two different regions. The cost function to be minimized is the sum of the cost of energy used. Taking into account the unit costs, it can be put in the form:

$$\min Z = C_{ph}E_{ph} + C_{th}E_{th} + C_{wi}E_{wi} + C_{Bi}\ E_{Bi} \tag{2}$$

Table 1. Distribution of resources obtained for the region (a and b).

	Region (a)	**Region (b)**
The general form	$\min Z = 0.3\,E_{ph} + 0.08\,E_{wi} + 0.2\,E_{th} + 0.1\,E_{bio}$ $E_{ph} + E_{wi} + E_{th} + E_{bio} = 400\ (kWh/an)$ $E_{ph} \le 110\ (kWh/an)$ $E_{wi} \le 140\ (kWh/an)$ $E_{th} \le 70\ (kWh/an)$ $E_{bio} \le 100\ (kWh/an)$	$\min Z_T = 0.3\,E_{ph} + 0.08\,E_{wi} + 0.2\,E_{th} + 0.1\,E_{bio}$ $E_{ph} + E_{wi} + E_{th} + E_{bio} = 400\ (kWh/an)$ $E_{ph} \le 80\ (kWh/an)$ $E_{wi} \le 100\ (kWh/an)$ $E_{th} \le 120\ (kWh/an)$ $E_{bio} \le 140\ (kWh/an)$

Results

Result in ( )		
Result in en%	$E_{ph} = 22.5$ $E_{wi} = 35$ $E_{th} = 17.5$ $E_{bio} = 25$	$E_{ph} = 10$ $E_{wi} = 25$ $E_{th} = 30$ $E_{bio} = 35$
Min Z (€/an)	62.2	58

Comments

* For the region (a), we obtain the following distribution; wind has the greatest capacity at the lowest cost 35%. Other resources were used at low percentage: 25% for biomass, 22.5% for photovoltaic, 17.5% for and thermal.

* For the region (b), we obtain the following distribution; biomass has the greatest capacity at the lowest cost 35%. Other resources were used at low percentage: 30% for thermal, 25% for wind and 10% for photovoltaic.

5 User interface

A graphical user interface (Figure 2) has been developed by means of Java software. It allows users to enter the unit cost, the capacity of each energy as well as the demand. The program outputs the optimal allocation of resources.

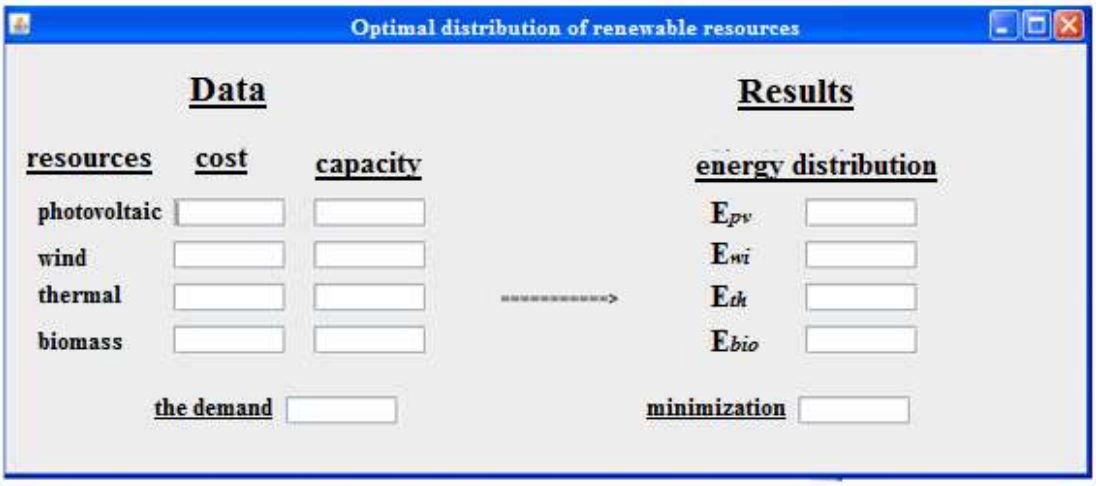

Figure 2. Graphical User Interface

6 Conclusion

After designing and implementing low-power experimental renewable energy systems (thermal solar photovoltaic, solar, wind, and biomass), we focused on optimizing the distribution of combined renewable energy resources (RE). To achieve this, we modeled the problem as a linear program, solved using the simplex method. Furthermore, we developed a graphical user interface to facilitate the simulation and analysis of hybrid renewable energy systems (RES). This approach is illustrated through two practical examples

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Evolution of nonconvex integro-differential inclusion

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Abstract

The paper analyzes and provides the existence of solutions to integro-differential sweeping processes involving perturbations with multimappings. In this variant, the non-convex sets are supposed to have only prox-regular sets.

In this paper aims to study the following integro-differential inclusion:

$$\begin{cases} -\dot{x}(t) \in N_{C(t)}(x(t)) + G(t, x(t)) + \int_{T_0}^{t} g(t, s, x(s))ds \text{ a.e. } t \in I := [T_0, T] \\ x(T_0) = x_0 \in C(T_0), \end{cases}$$

in a Hilbert space H, where $g : I \times I \times H \to H$ is given mapping and $C : I \rightrightarrows H$ and $G : I \times H \rightrightarrows H$ are given multimappings.

Keywords: Sweeping process, Voltera-type integral, proximal normal cone.
2020 Mathematics Subject Classification: Primary 34A60, 49J24, 49J52.

1 Introduction

This paper deals with the Cauchy problem for the time dependent evolution inclusion

$$\begin{cases} -\dot{x}(t) \in N_{C(t)}(x(t)) + G(t, x(t)) + \int_{T_0}^{t} g(t, s, x(s))ds \text{ a.e. } t \in I \\ x(T_0) = x_0 \in C(T_0), \end{cases} \tag{1.1}$$

where the state variable x belongs to H for each $t \in [T_0, T]$, and $x(0) = x_0$ is the initial condition, $C(t) \subset H$ is a (non-convex) Prox-regular moving set, G has nonempty, closed, convex values and is upper semicontinuous with respect to its second variable and g is given mapping.

This paper is inspired from the article of Gaouir-Haddad-Thibault [1] where the Problem (1.1) is called *integro-differential sweeping process*.

The present paper is devoted to the existence of solutions of integro-differential sweeping processes with the presence of a multimapping in the right hand-side.

Section 2: Subsection 2.1 and 2.2 recalls some hypotheses, basic concepts and results and Subsection 2.3 develops the analysis and the proof.

2 Main results

2.1 Hypotheses on the data

In this subsection we collect the basic hypotheses and additional results which will be utilized along the section. We keep H as a real Hilbert space and we consider the interval $I = [T_0, T]$.
Hypotheses on the multimapping $C : I \rightrightarrows H$:
C is a multimapping with nonempty closed values for which the following conditions will be involved :

(C_1) $C(t)$ varies in an absolutely continuous way, i.e. there exists an absolutely continuous function $\zeta : [T_0, T] \to \mathbb{R}$, which is monotone increasing and

$$d(y, C(t)) \leq d(y, C(s)) + |\zeta(t) - \zeta(s)| \text{ for all } y \in H \text{ and } s, t \in I,$$

that is, if the sets $C(t)$ are bounded

$$\text{haus}(C(t), C(s)) \leq |\zeta(t) - \zeta(s)| \text{ for all s, t} \in \text{I},$$

where haus$(\cdot, \cdot)$ denotes the classical Hausdorff-Pompeiu distance;

(C_2) there exists $r > 0$ such that $C(t)$ is r-prox-regular for each $t \in I$.

Hypotheses on the multimapping $G : [T_0, T] \times H \rightrightarrows H$:
G is a multimapping for which the following conditions will be considered :

$(\mathcal{H}_0^G)$ G takes nonempty closed and convex values;

$(\mathcal{H}_1^G)$ for every $x \in H$ the multimapping $G(\cdot, x)$ is Lebesgue measurable;

$(\mathcal{H}_2^G)$ for a.e. $t \in I$ the multimapping $G(t, \cdot)$ is upper semicontinuous;

$(\mathcal{H}_3^G)$ for some compact subset $K \subset \mathbb{B}$ and some nonnegative function $c \in L^1(I, \mathbb{R})$ one has for all $x \in \cup_{s \in I} C(s)$ the inclusion

$$G(t, x) \subset c(t)(1 + \|x\|)K.$$

Hypotheses on the mapping $g : I^2 \times H \to H$:
For the mapping g the following conditions will be crucial:

$(\mathcal{H}_0^g)$ $g(\cdot, \cdot, x)$ is Bochner measurable on $I \times I$ for each $x \in H$;

$(\mathcal{H}_1^g)$ there exists a non-negative function $\beta(\cdot, \cdot) \in L^1(Q_\Delta, \mathbb{R})$ such that

$$\|g(t, s, x)\| \leq \beta(t, s)(1 + \|x\|), \text{ for all } (t, s) \in Q_\Delta \text{ and } x \in H;$$

$(\mathcal{H}_2^g)$ for each real $\eta > 0$ there exists a non-negative function $L^\eta(\cdot)$ λ-integrable on I such that for all $(t, s) \in Q_\Delta$ and $(x, y) \in B[0, \eta] \times B[0, \eta]$,

$$\|g(t, s, x) - g(t, s, y)\| \leq L^\eta(t)\|x - y\|.$$

Above Q_Δ stands for the set

$$Q_\Delta := \{(t, s) \in I \times I : s \leq t\}.$$

2.2 basic concepts and results

Lemma 2.1. *Given the real Hilbert space H, let $h : J \to H$ be a mapping on an interval J with nonempty interior and $\Gamma : J \rightrightarrows H$ be a multimapping with nonempty closed values and which is locally absolutely continuous on J, that is, for some locally absolutely continuous function $\rho : J \to \mathbb{R}$*

$$d(y, \Gamma(t)) \leq d(y, \Gamma(s)) + |\rho(t) - \rho(s)| \quad \text{for all } s, t \in J.$$

Let $x : J \to H$ be a locally absolutely continuous mapping such that for almost every $t \in J$

$$\dot{x}(t) + h(t) \in -N_{\Gamma(t)}(x(t)),$$

where $N_{\Gamma(t)}(\cdot)$ is the proximal normal cone. Then, for almost every $t \in J$ one has

$$\|\dot{x}(t) + h(t)\| \leq |\dot{\rho}(t)| + \|h(t)\|.$$

Proposition 2.1. *With the real Hilbert space H assume that G satisfies $(\mathcal{H}_0^G)$, $(\mathcal{H}_1^G)$, $(\mathcal{H}_2^G)$ and $(\mathcal{H}_3^G)$ and C satisfies (C_1) and (C_2). Let $h : [T_0, T] \longrightarrow H$ be a mapping Bochner λ-integrable on $[T_0, T]$. Then for any $x_0 \in C(T_0)$ there exists at least an absolutely continuous solution $x(\cdot) : I \longrightarrow H$ of the following differential inclusion*

$$-\dot{x}(t) \in N_{C(t)}(x(t)) + G(t, x(t)) + h(t) \quad \text{a.e.} \quad t \in [T_0, T], \quad x(T_0) = x_0$$

and there exists a Bochner λ-integrable mapping $z(\cdot) : I \longrightarrow H$ with for a.e. $t \in I$

$$z(t) \in G(t, x(t)), \ \dot{x}(t) + z(t) + h(t) \in -N_{C(t)}(x(t))$$

such that the inequality

$$\left\|\dot{x}(t) + z(t) + h(t)\right\| \leq |\dot{\zeta}(t)| + \|z(t)\| + \|h(t)\|$$

holds for a.e. $t \in I$.

2.3 Main theorem

Theorem 2.1. *Let H be a real Hilbert space and assume that $(\mathcal{H}_0^G)$, $(\mathcal{H}_1^G)$, $(\mathcal{H}_2^G)$, $(\mathcal{H}_3^G)$, $(\mathcal{H}_0^g)$, $(\mathcal{H}_1^g)$, $(\mathcal{H}_2^g)$, (C_1), (C_2) are satisfied. Then for any initial point $x_0 \in H$, with $x_0 \in C(T_0)$ there exists at least an absolutely continuous solution $x : [T_0, T] \longrightarrow H$ of the differential inclusion (1.1) and there exists a Bochner λ-integrable mapping $z : [T_0, T] \to H$ with $z(t) \in G(t, x(t))$ for a.e. $t \in [T_0, T]$.*

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Model-Based FDI Using Adaptive gain High-Order Sliding Mode differentiator for PEM Fuel Cell Air Feed System

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Abstract
Oxygen starvation during rapid load variations is a critical issue in Proton Exchange Membrane (PEM) fuel cell (FC) operations. To mitigate this, precise regulation of the oxygen excess ratio is essential. Addressing this challenge, we propose a robust model based fault diagnosis approach using adaptive gain sliding mode differentiators for PEMFC air feed systems. This method relies on the algebraic manipulation of measured outputs and their derivatives,The objective is to synthesise an observer to determine the unknown quantities which is the fault signals based on the known quantities demonstrating high resilience to noise and uncertainties without needing prior knowledge of higher-order derivative bounds. By implementing this innovative approach, we significantly enhance the reliability and operational efficiency of PEMFC systems.

Keywords: Proton Exchange Membrane Fuel Cell (PEMFC), Fault Diagnosis, Adaptive Gain Sliding Mode Differentiators.

1 Introduction

PEMFC is a promising energy conversion technology due to its high efficiency and low environmental impact. However, one significant operational challenge is oxygen starvation, especially during fast load variations, which can lead to reduced performance and potential damage to the fuel cell. Ensuring an optimal oxygen excess ratio is crucial to prevent oxygen starvation and maintain efficient operation[**3**]. PEMFC systems are prone to various faults, which can impair their performance and reliability. Traditional fault diagnosis methods often require prior knowledge of system parameters and can be sensitive to noise and uncertainties.To address these challenges, a robust fault diagnosis approach based on adaptive gain sliding mode differentiators is proposed. This methodology leverages the algebraic manipulation of measured outputs and their derivatives.

2 PEMFC Modeling

PEMFCs are a complex non-linear systems, necessitating Sophisticated modeling for comprehensive,understanding, optimization, and monitoring of their performance. A fourth-order state-space model under certain assumptions which represents a simplified version of the ninth-order model[**1**,**2**]

is proposed in this work, and we will take the dynamics of the faulty states (the supply manifold air pressure), expressed as follows:

$$\dot{x} = \alpha(y,t); \tag{2.1}$$

$$y = ax(t); \tag{2.2}$$

$$\dot{x} = \left(\sigma_6 \left(1 + \sigma_7 \left[\left(\frac{y}{\sigma_4}\right)^{\sigma_5} - 1 \right] \right) (W_{cp} - \sigma_8 (y - P_{ca})) \right) \tag{2.3}$$

The constants σ_i, where i ranges from 1 to 10, represent the parameters of the model and will be illustrated in the Poster. The system is characterized by its outputs $y \in \mathbb{R}^2$,defined as: $y = \begin{bmatrix} y_1 & y_2 \end{bmatrix} = \begin{bmatrix} P_{sm} & W_{cp} \end{bmatrix}^T$

2.1 Integration of faults into the PEMFC air feed System

The fault, denoted as $f(t)$, involves an increment, $\Delta\sigma_8$, in the supply manifold's outlet flow constant, $\sigma_8 := k_{sm,out}$. This adjustment models a change in the outlet airflow within the supply manifold, effectively simulating as a sudden air leak[**4**].

$$W_{ca,in} = (\sigma_8 + \Delta\sigma_8)(y_2 - P_{ca}) \tag{2.4}$$

The f_2 signal is given as:

$$f_2 = \Delta\sigma_8 = \begin{cases} k_{sm,out} \cdot 0.5, & if \quad t \in [4,10] \\ k_{sm,out} \cdot 1, & if \quad t \in [10,20] \\ 0, & else \end{cases} \tag{2.5}$$

After integrating faults, the system equation undergoes a modification to encompass their effects. and presenting a revised expression:

$$\dot{x} = \alpha(y,t) + \beta(y,t)f; \tag{2.6}$$

$$y = ax(t), \tag{2.7}$$

Where

$$\beta = \begin{cases} -\sigma_1 y \\ -\sigma_6(W_{cp} - p_{ca}) \end{cases} \tag{2.8}$$

3 Methodology and results

The objective is to synthesise an observer to determine the unknown quantities f based on the known quantities α,β and the output y, starting with the first and second derivatives of y:

$$\dot{y} = a\,(\alpha + \beta f) \tag{3.1}$$

$$\ddot{y} = a\left(\dot{\alpha} + \dot{\beta} f\right) \tag{3.2}$$

Executing the division of $\dot{y}$ by $\ddot{y}$ and subsequently engaging in algebraic manipulation, the equations are meticulously rearranged. This rearrangement aims to methodically isolate and emphasize the terms involving the unknown quantities f.

$$\frac{\ddot{y}}{\dot{y}} = \frac{\dot{\alpha} + \dot{\beta} f}{\alpha + \beta f} \tag{3.3}$$

With

$$\dot{y} \neq 0 \tag{3.4}$$

Identifying the faults:

$$\hat{f} = \frac{\ddot{y}\alpha - \dot{y}\dot{\alpha}}{\dot{y}\dot{\beta} - \ddot{y}\beta} \tag{3.5}$$

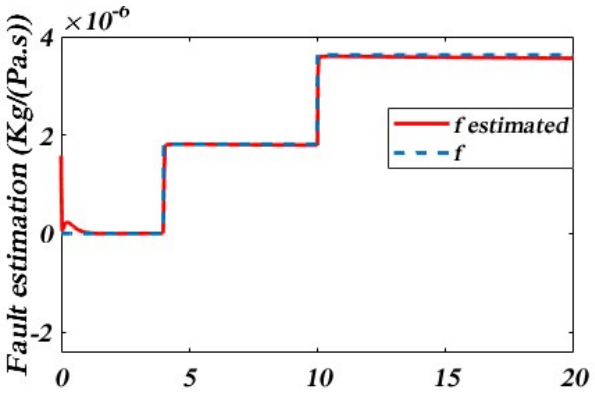

Figure 1: Fault f(t) estimation.

we can see in (3.5) the fault is represented as a function of the first and second derivatives of the system output y and the first derivatives of α and β. These derivatives are estimated using an adaptive robust high-order sliding mode differentiation technique follows from[**3**]. This approach is distinguished by its capability to pinpoint and isolate faults with precision. To robustly validate the efficacy of our methodology, we developed a PEMFC model within the MATLAB/Simulink environment, meticulously incorporating fault scenario of a sudden air leak. Figure 1 provides a comprehensive visualization of the faithful reconstruction of fault signal, affirming the robustness and effectiveness of our methodology in fault detection and isolation.

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Impulsive Fractional Stochastic Differential Equations with Delay

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Abstract
We concentrate our interest to investigate a fractional stochastic differential equation with delay and non-instantaneous impulses involving the Riemann-Liouville derivative of order $\alpha \in (1/2, 1)$ with a fixed lower bound. In our main results, the integral representation of the mild solution is presented, and the existence of unique solution are established using Banach's fixed point theorem with some appropriate assumptions. We provide an example to illustrate the condition of contraction.

Keywords: Riemann-Liouville derivative, Non-instantaneous impulses, Fractional differential equations with delay, Stochastic calculus, Banach fixed point theorem.
2020 Mathematics Subject Classification: 35A01, 34A37, 60H15, 60H20.

1 Introduction

Many different real-world processes and phenomena are modeled using delay differential equations, where the derivative of the unknown variable at a current time is expressed in terms of the variable or its derivatives at previous times, such that the delays may be constants, functions of time (time-dependent delays), or functions of variables (state-dependent delays). Moreover, many processes experience several abrupt changes in their state at discrete points, known as impulses, which can be instantaneous or non-instantaneous. It is necessary to model these processes by impulsive differential equations. The existence of the stochastic term (a random term) is important due to the possibility of unpredictability in the characteristics of natural systems. For this reason, stochastic differential equations are considered important tools in modeling and simulating real phenomena and have been applied in a variety of fields. During the last decades, some papers have been presented on fractional stochastic differential equations, especially with Riemann-Liouville derivatives (see, for example, [11, 8]). Therefore, and based on the above facts, we propose in this paper the following fractional stochastic differential equation with delay and non-instantaneous impulses:

$$
\begin{aligned}
&{}^{R}_{s_0}D^{\alpha}_{t}u(t) = f(t,u(t),u(t-\tau)) + g(t,u(t),u(t-\tau))\frac{dw(t)}{dt},\ t\in(s_j,t_{j+1}],\ j=0,1,\dots,m, && (1.1)\\
&u(t) = \Psi_j(t,u(t_j^-)),\quad t\in(t_j,s_j],\quad j=1,2,\dots,m, && (1.2)\\
&{}_{s_j}I_t^{1-\alpha}u(t)|_{t=s_j} = \Psi_j(s_j,u(t_j^-)),\quad j=1,2,\dots,m, && (1.3)\\
&{}_{0}I_t^{1-\alpha}u(t)|_{t=0} = \varphi(0),\quad \text{and}\quad u(t)=\varphi(t),\quad t\in[-\tau,0]. && (1.4)
\end{aligned}
$$

where ${}^{R}_{s_0}D^{\alpha}_{t}$ denotes the Riemann-Liouville derivative of order $\frac{1}{2}<\alpha<1$ with a fixed lower bound $s_0=0$, $t\in J=(s_0,b]$; the non-negative points $\{s_j\}_{j=0}^m$ and $\{t_{j+1}\}_{j=0}^m$ are given such that $0=s_0<t_1<s_1<t_2<\cdots<s_m<t_{m+1}=b$. The unknown $u(\cdot)$ is a random variable takes its values in a real separable Hilbert space $\mathcal{H}$. The values $u(t_j)=u(t_j^-)=\lim_{\epsilon\to0}u(t_j-\epsilon)$ and $u(t_j^+)=\lim_{\epsilon\to0}u(t_j+\epsilon)$ denote the left and right hand limits of $u(t)$ at t_j for $j=1,2,\dots,m$, respectively. The process jumps to the states $u(t_j^+)=\lim_{\epsilon\to0}u(t_j+\epsilon)$ at the points t_j where the impulsive functions Ψ_j begin and continue along the interval $(t_j,s_j]$ for $j=1,2,\dots,m$. $f:\cup_{j=0}^m[s_j,t_{j+1}]\times\mathcal{H}\times\mathcal{H}\to\mathcal{H}$, $g:\cup_{j=0}^m[s_j,t_{j+1}]\times\mathcal{H}\times\mathcal{H}\to L(\mathcal{K},\mathcal{H})$, $\Psi_j:[t_j,s_j]\times\mathcal{H}\to\mathcal{H}$ for $j=1,2,...,m$, and $\varphi:[-\tau,0]\to\mathcal{H}$ are appropriate functions. $L(\mathcal{K},\mathcal{H})$ is the space of all bounded linear operators from $\mathcal{K}$ to $\mathcal{H}$, where $\mathcal{K}$ is a real separable Hilbert space. Suppose that $\{w(t)\}_{t\geq0}$ is a $\mathcal{K}$-valued Brownian motion (or Wiener process) with a finite trace nuclear operator $Q\geq0$ define on a complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq0},P)$.

2 Main results

In the following, we will give the integral representation of the mild solution to the system (1.1)-(1.4).

Lemma 2.1. *The stochastic process $u\in\mathcal{PC}_{1-\alpha}(J,\mathcal{H})$ is a mild solution to (1.1)-(1.4) if the following integral equation is satisfied:*

$$
u(t)=\begin{cases}
\dfrac{(t-s_j)^{\alpha-1}}{\Gamma(\alpha)}\Psi_j(s_j,u(t_j^-)) + \dfrac{1}{\Gamma(\alpha)}\displaystyle\int_{s_j}^{t}\frac{f(s,u(s),u(s-\tau))}{(t-s)^{1-\alpha}}ds\\
\quad+\dfrac{1}{\Gamma(\alpha)}\displaystyle\int_{s_j}^{t}\frac{g(s,u(s),u(s-\tau))}{(t-s)^{1-\alpha}}dw(s)\\
\quad+\dfrac{1}{\Gamma(\alpha)}\displaystyle\int_{s_j}^{t}\frac{\Phi_j(s,u(s))}{(t-s)^{1-\alpha}}ds,\quad t\in(s_j,t_{j+1}],\ j=0,1,\dots,m\\
\Psi_j(t,u(t_j^-)),\quad t\in(t_j,s_j],\ j=1,2,\dots,m.
\end{cases} \qquad (2.1)
$$

where $\Psi_0=\varphi(0)$, $\Phi_0=0$ and for $j=1,\dots,m$,

$$
\begin{aligned}
\Phi_j(t,u(t)) = \frac{\alpha}{\Gamma(1-\alpha)}\sum_{k=0}^{j-1}\Big(&\int_{s_k}^{t_{k+1}}(t-s)^{-\alpha-1}u(s)ds\\
&+\int_{t_{k+1}}^{s_{k+1}}(t-s)^{-\alpha-1}\Psi_{k+1}(s,u(t_{k+1}^-))ds\Big). \qquad (2.2)
\end{aligned}
$$

Before state our main existence result of unique solution, we introduce some hypotheses for the functions f, g and Ψ_j as follows:
(H1) The function $f : \cup_{j=0}^{m}[s_j, t_{j+1}] \times \mathcal{H} \times \mathcal{H} \to \mathcal{H}$ satisfies the following condition:
There exist two constants $L_f, M_f > 0$, such that for all $t \in [s_j, t_{j+1}]$, $j = 0, 1, \ldots, m$ and $x_1, x_2, y_1, y_2 \in \mathcal{H}$, we have

$$\mathbb{E}\left\|f(t, x_1, y_1) - f(t, x_2, y_2)\right\|^2 \leq L_f \mathbb{E}\left\|x_1 - x_2\right\|^2 + M_f \mathbb{E}\left\|y_1 - y_2\right\|^2.$$

(H2) The function $g : \cup_{j=0}^{m}[s_j, t_{j+1}] \times \mathcal{H} \times \mathcal{H} \to L(\mathcal{K}, \mathcal{H})$ satisfies the following condition:
There exist two constants $L_g, M_g > 0$, such that for all $t \in [s_j, t_{j+1}]$, $j = 0, 1, \ldots, m$ and $x_1, x_2, y_1, y_2 \in \mathcal{H}$, we have

$$\mathbb{E}\left\|g(t, x_1, y_1) - g(t, x_2, y_2)\right\|^2 \leq L_g \mathbb{E}\left\|x_1 - x_2\right\|^2 + M_g \mathbb{E}\left\|y_1 - y_2\right\|^2.$$

(H3) For $j = 1, 2, \ldots, m$, the impulsive functions $\Psi_j : [t_j, s_j] \times \mathcal{H} \to \mathcal{H}$ are continuous and satisfy the following condition:

$$\mathbb{E}\left\|\Psi_j(t, x) - \Psi_j(t, y)\right\|^2 \leq \lambda_{\Psi_j} \mathbb{E}\left\|x - y\right\|^2, \quad \forall x, y \in \mathcal{H}, \quad \text{with} \quad \lambda_{\Psi_j} \in (0, 1].$$

(H4) For $j = 1, 2, \ldots, m$, the impulsive functions $\Psi_j : [t_j, s_j] \times \mathcal{H} \to \mathcal{H}$ are of order $(s_j - t)^{\alpha+1}$, i.e.,

$$\Psi_j(t, x) = (s_j - t)^{\alpha+1} \psi_j(t, x), \quad \forall x \in \mathcal{H},$$

where the functions ψ_j satisfy $\mathbb{E}\left\|\psi_j(t, x) - \psi_j(t, y)\right\|^2 \leq \lambda_{\psi_j} \mathbb{E}\left\|x - y\right\|^2$, $\forall x, y \in \mathcal{H}$, with $\lambda_{\psi_j} \in (0, 1]$.

Theorem 2.1. *Assume that conditions (H1)-(H4) hold. Then (1.1)-(1.4) has a unique solution $u \in \mathcal{PC}_{1-\alpha}([-\tau, b], \mathcal{H})$ if the following condition holds:*

$$\begin{aligned}
\eta \;=\; & \max_{j=1,\ldots,m} \Big(\frac{16\lambda_{\Psi_j}(t_j - s_{j-1})^{2(\alpha-1)}}{\Gamma^2(\alpha)} + \frac{16L_f\Gamma(2\alpha-1)(t_{j+1} - s_j)^{2\alpha}}{\Gamma(\alpha+1)\Gamma(3\alpha-1)} \\
& + \frac{16M_f(s_j - \tau)^{2(\alpha-1)}(t_{j+1} - s_j)^2}{\Gamma^2(\alpha+1)} + \frac{16L_g\beta(2\alpha-1, 2\alpha-1)(t_{j+1} - s_j)^{2\alpha-1}}{\Gamma^2(\alpha)} \\
& + \frac{16M_g(s_j - \tau)^{2(\alpha-1)}(t_{j+1} - s_j)}{(2\alpha-1)\Gamma^2(\alpha)} \\
& + \frac{16 \times 2^j}{\Gamma^2(\alpha)\Gamma^2(1-\alpha)} \Big(\sum_{k=0}^{j-1} \frac{(t_{j+1} - s_j)^2}{(2\alpha-1)(s_j - t_{k+1})^2} \\
& + \sum_{k=0}^{j-2} \frac{\lambda_{\Psi_{k+1}}(t_{j+1} - s_j)^2(t_{k+1} - s_k)^{2(\alpha-1)}}{\alpha^2(s_j - s_{k+1})^{2\alpha}} + \frac{\lambda_{\psi_j}(s_j - t_j)^2(t_{j+1} - s_j)^2}{(t_j - s_{j-1})^{2(1-\alpha)}} \Big) \Big) < 1. \qquad (2.3)
\end{aligned}$$

Here, we give some references that motivated our study:

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A class of viscoelastic wave equation: Global existence and finite time blow up

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Abstract
A new class of nonlinear viscoelastic wave equation is studied. Under appropriate conditions imposed on h, the global existence of solutions with any initial data is proved when $m \geq p$, and a finite time blow−up with negative initial energy is obtained when $p > m$.

Keywords: Global existence, Blow-up, Source term, Wave equation, Viscosity
2020 Mathematics Subject Classification: Primary 35R11; 35A01; 34A08; 35C06; 34K37.

1 Introduction

In this section, we are in the process of studying the following non-linear viscoelastic wave equation:

$$\begin{cases} u_{tt} - \Delta u - \Delta u_{tt} + \int_0^t h(t-s)\Delta u(s)ds + cu_t \left|u_t\right|^{m-2} = du \left|u\right|^{p-2}, x \in \Omega, t > 0 \\ u(x,t) = 0, & x \in \partial\Omega,\ t \geq 0 \\ u(x,0) = u_0(x), u_t(x,0) = u_1(x), & x \in \Omega, \end{cases}$$

here Ω be an open bounded Lipschitz domain in $\mathbb{R}^n$ $(n \geq 1)$, with a Lipschitz-countinuous boundary $\partial\Omega$, $p > 2, m \geq 1$, and c, d are strictly positive constants. Our chapter is divided as follows:
- In the first section, we present some assumptions needed in our chapter.
- In the second section, we show that solutions with arbitrary data continue to exist globally if $m \geq p$.
- In the third section, we prove a finite time blow-up for solutions with negative initial energy if $m < p$.
We study in this work the interaction between the damping and source terms in the presence of the viscoelastic and dispersion terms when $c = d = 1$. Our first intent is to itemize an appropriate domain for the parameters m, p, where the damping term dominates over the source and the global solution exists for any initial data. Secondly, we define another domain, where the blowup of the solution occurs for a finite time because the influence of the source is stronger.

2 Global Existence Result

We clarify in this section that the solution is global if the exponent $m \geq p$

Theorem 2.1. *Let $E_0 < 0, 2 \leq p \leq m$ and let the condition*

$$m \leq \frac{2\,(n-1)}{n-2}, \quad n \geq 3,$$

hold. Then our problem admits a unique global solution

$$u \in C\left([0,\infty), H_0^1(\Omega)\right), \quad u_t \in C\left([0,\infty), H_0^1(\Omega)\right) \cap L^{m+1}\left((\Omega) \times (0,\infty)\right),$$

for any

$$u_0 \in H_0^1(\Omega), u_1 \in L^2(\Omega).$$

3 Finite-Time Blow-Up

In order to carry the proof of our result, we need the following:

Theorem 3.1. *Let $m > 1, p > max\{2, m\}$. Assume that*

$$E_0 = \frac{1}{2}\|u_1\|_2^2 + \frac{1}{2}\|\nabla u_0\|_2^2 + \frac{1}{2}\|\nabla u_1\|_2^2 - \frac{d}{p}\|u_0\|_p^p < 0.$$

Then there exist a finite time T^* such that

$$T^* \leq \frac{1-\alpha}{\Gamma\alpha[L(0)]^{\alpha/(1-\alpha)}},$$

where Γ, α $(\alpha < 1)$ are positive constant and L is given below.

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Risk-sensitive Stochastic Variational Inequality via Malliavin Calculus

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Abstract
In this work, we study the risk-sensitive optimal control problem for a backward stochastic system driven by a fractional Brownian motion with Hurst parameter H greater than one-half. We make the Malliavin derivative interfere to establish the risk-sensitive necessary optimality conditions, when the control domain is assumed convex and the driver of the system depends on the control variable.

Keywords: Backward Stochastic Differential Equation, Fractional Brownian motion, Malliavin derivative.
2020 Mathematics Subject Classification: Primary 60G22, 60H07, 60H30.

1 Introduction

In this paper, we intend to establish necessary optimality conditions of Pontryagin's stochastic maximum principle, for risk-sensitive performance functionals, for a system governed by a backward stochastic differential equation (BSDEs in short) driven by a fractional Brownian motion (FBM in short) with Hurst parameter $H \in (\frac{1}{2}, 1)$.

2 Problem formulation

We consider the following controlled fractional BSDE

$$\begin{cases} dy_t^v &= -g(t, \eta(t), y_t^v, z_t^v, v_t)dt + z_t^v dW_t^H \\ y_T^v &= \xi. \end{cases} \tag{2.1}$$

We denote by $\mathcal{U}$ the set of all admissible controls, which we suppose convex and define the criterion to be minimized, with initial risk-sensitive cost, as follows

$$J^\theta(v) = \mathbb{E}\left[exp\theta\left(\psi\left[y^v(0)\right] + \int_0^T f\left(t, \eta(t), y_t^v, z_t^v, v_t\right) dt\right)\right]. \tag{2.2}$$

We seek an admissible control $u \in \mathcal{U}$ such that

$$J^\theta(u) = \inf_{v \in \mathcal{U}} J^\theta(v). \tag{2.3}$$

3 Risk-Sensitive Necessary Optimality Conditions

The main result of this paper is given in the following theorem.

Theorem 3.1. *If (y, z, u) is the optimal solution to the risk-sensitive control problem $\{(2.1), (2.2), (2.3)\}$, there exist pairs of $\mathcal{F}_T^H$-adapted processes $\left(V^\theta, l\right)$ and (p, q), such that*

$$(v_t - u_t)\widetilde{H}_v^\theta(t) \geq 0, \tag{3.1}$$

for all $v \in \mathcal{U}$, almost every $0 \leq t \leq T$ and $\mathbb{P}$-almost surely, where

$$\widetilde{H}^\theta(t, y_t^u, z_t^u, \overrightarrow{p}, \overrightarrow{q}, u_t) = \theta V^\theta(t) H\left(t, \gamma_t^u, y_t^u, z_t^u, \begin{pmatrix} \widetilde{p}_2(t) \\ \widetilde{q}_2(t) \end{pmatrix}, V^\theta(t), l_t, u_t\right).$$

4 Conclusion

This paper presents a solution to stochastic optimization problem for a BSDE driven by a fractional Brownian motion with Hurst parameter $H \in (\frac{1}{2}, 1)$, an exponential utility cost functional is used.

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Carleman estimates: Application to controllability problems

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Abstract In this work, we represent one of the applications of Carleman estimates to controllability problems . Where we use it to achieve the null controllability of linear heat equation with mixed boundary conditons.

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Keywords: Carleman estimates,parabolic equations,null controllability, weight functions
2020 Mathematics Subject Classification: 93B05,93C20.

1 Introduction

Overall, Carleman estimates provide powerful tools for analyzing complex mathematical systems and have numerous applications across areas such as photography.

Control theory is a branch of engineering and mathematics that deals with the analysis and design of controllable systems to achieve the required behavior. This theory is concerned with creating systems capable of maintaining or changing their condition based on feedback, and involves using mathematical models to describe system behaviour and developing control algorithms to handle system input in order to achieve the desired output. The goal of control theory is to develop techniques and tools that can be used to design systems capable of achieving specific performance goals, such as stability, response, and accuracy, control theory uses all disciplines.

An example of steering a vehicle, driving an aircraft or satellite to a geostationary orbit, improving the flow of information into a network, coding and decoding a digital image or SMS, regulating the thermostat, refining oil, controlling the pH of chemical reactions or optimising profits from stock market flows... Control can also reduce pain and prolong life. For example, a blood pressure regulator is designed to maintain this pressure at a constant and appropriate level; we can also control an epidemic such as studying brain tumor treatment or performing laser surgery. Plus robots. It is also used in areas such as economics, biology, and social sciences to model and control complex systems.

2 Main results

Let $\Omega \subset \mathbb{R}^n$, $n \geq 1$ be a bounded spatial domain with smooth boundary Γ of class C^2 and let $T > 0$. We denote $Q = \Omega \times]0,\ T[$, and $\Sigma = \Gamma \times]0,\ T[$ where Σ_1 is a piece of the boundary Σ and $\Sigma_2 = \Sigma \backslash \Sigma_1$. Consider the following parabolic equation:

$$\begin{cases} \partial_t u - \Delta u + a_0 u = h_0 \chi_{\mathcal{O}} + w\chi_\omega & \text{in } Q, \\ u = 0 & \text{on } \Sigma_1, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \Sigma_2, \\ u(x,0) = 0 & \text{in } \Omega, \end{cases} \tag{2.1}$$

with $a_0 \in L^\infty(Q)$, $\mathcal{O} \subset \Omega$ be some non-empty open subset, is called an observatory set, and ω be an open and non-empty subset of Ω.
Where $h \in L^2(Q)$, and whereχ_O and χ_ω denote the characteristic functions of $\mathcal{O}$ and ω respectively. Then problem (2.1)admits a unique solution u that satisfies $u \in C([0,\ T];\ L^2(\Omega)) \cap L^2(0,\ T;\ H_0^1(\Omega))$.

Theorem 2.1. *Let $u \in \mathcal{V}$ say defined in* (3.3)*, then there exists a positive constant $C = C(\Omega; \omega; O; T; a_0)$ such that*

$$\int_Q \frac{1}{\rho^2} |u|^2 \, dxdt \leq C \left[\int_Q |Lu|^2 \, dxdt + \int_0^T \int_\omega |u|^2 \, dxdt \right], \tag{2.2}$$

where $\rho \in C^2(Q)$ positive with $\frac{1}{\rho}$ bounded.
This inequality is called observability inequality resulting from Carleman simplification estimates (see[6]) .
Carleman simplification estimates. We will provide a little glimpse of how to simplify before launching into the previous theorem proof.
The first step is to set $L = \partial_t - \Delta + a_0 I$, and its formal adjoint $L^* = -\partial_t - \Delta + a_0 I$, where I is the identity operator.
And this space within which we will work is as follows:

$$\mathcal{V} = \left\{ \upsilon \in C^\infty(Q) \text{ such that : } \upsilon|_{\Sigma_1} = \left.\frac{\partial \upsilon}{\partial t}\right|_{\Sigma_1} = 0 \text{ and } \left.\frac{\partial \upsilon}{\partial \nu}\right|_{\Sigma_2} = 0 \right\}. \tag{2.3}$$

The second step is to take [5] in the following form, which represents the Carleman estimate

$$\begin{aligned} & 2s^3\lambda^4 \int_Q \varphi^3 e^{-2s\eta} |u|^2 \, dxdt + 4s^2\lambda \int_{\Sigma_2} \varphi \frac{\partial \eta}{\partial t} \frac{\partial \psi}{\partial \nu} e^{-2s\eta} |u|^2 \, d\gamma dt \\ & -4s^3\lambda^3 \int_{\Sigma_2} \varphi^3 |\nabla \psi|^2 \frac{\partial \psi}{\partial \nu} e^{-2s\eta} |u|^2 \, d\gamma dt - 4s^2\lambda^3 \int_{\Sigma_2} \varphi^2 |\nabla \psi|^2 \frac{\partial \psi}{\partial \nu} e^{-2s\eta} |u|^2 \, d\gamma dt \\ & -2s\lambda \int_{\Sigma_2} \varphi \frac{\partial \psi}{\partial \nu} e^{-2s\eta} \partial_t u \; u d\gamma dt - 4s\lambda \int_{\Sigma_1} \varphi \nabla \psi e^{-2s\eta} \nabla u \frac{\partial u}{\partial \nu} d\gamma dt \\ & +2s\lambda \int_{\Sigma} \varphi \frac{\partial \psi}{\partial \nu} e^{-2s\eta} |\nabla u|^2 \, d\gamma dt \\ \leq \; & C \left(\int_Q e^{-2s\eta} |\partial_t u - \Delta u|^2 \, dxdt + s^3\lambda^4 \int_0^T \int_\omega \varphi^3 e^{-2s\eta} |u|^2 \, dxdt \right). \end{aligned} \tag{2.4}$$

In the third step, we will prove the theorem (2.1).
At the beginning of the work, we start by finding the adjoint equation of the equation in the system (2.1) by $q(x,t)$ we find this equation now

$$\begin{cases} -\partial_t q - \Delta q + a_0 q = h_{\mathcal{O}}\chi_{\mathcal{O}} + w\chi_\omega & \text{in } Q, \\ q = 0 & \text{on } \Sigma_1, \\ \frac{\partial q}{\partial \nu} = 0 & \text{on } \Sigma_2, \\ q\,(x,0) = 0 & \text{in } \Omega, \end{cases} \tag{2.5}$$

or

$$L^* q = h_O\chi_O + w\chi_\omega \qquad \text{in } Q. \tag{2.6}$$

In our case, we write the weak formulation of the following equation (2.7), by multiplying the previous equation in the function $\upsilon \in \mathcal{V}$, and integrating on Q, we obtain

$$\int_0^T \int_\Omega L^* q \cdot \upsilon \; dxdt = \int_0^T \int_\Omega (h_O\chi_O + w\chi_\omega)\, \upsilon \; dxdt \qquad \text{in } Q. \tag{2.7}$$

This is all for proof of the controllability property by condition

$$q\,(x,\, T) = 0 \quad , \text{in } \omega. \tag{2.8}$$

Using integration by parts and the Green formula, we have

$$\int_0^T \int_\Omega (-\partial_t q - \Delta q + a_0 q\ )\, \upsilon \; dxdt = \int_0^T \int_\Omega h_O\chi_O \upsilon \; dxdt + \int_0^T \int_\omega w\chi_\omega \upsilon \; dxdt$$

$$\int_0^T \int_\Omega -\partial_t q \; \upsilon \; dxdt - \int_0^T \int_\Omega \Delta q \; \upsilon \; dxdt + \int_0^T \int_\Omega a_0 q \; \upsilon \; dxdt = \int_0^T \int_\Omega h_O\chi_O \upsilon \; dxdt + \int_0^T \int_\omega w\upsilon \; dxdt$$

$$\int_0^T \int_\Omega q \; \partial_t \upsilon \; dxdt - \int_0^T \int_\Omega q \; \Delta \upsilon \; dxdt + \int_0^T \int_\Omega a_0 q \; \upsilon \; dxdt - \int_\Omega q\,(x,\ T)\, \upsilon\,(x,\ T)\, dxdt$$

$$= \int_0^T \int_\Omega h_O\chi_O \upsilon \; dxdt + \int_0^T \int_\omega w\upsilon \; dxdt.$$

Then

$$\int_0^T \int_\Omega q\,(\partial_t \upsilon - \ \Delta \upsilon \ + a_0 \upsilon\ )\, dxdt - \int_\Omega q\,(x,\ T)\, \upsilon\,(x,\ T)\, dxdt = \int_0^T \int_\Omega h_O\chi_O \upsilon \; dxdt + \int_0^T \int_\omega w\upsilon \; dxdt$$

$$\int_0^T \int_\Omega q\,(\partial_t - \ \Delta \ + a_0\ )\, \upsilon \; dxdt - \int_\Omega q\,(x,\ T)\, \upsilon\,(x,\ T)\, dxdt \ = \ \int_0^T \int_\Omega h_O\chi_O \upsilon \; dxdt + \int_0^T \int_\omega w\upsilon \; dxdt$$

$$\int_0^T \int_\Omega qL\upsilon \; dxdt - \int_\Omega q\,(x,\ T)\, \upsilon\,(x,\ T)\, dxdt \ = \ \int_0^T \int_\Omega h_O\chi_O \upsilon \; dxdt + \int_0^T \int_\omega w\upsilon \; dxdt.$$

We assume that $q = Lu$, and $w = -u\chi_\omega$ with $u \in \mathcal{V}$, we obtain

$$\int_0^T \int_\Omega LuL\upsilon \; dxdt + \int_0^T \int_\omega u\chi_\omega \upsilon \; dxdt - \int_\Omega q\,(x,\ T)\, \upsilon\,(x,\ T)\, dxdt = \int_0^T \int_\Omega h_O \upsilon \; dxdt.$$

We further set

$$a(u,\ v) = \int_0^T \int_\Omega LuLv\ dxdt + \int_0^T \int_\omega u\chi_\omega v\ dxdt,$$

and

$$l(v) = \int_0^T \int_\Omega h_O \chi_O v\ dxdt.$$

Our null controllability problem becomes

$$a(u,\ v) - \int_\Omega q(x,\ T)\, v(x,\ T)\, dxdt = l(v) \qquad \text{for all } v \text{ in } V. \tag{2.9}$$

We consider the following subspace V to be a Hilbert space for the scalar product $a(u;\ v)$ and the associated norm

$$v \longrightarrow \|v\|_V = \sqrt{a(v;v)},$$

and V be the completion of $\mathcal{V}$.

Remark 2.1. We can precise the structure of the elements of V, let $L^2_\rho(Q)$ be the weighted Hilbert space defined by

$$L^2_\rho(Q) = \left\{ v \in L^2(Q) \text{ such that } \int_Q \frac{1}{\rho^2} |v|^2\, dxdt \right\},$$

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Well-posedness of nonconvex sweeping processes

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Abstract
By using a regularization method, we study in this paper the well-posedness in the sense of existence and uniqueness of a solution of a new variant of non-convex sweeping processes. This problem is perturbed by combining a multivalued maximal monotone operator and an integral forcing term that depends on two time variables.

Keywords: Sweeping process, non-convex sets, maximal monotone operator, normal cone.
2020 Mathematics Subject Classification: 47J22, 49J40, 34G25.

1 Introduction

The general class of differential inclusion known as the sweeping process has been introduced and thoroughly studied by J.J. Moreau in a series of papers. In this work, we are interested in a variant of sweeping processes associated with maximal monotone operators of following form

$$\begin{cases} -\dot{x}(t) \in N_{C(t)}(x(t)) + Bx(t) + \int_0^t g(t,s,x(s))ds, \qquad \text{a.e. } t \in [0,T] \\ x(0) = x_0 \in C(0), \end{cases} \tag{1.1}$$

where $B : H \rightrightarrows H$ is a set-valued maximal monotone operator and $N_{C(t)}$ denotes the proximal normal cone operator of the closed, prox-regular set $C(t)$ in Hilbert space H. The aim of this work is to show that (1.1) has a unique solution. We achieve this through the regularization technique. Namely, we approach the differential inclusion with a penalized one, depending on a parameter whose existence is easier to establish, and then study the limit when the parameter goes to zero.

2 Main results

First, let us make the following assumptions:

($\mathcal{H}_1$) The set-valued mapping $C : [0,T] \rightrightarrows H$ has non-empty, closed and r-prox-regular values for some constant $r \in]0, +\infty]$, and there exists an absolutely continuous function $v(\cdot) : [0,T] \to \mathbb{R}$ such that

$$C(t) \subset C(s) + |v(t) - v(s)|\mathbb{B}, \quad \forall t, s \in [0,T].$$

($\mathcal{H}_2$) The set-valued mapping $B : H \rightrightarrows H$ is a maximal monotone operator.

($\mathcal{H}_3$) $g : Q_\Delta \times H \longrightarrow H$ is a measurable mapping such that, There is a non-negative function $\beta(\cdot,\cdot) \in L^1(Q_\Delta, \mathbb{R}_+)$ satisfies:

$$\|g(t,s,x)\| \leq \beta(t,s)(1 + \|x\|), \text{ for all } (t,s) \in Q_\Delta \text{ and for each } x \in \mathcal{R}(C(t)),$$

where

$$Q_\Delta := \{(t,s) \in [0,T] \times [0,T] : s \leq t\}.$$

($\mathcal{H}_4$) For any real $\eta > 0$, there exists a non-negative function $L^\eta(\cdot) \in L^1([0,T], \mathbb{R}_+)$ such that

$$\|g(t,s,x) - g(t,s,y)\| \leq L^\eta(t)\|x - y\|,$$

for all $(t,s) \in Q_\Delta$ and for each $(x,y) \in B[0,\eta] \times B[0,\eta]$.

We come now to our main results in this work which gives the existence result of (1.1).

Theorem 2.1. *Let the assumptions* ($\mathcal{H}_1$), ($\mathcal{H}_2$), ($\mathcal{H}_3$) *and* ($\mathcal{H}_4$) *hold. Under the following additional conditions:*

1. $\mathcal{R}(C(t)) \subset \mathrm{Dom}(A)$,
2. *there exists a non-negative functions* $\alpha(\cdot), \delta(\cdot) \in L^2([0,T], \mathbb{R}_+)$ *with* $\alpha(\cdot)$ *is a continuous function, such that*

$$\|A^\circ x\| \leq \alpha(t)\|x\| + \delta(t), \text{ for a.e. } t \in [0,T] \text{ and } x \in C(t),$$

3. *there exists* $\gamma_1(\cdot), \gamma_2(\cdot) \in L^1([0,T], \mathbb{R}_+)$ *such that*

$$\beta(t,s) \leq \gamma_1(t).\gamma_2(s) \text{ for a.e. } t, s \in [0,T],$$

for each $x_0 \in C(0)$, *the differential inclusion* (1.1) *admits, at least, an absolutely continuous solution* $x(\cdot) : [0,T] \to H$.

Under some additional assumptions, we can prove a uniqueness result. The following theorem is in this sense.

Theorem 2.2. *Let the assumptions* ($\mathcal{H}_1$), ($\mathcal{H}_2$), ($\mathcal{H}_3$) *and* ($\mathcal{H}_4$) *hold and assume that*

$$\mathcal{R}(C) \subset \mathrm{int}(\mathrm{Dom}(A)). \tag{2.1}$$

Then, for given initial condition $x_0 \in C(0)$, *the problem* (1.1) *has only one absolutely continuous solution.*

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A New Riesz Potential Dissipation for Wave and Beam Equations

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Abstract
We introduce a mathematical model in $\mathbb{R}^n$ for linear classical wave and beam equations with structural damping of negative order, namely, the Riesz potential of frictional damping of the form $(-\Delta)^{-\delta}u_t(t,x)$, $t\in[0,\infty)$, $x\in\mathbb{R}^n$ and $0<\delta$.
Our approach is based on the Fourier analysis and the main purpose is to derive the L^2-energy estimates for the solution. In comparison to the well-known results for classical waves with frictional damping $u_t(t,x)$, the proposed damping produces the regularity-loss type and leads to a new fast decay rate. As a result, according to the fractional power $\gamma\in\mathbb{R}$, we will introduce a new classification of the damping mechanism $(-\Delta)^{\gamma}u_t$.

Keywords: Wave equation, beam equation, structural damping, dispersion terms, global existence, blow up, critical exponent.
2020 Mathematics Subject Classification: Primary 42B10, 35G10, 35B45.

1 Introduction

We are interested to know how, mathematically, structural damping of negative order, or the Riesz potential of frictional damping $(-\Delta)^{-\delta}u_t(t,x)$ affects the qualitative properties of solutions to linear classical wave and beam equations. What we have in mind are the following two models:

$$\begin{cases} u_{tt}(t,x)+(-\Delta)^{\sigma}u(t,x)+au_t(t,x)+b(-\Delta)^{-\delta}u_t(t,x)=0, & x\in\mathbb{R}^n,\ t>0,\\ \\ u(0,x)=u_0(x), \quad u_t(0,x)=u_1(x), & x\in\mathbb{R}^n, \end{cases} \tag{1.1}$$

if only one damping is used $(a=0,b=1)$ or double damping $(a=1,b=1)$, where $\sigma\geq 1$, $\delta>0$. Here, $u_0(x)$ represents the initial displacement and $u_1(x)$ the initial velocity, and the notations $(-\Delta)^{\sigma}$ and $(-\Delta)^{-\delta}$ denote the fractional Laplacian operator and the Riesz potential with symbols $|\xi|^{2\sigma}$ and $|\xi|^{-2\delta}$ respectively. In the Fourier analysis they can be defined as follows

$$(-\Delta)^{\theta}f(x)=\mathcal{F}^{-1}\left(|\xi|^{2\theta}\mathcal{F}\left(f\right)\left(\xi\right)\right)(x),\ \xi\in\mathbb{R}^n,\ \ |\xi|^2=\xi_1^2+\cdots+\xi_n^2,\ \ \theta=\sigma,-\delta.$$

This definition means that the Riesz potential operator is the inverse operation of the fractional Laplace operator for an appropriate function.
Model (1.1) is the classical wave equation with damping mechanisms when $\sigma = 1$ and the classical beam equation with damping mechanisms when $\sigma = 2$. These two equations are very important in real-world applications, this motivates us to investigate them using our new nonlocal damping $(-\Delta)^{-\delta}u_t(t,x)$. We should mention that the case $\delta \leq 0$ has been widely studied by many mathematicians, where the damping mechanism is well known as structural damping when $-\delta \in (0,\sigma)$ or viscoelastic damping when $-\delta = \sigma$ and very strong damping when $\sigma < -\delta$. Some qualitative properties of solutions has been well investigated, such as, well posdness, decay estimates and regularity-loss type.
From these published results, it is clear that the phenomenon that the decay rate corresponding to u_0 is always better than that of u_1 holds true for all positive powers of fractional damping $-\delta \in [0,\infty)$ with the exception of $\delta = 0$, when they are equal.

A natural question arises: Does this phenomena hold true for $\delta > 0$?

2 Main results

Before state our main results, let us define the energy function $E(u)(t)$ or $E(\hat{u})(t)$ associated to models (1.1) as follows:

$$E(u)(t) = \frac{1}{2}\|u_t(t,\cdot)\|_{L^2}^2 + \frac{1}{2}\|(-\Delta)^{\sigma/2}u(t,\cdot)\|_{L^2}^2 = \frac{1}{2}\|\hat{u}_t(t,\cdot)\|_{L^2}^2 + \frac{1}{2}\||\cdot|^{\sigma}\hat{u}(t,\cdot)\|_{L^2}^2 = E(\hat{u})(t).$$

In (1.1)(a=0,b=1), the term whose action may dissipates this energy $E(u)$ is $(-\Delta)^{-\delta}u_t$. In fact, for a fixed u we have,

$$\frac{dE(u)(t)}{dt} = -\|(-\Delta)^{-\delta/2}u_t(t,\cdot)\|_{L^2}^2 = -\||\cdot|^{-\delta}\hat{u}_t(t,\cdot)\|_{L^2} = \frac{dE(\hat{u})(t)}{dt} \leq 0,$$

this indicates that for all $t > 0$, the energy $E(u)(t)$ is a non-increasing function and tends to 0 as time t tends to ∞, its decay rate is determined below.
One can use this energy method in order to prove the well-posedness of solutions to (1.1) in Sobolev space of fractional order as in [**?**, page 193] for classical wave model, or in [**?**, Theorem 4], [**?**, Proposition 1.2] for damped wave models. Our main results are respectively.

Theorem 2.1. *Let $u_0 \in H^s$ and $u_1 \in H^{s-\sigma}$ with nonnegative real number s. Then, the Cauchy problem (1.1) is well posed in $C([0,\infty), H^s) \cap C^1([0,\infty), H^{s-\sigma})$.*

Theorem 2.2. *Let us consider the Cauchy problem (1.1)(a=0,b=1) with $\sigma \geq 1$ and $\delta > 0$. Let us assume the data $u_0 \in L^1 \cap H^l$, $u_1 \in L^1 \cap H^{l-\sigma}$, $l \geq \sigma$. The solution u satisfies the estimates:*

$$\|u(t,\cdot)\|_{L^2} \lesssim (1+t)^{-\frac{n}{4(\sigma+\delta)}} \|u_0\|_{L^1} + (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\delta}{\sigma+\delta}} \|u_1\|_{L^1}$$
$$+(1+t)^{-\frac{l}{2\delta}} \left(\||D|^l u_0\|_{L^2} + \||D|^{l-\sigma} u_1\|_{L^2} \right). \tag{2.1}$$

Theorem 2.3. *Let us consider the Cauchy problem (1.1)(a=0,b=1) with $\sigma \geq 1$ and $\delta > 0$. Let us assume the data $u_0 \in L^1 \cap H^{l+\sigma}$, $u_1 \in L^1 \cap H^l$, $l \geq 0$. The energy function $E(u)(t)$ satisfies the estimates:*

$$\left\|(-\Delta)^{\sigma/2} u(t,\cdot)\right\|_{L^2} \lesssim (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma}{2(\sigma+\delta)}} \|u_0\|_{L^1} + (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma+2\delta}{2(\sigma+\delta)}} \|u_1\|_{L^1},$$
$$+(1+t)^{-\frac{l}{2\delta}} \left(\||D|^{l+\sigma} u_0\|_{L^2} + \||D|^l u_1\|_{L^2} \right), \tag{2.2}$$
$$\|u_t(t,\cdot)\|_{L^2} \lesssim (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma}{\sigma+\delta}} \|u_0\|_{L^1} + (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma+2\delta}{\sigma+\delta}} \|u_1\|_{L^1}$$
$$+(1+t)^{-\frac{l}{2\delta}} \left(\||D|^{l+\sigma} u_0\|_{L^2} + \||D|^l u_1\|_{L^2} \right). \tag{2.3}$$

Theorem 2.4. *Let us consider the Cauchy problem (1.1)(a=1,b=1) with $\sigma \geq 1$ and $\delta > 0$. Let us assume the data $u_0 \in L^1 \cap H^\sigma$, $u_1 \in L^1 \cap L^2$. The solution u and its energy $E(u)(t)$ satisfy the estimates:*

$$\|u(t,\cdot)\|_{L^2} \lesssim (1+t)^{-\frac{n}{4(\sigma+\delta)}} \|u_0\|_{L^1\cap L^2} + (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\delta}{\sigma+\delta}} \|u_1\|_{L^1\cap L^2},$$
$$\left\|(-\Delta)^{\sigma/2} u(t,\cdot)\right\|_{L^2} \lesssim (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma}{2(\sigma+\delta)}} \|u_0\|_{L^1\cap H^\sigma} + (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma+2\delta}{2(\sigma+\delta)}} \|u_1\|_{L^1\cap L^2},$$
$$\|u_t(t,\cdot)\|_{L^2} \lesssim (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma}{\sigma+\delta}} \|u_0\|_{L^1\cap H^\sigma} + (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\sigma+2\delta}{\sigma+\delta}} \|u_1\|_{L^1\cap L^2}.$$

Remark 2.1. The estimates in the second and third theorems show that our model (1.1)(a=0,b=1) is of the regularity-loss type exactly as in the model (1.1)(a=0,b=1) with $\sigma < -\delta$. The regularity-loss type happens as a result of the presence of the factor $e^{-|\xi|^{-2\delta}t}$ in the high frequency zone, which causes some polynomial decay rather than exponential decay.

Remark 2.2. One can see that the decay rate corresponding to u_1 is always better than that of u_0 for all $0 < \delta$, this new phenomenon occurs due to the special structure of the solution and the characteristics roots (see the proof below). Since Duhamel's principle is always dependent on the kernel of u_1 and used in the treatment of semilinear model, choosing $u_1 = 0$ is generally undesirable. Hence, we obtain the following double-decay rate phenomenon for the Cauchy problem (1.1)(a=1,b=1):

$$\|u(t,\cdot)\|_{L^2} \lesssim \begin{cases} (1+t)^{-\frac{n}{4(\sigma+\delta)}} \left(\|u_0\|_{L^1\cap L^2} + \|u_1\|_{L^1\cap L^2} \right), & \text{if } u_0 \neq 0,\ u_1 \neq 0, \\ (1+t)^{-\frac{n}{4(\sigma+\delta)}-\frac{\delta}{\sigma+\delta}} \|u_1\|_{L^1\cap L^2}, & \text{if } u_0 = 0,\ u_1 \neq 0. \end{cases}$$

Remark 2.3. Some interactions between frictional and structural damping of negative order are shown in the following points:

- First, the solution to (1.1)(a=1,b=1) inherits the same decay rate as the solution to the problem (1.1)(a=0,b=1).
- Second, the solution to (1.1)(a=1,b=1) has the same regularity as the solution to the classical damped wave and beam equations, where the regularity-loss type disappears. This is remarkable when compared to the results of Ikehata-Kitazaki [5].
- Third, from Remark 2.2, the solution and its energy to (1.1)(a=1,b=1) satisfy fast decay rate than those in classical damped wave and beam equations, provided that

$$u_0 = 0, \quad n < 4\sigma.$$

In this case, the structural damping of negative enhance the damping effect in the model (1.1)(a=1,b=1) without higher regularity on the data. In other words, if $u_0 \neq 0$, the decay rates corresponding to the frictional damping in Theorem **??** are better than those for structural damping of negative order.

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On Caputo boundary value problems for fractional differential equations

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Abstract
In this work, we will study the existence and uniqueness of the solution of a fractional boundary value problem for differential equation with integral boundary conditions, by using Schauder fixed point theorem. An example is given to illustrate our main results.

Keywords: Caputo-Hadamard derivative, fractional differential equations, fixed point, existence
2010 Mathematics Subject Classification: Primary 34A08,34B15,34G20,47H08.

1 Introduction

In recent years, the fractional differential equations have been attracted the interest of many authors, due their applications in various fields of sciences, as in physics, chemistry, hydrology, biophysics, thermodynamics, blood flow problems, statistical mechanics, and control theory.

2 Main results

Lemma 2.1. *A function x is a solution of the problem* (**??**) *if and only if it is a solution of the following integral equation*

$$x(t) = \frac{(at+b)}{a+b}\int_0^1 g(s)ds + \int_0^1 G(t,s)f(s,x(s))ds, \tag{2.1}$$

Where G is the Green function given by:

$$G(t,s) = \frac{1}{\Gamma(\alpha)}\begin{cases}(t-s)^{\alpha-1} - \dfrac{at+b}{a+b}(1-s)^{\alpha-1}, & 0 \le s \le t,\\ \dfrac{at+b}{a+b}(1-s)^{\alpha-1}, & t \le s \le 1 \quad .\end{cases} \tag{2.2}$$

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Fractal model for predicting the spread of certain pandemics in society

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Abstract
In this paper, we discuss and provide some analytical studies for a mathematical model of fractional-order SIRD for COVID-19 in the sense of the Caputo-Katugampola derivative. We compute and derive several stability results based on some parameters that satisfy some conditions which prevent the pandemic from occurring. The paper also investigates the problem of the existence and uniqueness of solutions for the SIRD model. It does so by applying the properties of Schauder's and Banach's fixed point theorems.

Keywords: COVID-19; SIRD model; fractional derivative; existence and uniqueness.
2020 Mathematics Subject Classification: Primary 34A08; 62P10; 92-10; 92B05; 93-10.

1 Introduction

The coronavirus pandemic was a major worldwide challenge in 2020. The severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) is the virus responsible for COVID-19 infection. Wuhan city in China is reported to be where this virus originated.
A better understanding and evaluation of the existence, stability and control of infectious diseases can be acquired through modeling them mathematically. However, mathematical models' classical approaches are not highly accurate in modeling such diseases; hence, the introduction of fractional differential equations for handling these problems becomes necessary.
Our model requires the division of the total population $\mathcal{N}$ into four epidemiological classes:

$\mathfrak{S}$: Susceptible class, $\mathfrak{I}$: Infected class, $\mathfrak{R}$: Recovered class, and $\mathfrak{D}$: Death class.

The parameters are positive and could be described as follows:
∘ β is the average number of contacts per person per time t,
∘ γ is the recovery rate,
∘ κ is the death rate.

For $0 < \alpha < 1$ and $\rho > 0$,

$$\begin{cases} {}^{C}\mathcal{D}_{0^+}^{\alpha,\rho}\mathfrak{S}(t) &= -\beta\frac{\mathfrak{S}(t)\mathfrak{I}(t)}{\mathcal{N}_0}, \\ {}^{C}\mathcal{D}_{0^+}^{\alpha,\rho}\mathfrak{I}(t) &= \beta\frac{\mathfrak{I}(t)\mathfrak{S}(t)}{\mathcal{N}_0} - (\gamma+\kappa)\mathfrak{I}(t), \\ {}^{C}\mathcal{D}_{0^+}^{\alpha,\rho}\mathfrak{R}(t) &= \gamma\mathfrak{I}(t), \\ {}^{C}\mathcal{D}_{0^+}^{\alpha,\rho}\mathfrak{D}(t) &= \kappa\mathfrak{I}(t), \end{cases} \tag{1.1}$$

along with the positive initial conditions

$$\mathfrak{S}(0) = \mathfrak{S}_0,\ \mathfrak{I}(0) = \mathfrak{I}_0,\ \mathfrak{R}(0) = \mathfrak{R}_0,\ \mathfrak{D}(0) = \mathfrak{D}_0, \tag{1.2}$$

where $\mathcal{N}_0 > 0$ is the initial total population at the moment $t = 0$, with $0 \leq t \leq T < \infty$.
It is essential to find the solution of (1.1) with an efficient technique. Considering this point, our main goal in this work is to determine the main properties of the solution for the system of non-linear fractional differential equations (1.1), checking the considered model's stability necessitates establishing some existence results.

2 Main Results

Lemma 2.1. *The solution of model (1.1) is restricted to the feasible region given by*

$$u = \left\{(\mathfrak{S},\mathfrak{I},\mathfrak{R},\mathfrak{D}) \in \mathbb{R}_+^4,\ 0 \leq \mathcal{N}(t) \leq \mathcal{N}_0\right\},$$

and the pandemic will occur if $\mathfrak{S}_0 > \frac{\gamma+\kappa}{\beta}\mathcal{N}_0$, *where* $\frac{\gamma+\kappa}{\beta}$ *is called threshold phenomenon or critical community size for the pandemic.*

Theorem 2.1. *The disease-free equilibrium point of (1.1) is*

$$u^* = \left(\frac{\gamma+\kappa}{\beta}\mathcal{N}_0, 0, \mathfrak{R}_0, 0\right).$$

Theorem 2.2. *If the susceptible class* $\mathfrak{S}(t) < \mathfrak{S}^*$, *the pandemic free equilibrium point of (1.1) is locally asymptotically stable and is unstable if* $\mathfrak{S}(t) > \mathfrak{S}^*$.

Let $u = (\mathfrak{S},\mathfrak{I},\mathfrak{R},\mathfrak{D}) \in E$, where $E = [C([0,T],\mathbb{R}_+)]^4$ is a Banach space equipped with the norm

$$\|u\|_E = \|\mathfrak{S}\|_\infty + \|\mathfrak{I}\|_\infty + \|\mathfrak{R}\|_\infty + \|\mathfrak{D}\|_\infty$$

and let $f = (f_1, f_2, f_3, f_4)$, be such that

$$\begin{cases} f_1(t,u(t)) &= -\beta\frac{\mathfrak{S}(t)\mathfrak{I}(t)}{\mathcal{N}_0}, \\ f_2(t,u(t)) &= \beta\frac{\mathfrak{I}(t)\mathfrak{S}(t)}{\mathcal{N}_0} - (\gamma+\kappa)\mathfrak{I}(t), \\ f_3(t,u(t)) &= \gamma\mathfrak{I}(t), \\ f_4(t,u(t)) &= \kappa\mathfrak{I}(t), \end{cases}$$

it is clear that the function $f \in ([0,T]\times E)^4$ is continuous.

By applying the fractional integral to both sides of the system (1.1), we get

$$\begin{cases} \mathfrak{S}(t) &= \mathfrak{S}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho-\tau^\rho)^{1-\alpha}} f_1(\tau, u(\tau))\, d\tau, \\ \mathfrak{I}(t) &= \mathfrak{I}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho-\tau^\rho)^{1-\alpha}} f_2(\tau, u(\tau))\, d\tau, \\ \mathfrak{R}(t) &= \mathfrak{R}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho-\tau^\rho)^{1-\alpha}} f_3(\tau, u(\tau))\, d\tau, \\ \mathfrak{D}(t) &= \mathfrak{D}_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho-\tau^\rho)^{1-\alpha}} f_4(\tau, u(\tau))\, d\tau, \end{cases}$$

By choosing $u_0 = (u_1, u_2, u_3, u_4) = (\mathfrak{S}_0, \mathfrak{I}_0, \mathfrak{R}_0, \mathfrak{D}_0)$, we get

$$u(t) = u_0 + \frac{\rho^{1-\alpha}}{\Gamma(\alpha)} \int_0^t \frac{\tau^{\rho-1}}{(t^\rho-\tau^\rho)^{1-\alpha}} f(\tau, u(\tau))\, d\tau. \tag{2.1}$$

In what follows, we present the principal theorems:

Theorem 2.3. *Let $\beta, \gamma, \kappa, \alpha, \rho, T \in \mathbb{R}_+$, be such that $\alpha \in (0,1)$ and*

$$T < \left(\frac{\rho^\alpha \Gamma(\alpha+1)}{4(\beta+\gamma+\kappa+3)} \right)^{\frac{1}{\rho\alpha}}, \tag{2.2}$$

then the problem (1.1)–(1.2) has at least one solution on $[0,T]$.

Theorem 2.4. *Let $\alpha \in (0,1)$ and $\beta, \gamma, \kappa, \rho, \mu \in \mathbb{R}_+^*$, be such that*

$$\mu = \max\{\beta+\gamma+\kappa+2, \gamma+3, \kappa+3\}.$$

If

$$\frac{4\mu T^{\rho\alpha}}{\rho^\alpha \Gamma(\alpha+1)} < 1, \tag{2.3}$$

then the problem (1.1)–(1.2) admits a unique solution on $[0,T]$.

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Chaotic dynamic systems for symmetric encryption

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Abstract
Symmetric systems, also known as conventional systems, is based on reversible mathematical functions. In symmetric encryption, only one key is used to encrypt and decrypt data. When two parties communicate and use symmetric encryption to encrypt their data, they both use the same key for encryption and decryption. This is why symmetric encryption is also known as shared key encryption. logistic maps offer us good properties such us non-linearity and sensitivity to initial conditions. In this work, using chaotic logistic sequences, we will see how to realize and create a common secret key to use for the encryption and decryption of digital images.

Keywords: Symmetric encryption, share keys, Logistic maps, BB84 protocol, digital images.
2020 Mathematics Subject Classification: 08A70, 11C08, 34H10.

1 Introduction

Encryption ensures that even if sensitive information falls into the wrong hands, it cannot be understood by unauthorized persons. There are two types of encryption: asymmetric encryption and symmetric encryption. The difference between the two lies in the keys used for encryption and decryption. In asymmetric encryption, also known as public key encryption, there are two keys, one used for encryption and the other for decryption. In the case of symmetric encryption, only one key is used to encrypt and decrypt the encrypted data. When two parties communicate and use symmetric encryption to encrypt their data, they both use the same key for encryption and decryption. This is why symmetric encryption is also known as shared key encryption. It is therefore important that this key is kept secret from unauthorized persons. This is also the reason why symmetric encryption is also called secret key encryption. The security of symmetric encryption lies in the fact that the key remains secret for this we use the quantum protocol called $BB84$ to share some parameters secretly and we create a new matrix their element are the terms of some logistic maps, were theit initial conditions and parameters are same that are exchanged using $BB84protocol$. this new matrix will be considered the key of encryptino and decryption of digial images.

2 Logistic Maps

In mathematics, a logistic suite is a simple real suite, but whose recurrence is not linear. Its recurrence relationship is

$$x_{n+1} = \mu x_n(1 - x_n), \quad x_0 \in [0; 1].$$

Depending on the value of the parameter $\mu(in[0; 4]$ to ensure that the values of x remain in $[0; 1])$, it generates either a convergent sequence, a oscillating sequence, or a chaotic sequence.

Around $\mu = 3.57$, chaos sets in. No oscillation is yet found and small variations in the initial conditions lead to radically different results (snnsitivity to initial conditions).

3 Generate a new key

In this section we propose a new construction of an encryption key in the form of a square matrix their components are the terms of three logistic maps.

1. First we must secretly share some parameters of three logistic maps, using a famous known protocol colled The $BB84$ quantum protocol.

2. - Let (μ_1, x_0^1), (μ_2, x_0^2) and (μ_3, x_0^3) three couples of parameters shared parameters, where, $\{\mu_i : 3 < \mu_i < 4$, for $i \in \{1, 2, 3\}\}$ and $\{x_0^i : i \in \{1, 2, 3\}\}$ is the initial terms of sequences defiend as in (**??**), these couples will be exchanged between the two interlocutors through the quantum channal using the $BB84$ protocol.
   - After having exchanged these three couples of parameters, we introduce the logistic map defined in the previous section for getting the n terms of each sequence :

$$\{x_1^1, x_2^1, \cdots, x_n^1\} \quad , \quad \{x_1^2, x_2^2, \cdots, x_n^2, \cdots, x^2_{\frac{n^2-n}{2}}\} \quad and \quad \{x_1^3, x_2^3, \cdots, x_n^3, \cdots, x^3_{\frac{n^2-n}{2}}\}$$

   - We create the matrix

$$\mathcal{K} = \begin{pmatrix} x_1^1 & x_1^2 & x_2^2 & \cdots & \cdots & x_{n-2}^2 & x_{n-1}^2 \\ x_1^3 & x_2^1 & x_n^2 & \cdots & \cdots & x_{2n-4}^2 & x_{2n-3}^2 \\ x_2^3 & x_3^3 & x_3^1 & \cdots & \cdots & x_{3n-7}^2 & x_{3n-6}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x^3_{\frac{n^2-5n+4}{2}} & x^3_{\frac{n^2-5n+6}{2}} & x^3_{\frac{n^2-5n+8}{2}} & \cdots & \cdots & x_{n-1}^1 & x^2_{\frac{n^2-n}{2}} \\ x^3_{\frac{n^2-3n+2}{2}} & x^3_{\frac{n^2-3n+4}{2}} & x^3_{\frac{n^2-3n+6}{2}} & \cdots & \cdots & x^3_{\frac{n^2-n}{2}} & x_n^1 \end{pmatrix}$$

4 Application

Our application is in the encryption of digital images. We propose to use a symmetric algorithm to encrypt and decrypt an image using the common key shared in the previous section, for this we follow the following steps:

- Step 1: We convert the digital image wanted to encrypt into a matrix $\mathcal{M}$ of the same order n.
- Step 2: The sender encrypt $\mathcal{M}$ using the formula: $\mathcal{K} \times \mathcal{M} \times \mathcal{K}^{-1} = \mathcal{M}_c$=the cipher matrix.
- Step 3: The receiver decrypt $\mathcal{M}_c$ using the formula: $\mathcal{K}^{-1} \times \mathcal{M}_c \times \mathcal{K} = \mathcal{M}$=the initial matrix.

Proof: $\mathcal{K}^{-1} \times \mathcal{M}_c \times \mathcal{K} = \mathcal{M} = \mathcal{K}^{-1} \times (\mathcal{K} \times \mathcal{M} \times \mathcal{K}^{-1}) \times \mathcal{K} = \mathcal{M} = \mathcal{I} \times \mathcal{M} \times \mathcal{I} = \mathcal{M}$.

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Ordinary generating functions of the products of Gaussian numbers with symmetric functions in several variables

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Abstract
In this paper, we first derive some new generating functions of the products of $(p,q)-$numbers with symmetric function in several variables. After that, we give the new generating functions for the products of bivariate polynomials with symmetric function in several variables.

Keywords: Symmetric functions, Generating functions, Gaussian $(p,q)-$numbers,p-bivariate polynomials.
2020 Mathematics Subject Classification: Primary 05E05; Secondary 11B39.

1 Introduction

In recent years, there are many studies of the different $(p,q)-$numbers such as: Gaussian $(p,q)-$Fibonacci, Fibonacci $(p,q)-$ Like, Gaussian $(p,q)-$Lucas, Gaussian $(p,q)-$Pell, Gaussian $(p,q)-$Pell Lucas, Gaussian $(p,q)-$Jacobsthal and Gaussian $(p,q)-$Jacobsthal Lucas numbers.
Karaaslan and Yagmur in [5] introduced the Gaussian $(p,q)-$Pell numbers $\{GP_{p,q,n}\}_{n\in\mathbb{N}}$and Gaussian $(p,q)-$Pell Lucas numbers$\{GQ_{p,q,n}\}_{n\in\mathbb{N}}$ are defined, respectively, by

$$\begin{cases} GP_{p,q,n} = 2pGP_{p,q,n-1} + qGP_{p,q,n-2}, \text{ for } n \geq 2 \\ GP_{p,q,0} = i,\ GP_{p,q,1} = 1 \end{cases},$$

and

$$\begin{cases} GQ_{p,q,n} = 2pGQ_{p,q,n-1} + qGQ_{p,q,n-2}, \text{ for } n \geq 2 \\ GQ_{p,q,0} = 2 - 2ip,\ GQ_{p,q,1} = 2p + 2iq \end{cases},$$

On the other hand, the Gaussian $(p,q)-$Fibonacci numbers $\{GF_{p,q,n}\}_{n\in\mathbb{N}}$

$$\begin{cases} GF_{p,q,n} = pGF_{p,q,n-1} + qGF_{p,q,n-2}, \text{ for } n \geq 2 \\ GF_{p,q,0} = i,\ GF_{p,q,1} = 1 \end{cases}.$$

Moreover the Gaussian $(p,q)-$Lucas numbers $\{GL_{p,q,n}\}_{n\in\mathbb{N}}$ are defined by the following recurrence relation

$$\begin{cases} GL_{p,q,n} = pGL_{p,q,n-1} + qGL_{p,q,n-2}, \text{ for } n \geq 2 \\ GL_{p,q,0} = 2 - ip, \ GL_{p,q,1} = p2iq \end{cases}.$$

Definition 1.1. [4] The bivariate Fibonacci polynomials $F_n(x,y)$ are defined by the following recurrence relationas

$$\begin{cases} F_n(x,y) = xF_{n-1}(x,y) + yF_{n-2}(x,y), \text{ for } n \geq 2 \\ F_0(x,y) = 0, \ F_1(x,y) = 1 \end{cases}.$$

2 Main results

2.1 Main Theorems

In this section, we present two theorems on the complete symmetric functions.

Theorem 2.1. *[1]Let $P = \{p_1, p_2\}$ and $A = \{a_1, a_2, a_3, a_4\}$, be two alphabets, then we have*

$$\begin{aligned} \sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) S_n(p_1+p_2) z^n &= \frac{1-(p_1p_2)e_2^{(4)}z^2+[(p_1p_2)(p_1+p_2)]e_3^{(4)}z^3}{\prod_{i=1}^{4}(1-a_ip_1z)\prod_{i=1}^{4}(1-a_ip_2z)} \\ &- \frac{(p_1p_2)[(p_1+p_2)^2-p_1p_2]e_4^{(4)}z^4}{\prod_{i=1}^{4}(1-a_ip_1z)\prod_{i=1}^{4}(1-a_ip_2z)}. \end{aligned} \quad (2.1)$$

Theorem 2.2. *[1]Let $P = \{p_1, p_2\}$ and $A = \{a_1, a_2, a_3, a_4\}$, be two alphabets, then we have*

$$\begin{aligned} \sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) S_{n-1}(p_1+p_2) z^n &= \frac{e_1^{(4)}z-(p_1+p_2)e_2^{(4)}z^2+[(p_1+p_2)^2-p_1p_2]e_3^{(4)}z^3}{\prod_{i=1}^{4}(1-a_ip_1z)\prod_{i=1}^{4}(1-a_ip_2z)} \\ &- \frac{(p_1+p_2)[(p_1+p_2)^2-2p_1p_2]e_4^{(4)}z^4}{\prod_{i=1}^{4}(1-a_ip_1z)\prod_{i=1}^{4}(1-a_ip_2z)}. \end{aligned} \quad (2.2)$$

2.2 On the generating functions of numbers

In this part, we now consider the previous theorems in order to derive a new generating functions for the products of the symmetric functions in several variables with Gaussian $(p,q)-$Fibonacci, Gaussian $(p,q)-$Lucas numbers.
By changing $p_2$by $(-p_2)$ in relationsips (2.1) and (2.2), we obtain

$$\sum_{n=0}^{\infty} S_n(a_1,a_2,a_3,a_4) S_n(p_1,[-p_2]) z^n = \frac{\begin{array}{c}1+(p_1p_2)e_2^{(4)}z^2-[(p_1p_2)(p_1-p_2)]e_3^{(4)}z^3 \\ +p_1p_2[(p_1-p_2)^2+p_1p_2]e_4^{(4)}z^4\end{array}}{\prod_{i=1}^{4}(1-(p_1-p_2)a_iz-p_1p_2a_i^2z^2)}. \quad (2.3)$$

$$\sum_{n=0}^{\infty} S_n\left(a_1,a_2,a_3,a_4\right) S_{n-1}\left(p_1,[-p_2]\right) z^n = \frac{\begin{array}{c} e_1^{(4)} z-\left(p_1-p_2\right) e_2^{(4)} z^2+\left[\left(p_1-p_2\right)^2+p_1 p_2\right] e_3^{(4)} z^3 \\ -\left(p_1-p_2\right)\left[\left(p_1-p_2\right)^2+2 p_1 p_2\right] e_4^{(4)} z^4 \end{array}}{\prod_{i=1}^{4}\left(1-\left(p_1-p_2\right) a_i z-p_1 p_2 a_i^2 z^2\right)}. \quad (2.4)$$

Case1, Puting $\begin{cases} p_1-p_2=p \\ p_1 p_2=q \end{cases}$ in relationship (2.3) ana (2.4), we get

$$\sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) S_n(p_1+[-p_2]) z^n = \frac{1+q e_2^{(4)} z^2-p q e_3^{(4)} z^3+q[p^2+q] e_4^{(4)} z^4}{\prod_{n=1}^{4}\left(1-p a_i z-q a_i^2 z^2\right)}$$

$$\sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) S_{n-1}(p_1+[-p_2]) z^n = \frac{e_1^{(4)} z-p e_2^{(4)} z^2+\left(p^2+q\right) e_3^{(4)} z^3-p\left(p^2+2 q\right) e_4^{(4)} z^4}{\prod_{n=0}^{4}\left(1-p a_i z-q a_i^2 z^2\right)}$$

As a result, we obtain the following theorem

Theorem 2.3. *Provided that n be a natural number, the novel generating function for the product of Gaussian $(p,q)-$ Fibonacci numbers and symmetric function in multiple variables is*

$$\sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) G F_{p,q,n} z^n = \frac{\begin{array}{c} i+(1-i p)-[p-i(p+q)] e_2^{(4)} z^2-\left[\left(p^2+q\right)-i p^3\right] e_3^{(4)} z^3 \\ -\left[p\left(p^2+2 q\right)-i\left(p^4+3 p^2 q+q^2\right)\right] e_4^{(4)} z^4 \end{array}}{\prod_{n=1}^{4}\left(1-p a_i z-q a_i^2 z^2\right)} \quad (2.5)$$

Theorem 2.4. *Provided that n be a natural number, the novel generating function for the product of Gaussian $(p,q)-$ Lucas numbers and symmetric function in multiple variables is*

$$\sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) G L_{p,q,n} z^n = \frac{\begin{array}{c} (2-i p)+\left(i\left[p^2+2 q\right]-p\right) e_1^{(4)} z+\left[2 q+p^2-i p\left(3 q-p^2\right)\right] e_2^{(4)} z^2 \\ -\left[\left(3 p q+p^3\right)-i\left(4 p^2 q+2 q^2+p^4\right)\right] e_3^{(4)} z^3 \\ +\left[\left(2 q^2+4 p^2 q+p^4\right)-i\left(3 p^3 q+5 q^2 p+q^5\right)\right] e_4^{(4)} z^4 \end{array}}{\prod_{n=1}^{4}\left(1-p a_i z-q a_i^2 z^2\right)} \quad (2.6)$$

Theorem 2.5. *Let n be a natural number, the new generating function for the product of bivariate Fibonacci polynomials and the symmetric function in several variables is given by*

$$\sum_{n=0}^{\infty} S_n(a_1+a_2+a_3+a_4) F_n(x,y) z^n = \frac{e_1^{(4)} z-x e_2^{(4)} z^2+\left(x^2+y\right) e_3^{(4)} z^3-x\left(x^2+2 y\right) e_4^{(4)} z^4}{\prod_{n=0}^{4}\left(1-x a_i z-y a_i^2 z^2\right)} \quad (2.7)$$

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Optimal Design of Non-Conventional Dynamical System with Negative Stiffness for Enhanced Vibration Control of Damped Primary Structures

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Abstract
In this present paper, a minimax optimal design of a Non-Conventional Dynamic Vibration Absorber with negative stiffness (NCNS-DVA) applied to damped primary systems is proposed to reduce high amplitudes in the resonant range of oscillations. The differential equation is formulated, and the analytical solution of the system is derived. A closed-form solution for the optimal tuning coefficient is analytically obtained using the principles of Den Hartog's equal peak theory under the assumption that the structural damping ratio falls within the weak to moderate range. Subsequently, numerical determination of the optimal damping coefficient and the optimal negative stiffness parameter for the proposed non-conventional DVA with negative stiffness is achieved by solving a system of nonlinear equations formulated through the utilization of Chebyshev's equioscillation theorem. Extended simulations are conducted to examine the effectiveness of the optimally designed NCNS-DVA and the sensitivity of the optimal parameters. Finally, the vibration control performance of the proposed configuration is compared with those of two typical DVAs, which were presented by Liu and Pennestri, respectively. The comparison results demonstrate that the non-conventional DVA with negative stiffness significantly enhances vibration control by reducing the dynamic magnification response of damped primary structures and confining the normalized DAV stroke.

Keywords: dynamic vibration absorber, mini-max optimization, negative stiffness, dynamic response mitigation, stroke length
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

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Anomaly Detection in Eye Images Using Variational Autoencoders: A Comprehensive Implementation and Analysis

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Abstract
This study explores the application of Variational Autoencoders (VAEs) for anomaly detection in eye images, focusing on identifying abnormal conditions early for effective diagnosis and treatment. VAEs are employed to acquire hidden representations of normal eye images and reconstruct them with minimal error. The approach involves training a VAE architecture with encoder and decoder components on a dataset of eye images, optimizing model parameters to minimize reconstruction loss and align latent space distributions. Reconstruction errors are used as indicators of anomalies, with a defined threshold separating normal from abnormal images. Experimental results demonstrate the efficacy of the VAE-based approach in accurately reconstructing normal eye images while effectively identifying anomalies. The methodology contributes to enhancing diagnostic capabilities in medical imaging, particularly in detecting subtle abnormalities not easily discernible by traditional methods.

Keywords: Variational Autoencoder (VAE), Anomaly Detection, Medical Imaging, Eye Images, Reconstruction Error, Deep Learning, Image Processing
2020 Mathematics Subject Classification: 62M10, 68T10, 92C50,68T07,68U10.

1 Introduction

In medical imaging, early detection of anomalies plays a crucial role in improving patient outcomes through timely intervention and treatment. Specifically, in the field of ophthalmology, identifying subtle abnormalities in eye images can significantly impact diagnosis and patient care. Traditional methods often rely on manual inspection or predefined criteria, which may overlook nuanced deviations indicative of underlying conditions.
Advancements in deep learning, particularly with generative models like Variational Autoencoders , offer promising avenues for automated anomaly detection in medical images. VAEs are capable of learning complex data distributions and generating high-fidelity reconstructions, making them well-suited for capturing subtle deviations that may indicate anomalies.

This study focuses on leveraging VAEs to detect anomalies in eye images by training a model to encode normal image features into a latent space and decode them back to reconstruct the original images. The approach involves optimizing the VAE architecture to minimize reconstruction errors while regularizing the latent space to enhance anomaly detection sensitivity.
By evaluating the reconstruction errors of test images against a predefined threshold, anomalies—images with reconstruction errors beyond the threshold—are identified. This method not only automates the anomaly detection process but also provides quantitative insights into the presence and nature of anomalies in eye images.
The research aims to contribute to the field of medical imaging by offering a robust computational framework for early anomaly detection, thereby assisting healthcare professionals in making informed decisions and improving patient care outcomes.

2 Main results

1. **Effective Reconstruction:** The Variational Autoencoder (VAE) successfully learned to encode normal eye images into a latent space and decode them back with high fidelity. This was evidenced by the significant reduction in reconstruction loss over epochs of training, indicating the model's ability to accurately reconstruct eye images.

2. **Anomaly Detection Performance:** The VAE-based anomaly detection method demonstrated robust performance in identifying anomalies within eye images. By computing and analyzing reconstruction errors on a separate test dataset, anomalies—images with reconstruction errors exceeding a predefined threshold—were effectively detected.

3. **Quantitative Evaluation:** The study quantitatively evaluated the detection of anomalies based on reconstruction errors. The Reconstruction Error Histogram showed a distinct distribution, with anomalies exhibiting higher error values compared to normal images. This validated the effectiveness of using reconstruction errors as indicators of anomalies.

4. **Visual Confirmation:** Visual inspection of reconstructed and anomaly images provided qualitative confirmation of the VAE's ability to identify and reconstruct both normal and anomalous patterns in eye images.

5. **Application Potential:** The methodology has potential applications in enhancing diagnostic capabilities in ophthalmology and medical imaging more broadly. By automating anomaly detection using deep learning techniques, healthcare professionals can benefit from improved efficiency and accuracy in early anomaly identification.

6. **Future Directions:** Future research could explore refining the anomaly detection threshold and integrating additional clinical data to further enhance the model's performance and applicability in real-world medical settings.

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Positive periodic solutions of iterative functional differential equations

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Abstract
By using the fixed point method, some sufficient conditions are obtained on the existence of positive periodic solutions for iterative functional differential equations with a positive parameter λ. In the processus we invert the given equation into an equivalent integral equation. Then, we construct an appropriate mapping and use Krasnoselskii's fixed point theorem in a cone of a Banach space to show the existence of positive periodic solutions of the given equation. Easily verifiable sufficient conditions are established. For this type of equations, we combine different techniques to prove new results.

Keywords: Cone, fixed point, iterated differential equation, positive periodic solution.
2020 Mathematics Subject Classification: Primary 34K13, 34A12, 39B12, 34K40

1 Introduction

Iterative differential equations have distinctive characteristics as a particular type of state-dependent delay-differential equations. The origin of this type of equation dates back to the early 19th century, and to the best of our knowledge, their study began with a simple example but a very important one dealt with by Babbage [1] in 1815, and after Babbage's equation, the theory slowly evolved (see, [1]). Now, intensive scientific work has been carried out on various dynamical aspects of iterative differential equations and has been reported. In particular, a number of existence results, periodicity and continuous dependence ([5]) and references therein are investigated by many authors.
Qualitative properties of differential equation have taken great interest due to the importance of understanding the continuous phenomena in application of mathematics. Existence, uniqueness, stability, periodicity, and oscillation of solution of differential equations became topic of many studies in the existing literature. Iterative differential equation that belong to the class of functional equations are visited by several researchers because of their complicated dynamics. We refer to ([2], [5]) as inspiring studies on this topic. Finally, let us indicate a very useful Theorem 1 when trying to prove that the existence and positivity of a solution to a periodic problem.

Definition 1.1. (see, [3]) *) Let X be a Banach space and let Ω be a closed, nonempty subset of X. Ω is a cone if*

i) $\alpha u + \beta v \in \Omega$ for all $u, v \in \Omega$ and all $\alpha, \beta \geq 0$;
ii) $u, -u \in \Omega$ imply $u = 0$. The proof of Krasnoselskii's fixed point theorem stated below can be found in [3] .

Theorem 1.1. *(Guo-Krasnoselskii, [3]) Let X be a Banach space, and let $\Omega \subset X$ be a cone in X. Assume that Ω_1 and Ω_2 are open subsets of X with $0 \in \Omega_1$, $\overline{\Omega}_1 \subset \Omega_2$ and let*

$$\mathcal{P} : \Omega \cap \left(\overline{\Omega}_2 \setminus \Omega_1\right) \to \Omega,$$

be a completely continuous operator such that either

i) $\|\mathcal{P}u\| \leq \|u\|$ for $u \in \Omega \cap \partial\Omega_1$ and $\|\mathcal{P}u\| \geq \|u\|$ for $u \in \Omega \cap \partial\Omega_2$; or
ii) $\|\mathcal{P}u\| \geq \|u\|$ for $u \in \Omega \cap \partial\Omega_1$ and $\|\mathcal{P}u\| \leq \|u\|$ for $u \in \Omega \cap \partial\Omega_2$.
Then $\mathcal{P}$ has a fixed point in $\Omega \cap \left(\overline{\Omega}_2 \setminus \Omega_1\right)$.

2 Main results

As we mentioned previously, one of our main objectives in this part is to discuss the work carried out in [4]. More precisely, very recently, the authors focused on a specific type of differential equations with state-dependent delay with a positive parameter $\lambda > 0$, given in the following form:

$$x'(t) = a(t)x(t) - \lambda f(t, x(t), x\left(t - \tau(t, x(t))\right)), t \in \mathbb{R}, \tag{2.1}$$

where $\tau(t, x(t)) = t - x(t)$, and the functions $a : \mathbb{R} \to (0, \infty), f : \mathbb{R} \times (0, \infty) \times (0, \infty) \to (0, \infty)$ are continuous. By employing a cone theoretical fixed point theorem, they established some sufficient conditions for the existence of positive periodic solutions of the iterative functional differential equation (2.1). Their approch allowed them to obtain som results that are not covered by the existing literature.
For $\omega > 0$, we define the set P_ω as a set of all continuous scalar functions $x(t)$, periodic in t of period ω, i.e., $x(t + \omega) = x(t)$ for all $t \in \mathbb{R}$. Then $(P_\omega, \|.\|)$ is a Banach space when it is endowed by the supremum norm

$$\|x\| := \sup_{t \epsilon \mathbb{R}} |x(t)| = \sup_{t \epsilon [0,\omega]} |x(t)|.$$

Since we are searching for the existence of periodic solutions for the equation (2.1), we need to introduce the following assumptions:
(H1) For a positive integer ω, the functions a and f are ω- periodic in t, that is

$$a(t + \omega) = a(t), \ f(t + \omega, x, y) = f(t, x, y),$$

for all $t \in \mathbb{R}$, and $(x, y) \in (0, \infty) \times (0, \infty)$.
(H2) There exist positive constants α_1, α_2, such that for all $t \in \mathbb{R}, x, y, \widehat{x}, \widehat{y} \in \mathbb{R}$,

$$\|f(t, x, y) - f(t, \widehat{x}, \widehat{y})\| \leq \alpha_1 \|x - \widehat{x}\| + \alpha_2 \|y - \widehat{y}\|. \tag{2.2}$$

In this part, we will use the Krasnoselskii in a cone of a Banach space Theorem 1.1 (see, section 1) to prove the existence of at least one periodic solutions of the equation (2.1).

Throughout this section, we assume the conditions (H1) and (H2)) are satisfied. For convenience in the following discussion, we introduce the following limits: Throughout this section, we assume the conditions (H1)and (H2)) are satisfied. For convenience in the following discussion, we introduce the following limits:

$$\begin{aligned}
&\bullet\ \underline{f_0} \ :=\ \lim_{x\to 0^+}\min_{t\in[0,\omega]}\frac{f(t,x,y)}{x};\\
&\bullet\ \overline{f_0} \ :=\ \lim_{x\to 0^+}\max_{t\in[0,\omega]}\frac{f(t,x,y)}{x};\\
&\bullet\ \underline{f_\infty} \ :=\ \lim_{x\to \infty}\min_{t\in[0,\omega]}\frac{f(t,x,y)}{x};\\
&\bullet\ \overline{f_\infty} \ :=\ \lim_{x\to \infty}\max_{t\in[0,\omega]}\frac{f(t,x,y)}{x}.
\end{aligned}$$

Our main result is the following.

Theorem 2.1. *If $\underline{f_\infty}=\infty$ and $\overline{f_0}=0$ (or $\underline{f_0}=\infty$ and $\overline{f_\infty}=0$) for each $y\in(0,\infty)$, then the equation (2.1) has a positive ω -periodic solution for any λ satisfying*

$$\frac{1-\sigma}{\omega}\le\lambda\le\frac{1-\sigma}{\sigma\omega}, \tag{2.3}$$

where

$$\sigma:=\exp(-\int_0^\omega a(s)ds). \tag{2.4}$$

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Supersymmetry in quantum mechanics and applications to position-dependent mass systems

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Abstract
We study the exact solutions of the Schrödinger and Dirac and Klein-Gordon equation with position-dependent mass by using the method of the supersymmetric quantum mechanics (SUSYQM) and shape invariance approach. The energy spectra equation of the bound states for the target potentials are given.

Keywords: Schrödinger equation, Dirac equation, Klein-Gordon equation, Position-dependent mass, Supersymmetric, Shape invariance.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

The objective is to choose, for each potential considered, an adequate distribution function for the mass and proceed by a series of point transformations so as to transform the initial Schrödinger equation, which is a priori a non-standard equation in a new equivalent equation involving a particle of constant mass, placed in an effective supersymmetric and shape-invariant potential, and exactly soluble, we have already used the supersymmetry approach to solve exactly the problem of a particle placed in the potential of central Hulthen and whose mass distribution is also of the Hulthen type. We have initiated a similar study concerning the problem of Central Morse potential combined with a mass distribution of the generalized Hulthen type.

2 Main results

In this work we solved the Schrödinger equation for a position-dependent mass particle placed in a Morse-type potential. By choosing the function of the mass in an adequate form and after a point transformation, the problem is reduced to solving the Schrödinger equation for the generalized Hulthen potential. The energies of the bound states are then obtained as a transcendental equation using the supersymmetry approach in quantum mechanics.

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Optimal control analysis for the COVID-19 stochastic epidemic model with vaccination

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Abstract

The aim of this work is to study COVID-19 control strategies by taking effect of vaccination and treatment of infection into account. We construct a SVIR epidemic model for COVID-19 with stochastic perturbations. Moreover, we show that the model has a unique global positive solution. By using stochastic control theory we investigated the model for controls. The optimal control model, both with and without the control term, is numerically analyzed using Runge-Kutta method. At last, numerical simulation were employed to illustrate our results.

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Keywords: COVID-19 stochastic model, stochastic optimal control,
2020 Mathematics Subject Classification: 93E03, 93E20, 49K45, 37A50.

1 Introduction

The COVID-19 pandemic has dramatically impacted global health, economies, and daily life, necessitating the development of effective strategies to control its spread. Mathematical modeling has played a crucial role in understanding the dynamics of COVID-19 and in guiding public health interventions. Traditional epidemic models, such as the SIR model [3], have been widely used to simulate the transmission of the virus. However, the complex nature of the pandemic, including the availability and impact of vaccination, calls for more nuanced models. The SVIR model extends the basic SIR framework by incorporating the vaccinated population, providing a more comprehensive tool for analyzing the effects of vaccination on disease dynamics[4]. In real-world scenarios, the spread of COVID-19 is influenced by numerous stochastic factors, such as random variations in contact rates, transmission probabilities, and the efficacy of interventions. These random variations necessitate the use of stochastic models to more accurately capture the inherent uncertainties in the

epidemic's progression. Stochastic optimal control theory offers a powerful approach to designing intervention strategies that can effectively mitigate the impact of the virus while accounting for these uncertainties. The primary objective of this paper was to apply optimal control theory to a stochastic COVID-19 model. We evaluated two distinct control measures aimed at minimizing coronavirus infection and its subsequent spread within the community. The cost function incorporated the nonlinear effects of these controls. Using the stochastic Pontryagin Maximum Principle , we analytically characterized the optimal control [1]. Numerical results demonstrated that the proposed joint control measures effectively reduced the disease transmission rate and minimized the number of infections. The main contribution of this work is the application of stochastic optimal control techniques to the study of COVID-19. Its key utility lies in balancing the minimization of costs with the containment of the virus's spread. Based on the characteristics of the disease. We construct a stochastic model for the dynamics of COVID-19, we propose to divide the total population $N(t)$ into four various compartment susceptible $S(t)$, Vaccinated $V(t)$, infected $I(t)$ and recovered $R(t)$ i.e $N(t) = S(t) + V(t) + I(t) + R(t)$, we assume all variables, parameters of the model are non-negative, then we have:

$$\begin{aligned} dS(t) &= \Big[\Delta - \beta\frac{S(t)I(t)}{N} + \mu R(t) - (k+\delta)S(t)\Big]dt + \eta_1 S(t)dB_1(t) \\ dV(t) &= \Big[kS(t) - (1-\tau)\beta\frac{V(t)I(t)}{N} - \delta V(t)\Big]dt + \eta_2 V(t)dB_2(t) \\ dI(t) &= \Big[\beta\frac{S(t)I(t)}{N} + (1-\tau)\beta\frac{V(t)I(t)}{N} - (\alpha+\delta+\delta_0)I(t)\Big]dt + \eta_3 I(t)dB_3(t) \\ dR(t) &= \Big[\alpha I(t) - (\delta+\mu)R(t)\Big]dt + \eta_4 R(t)dB_4(t) \end{aligned} \tag{1.1}$$

where B_i $\{i = 1, ..., 4\}$ are independent Brownian motion and η_i $\{i = 1, ..., 4\}$ are the intensity of white noise. The biological meaning of the parameters are: Δ birth rate, k vaccination rate, β contact rate between infected and susceptible persons, τ vaccine effectiveness rate, μ loss of natural immunity, δ_0 mortality rate by infection, δ natural mortality rate, α the rate of recovery from infection.

2 Main results

2.1 Existence and uniqueness of positive solution

Theorem 2.1. *There is a unique solution (S(t), V(t), I(t), R(t)) of system on $t \geq 0$ for any initial value $(S(0), V(0), I(0), R(0)) \in \mathbb{R}^4_+$, and the solution will remain in $\mathbb{R}^4_+$ with probability one.*

2.2 Optimal control strategy

To minimizing the COVID-19 infection, we apply the control variable $u_1(t)$ *i.e.*, to vaccinate the class of susceptible individuals.To increase the recovery cases, we apply the control variable $u_2(t)$ to treatment of COVID-19 infected individuals.

Through what was mentioned above, the following control system could be derived :

$$\begin{aligned} dS(t) &= \Big[\Delta - \beta\frac{S(t)I(t)}{N} + \mu R(t) - (ku_1(t)+\delta)S(t)\Big]dt + \eta_1 S(t)dB_1(t) \\ dV(t) &= \Big[ku_1(t)S(t) - (1-\tau)\beta\frac{V(t)I(t)}{N} - \delta V(t)\Big]dt + \eta_2 V(t)dB_2(t) \\ dI(t) &= \Big[\beta\frac{S(t)I(t)}{N} + (1-\tau)\beta\frac{V(t)I(t)}{N} - (\alpha u_2 + \delta + \delta_0)I(t)\Big]dt + \eta_3 I(t)dB_3(t) \\ dR(t) &= \Big[\alpha u_2(t)I(t) - (\delta+\mu)R(t)\Big]dt + \eta_4 R(t)dB_4(t) \end{aligned} \tag{2.1}$$

with

$$S(0) = S_0 > 0,\ V(0) = V_0 \geq 0,\ I(0) = I_0 \geq 0,\ R(0) = R_0 > 0.$$

The main goal is to minimize the cost function,

$$J(u_1,u_2) = \mathbb{E}\Big[\int_0^{T_e}\Big[b_1 S + b_2 I + \frac{1}{2}(b_3 u_1^2(t) + b_4 u_2^2(t))\Big]dt + \frac{c_1}{2}S^2(T_e) + \frac{c_2}{2}V^2(T_e) + \frac{c_3}{2}I^2(T_e) + \frac{c_4}{2}R^2(T_e)\Big]$$

Subject to the system (2.1), where $b_i, c_i \ \ i = 1, ..., 4$ are non-negative real numbers. In other words, we seek to find the controls $u^* = (u_1^*, u_2^*)$, such that $\ J(u^*) = \min_{u\in\Omega} J(u)$
where $\ \Omega = \big\{u_i : [0,T_e] \to [0,1], \text{is Lebesgue integrable}, i = 1,2\big\}$.
we defined the Hamiltonian $H(X,u,A,Q)$ function as follows:

$$\begin{aligned} H =& a_1 S + b_2 I + \frac{1}{2}(b_3 u_1^2 + b_4 u_2^2) + A_1(\Delta - \beta\frac{SI}{N} + \mu R - (ku_1+\delta)S) + A_2(ku_1 S - (1-\tau)\beta\frac{VI}{N} - \delta V) + \\ & A_3(\beta\frac{SI}{N} + (1-\tau)\beta\frac{VI}{N} - (\alpha u_2 + \delta + \delta_0)I) + A_4(\alpha u_2 I - (\delta+\mu)R) + Q_1\eta_1 S + Q_2\eta_2 V + Q_3\eta_3 I + Q_4\eta_4 R \end{aligned}$$

where $A = (A_1, A_2, A_3, A_4)$ and $Q = (Q_1, Q_2, Q_3, Q_4)$ represent the vectors adjoint respectively.

Theorem 2.2. *Let (S^*, V^*, I^*, R^*) be the optimal solution associated to the optimal control (u_1^*, u_2^*) of problem (2.1), then exist adjoint function $A(t) \neq 0$ that satisfies the following*

$$dA = -\frac{\partial H}{\partial X}dt + Q(t)dB(t) \tag{2.2}$$

with terminal conditions

$$A_1(T_e) = -c_1 S(T_e), A_2(T_e) = -c_2 V(T_e), A_3(T_e) = -c_3 I(T_e), A_4(T_e) = -c_4 R(T_e) \tag{2.3}$$

and we have

$$(u_1^*, u_2^*) = \left(\min\Big\{\max\Big\{\frac{kS^*(A_1 - A_2)}{a_3}, 0\Big\}, 1\Big\}, \min\Big\{\max\Big\{\frac{\alpha I^*(A_3 - A_4)}{a_4}, 0\Big\}, 1\Big\}\right) \tag{2.4}$$

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Analysis of a stochastic HIV-1 model with cell-to-cell transmission and Black-Karasinski process

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Abstract
The objective of this research is to analyze a novel stochastic $HIV-1$ epidemic model incorporating the Black-Karasinski process. Initially, we introduce and demonstrate the global positivity of the stochastic model's unique solution. By developing appropriate Lyapunov functions, we derive the condition $\mathcal{R}_0^s > 1$, ensuring the existence of a unique stationary distribution with ergodic properties. Additionally, we establish comprehensive criteria for the exponential extinction of the model. To validate our theoretical findings, we conduct a series of numerical simulations, confirming the accuracy and feasibility of our analytical results.

Keywords: HIV-1 model, Stochastic Dynamics, Black-Karasinski Process, Stationary distribution, Extinction.
2020 Mathematics Subject Classification: 60G10, 60H05, 37A50.

1 Introduction

Understanding the dynamics of HIV-1 infection is critical for developing effective treatment and intervention strategies. While deterministic models have provided valuable insights into the mechanisms of viral spread[3, 4], they often overlook the inherent randomness present in biological systems. This study focuses on the threshold behavior in a stochastic model of HIV-1 infection, integrating virus-to-cell and cell-to-cell transmission, humoral immunity, and the Black-Karasinski process.
The inclusion of stochastic elements, such as the Black-Karasinski process, allows us to capture the random fluctuations in viral replication and immune response, providing a more comprehensive and realistic depiction of the infection dynamics[2, 1]. The integration of virus-to-cell and cell-to-cell transmission modes, along with humoral immunity, underscores the complexity of HIV-1 infection and the necessity of considering both deterministic and random modeling approaches.

By analyzing this stochastic model, we aim to identify critical conditions that determine the persistence or extinction of the virus, enhancing our understanding of the disease and informing the development of more effective treatment strategies. Jiazhe Lin et al[5]. introduced an HIV model that closely resembles the subsequent system:

$$\begin{cases} \frac{dx(t)}{dt} = \Lambda - dx(t) - \beta_1 x(t)v(t) - \beta_2 x(t)y(t), \\ \frac{dy(t)}{dt} = \beta_1 x(t)v(t) + \beta_2 x(t)y(t) - ay(t) - \alpha y(t), \\ \frac{dv(t)}{dt} = ky(t) - uv(t) - pv(t)z(t), \\ \frac{dz(t)}{dt} = cv(t)z(t) - bz(t) - \beta z(t) \end{cases} \tag{1.1}$$

In this dynamic system, various components interact to model the progression of an infection. The variable $x(t)$ represents uninfected cells, which are produced at a rate Λ and die at a rate d. The infected cell population is denoted by $y(t)$, and the viral population is represented by $v(t)$. B cells are indicated by the variable $z(t)$. The model includes two distinct infection rates: β_1 for virus-to-cell transmission and β_2 for cell-to-cell transmission. The system also incorporates death rates (a, u, b) for infected cells, viruses, and B cells, respectively. Additionally, α represents the disease-induced death rate of infected cells, while β signifies the disease-induced death rate of B cells. The parameter k denotes the production of free virus particles by each infected cell. Interactions involving B cells are described by terms such as $pv(t)z(t)$, representing the elimination of the virus by B cells, and $cv(t)z(t)$, representing the production of new B cells stimulated by antigens. The complete system is defined as follows:

$$\Pi = \left\{ (x, y, v, z) | x + y < \frac{\Lambda}{\mu_1}, v + \frac{p}{c} z < \frac{k\Lambda}{\mu_2} \right\}, \mu_1 = \min\{d, a\}, \mu_2 = \min\{d, a\} \min\{u, b\}.$$

Consequently, the ensuing stochastic model is derived as follows:

$$\begin{cases} dx(t) = [\Lambda - dx(t) - \beta_1(t)x(t)v(t) - \beta_2(t)x(t)y(t)]dt, \\ dy(t) = [\beta_1(t)x(t)v(t) + \beta_2(t)x(t)y(t) - ay(t) - \alpha y(t)]dt, \\ dv(t) = [ky(t) - uv(t) - pv(t)z(t)]dt, \\ dz(t) = [cv(t)z(t) - bz(t) - \beta z(t)]dt, \\ d(\ln \beta_1(t)) = \rho_1[\ln \bar{\beta}_1 - \ln \beta_1(t)]dt + \sigma_1 dB_1(t), \\ d(\ln \beta_2(t)) = \rho_2[\ln \bar{\beta}_2 - \ln \beta_2(t)]dt + \sigma_2 dB_2(t). \end{cases} \tag{1.2}$$

For ease of representation, let $\varphi(t) = \frac{\beta_1(t)}{\bar{\beta}_1}$ and $\varphi(t) = \frac{\beta_2(t)}{\bar{\beta}_2}$, making system (1.2) equivalently expressed as :

$$\begin{cases} dx(t) = [\Lambda - dx(t) - \bar{\beta}_1 e^{\varphi} x(t)v(t) - \bar{\beta}_2 e^{\Phi} x(t)y(t)]dt, \\ dy(t) = [\bar{\beta}_1 e^{\varphi} x(t)v(t) + \bar{\beta}_2 e^{\Phi} x(t)y(t) - ay(t) - \alpha y(t)]dt, \\ dv(t) = [ky(t) - uv(t) - pv(t)z(t)]dt, \\ dz(t) = [cv(t)z(t) - bz(t) - \beta z(t)]dt, \\ d\varphi(t) = -\rho_1 \varphi(t)dt + \sigma_1 dB_1(t), \\ d\Phi(t) = -\rho_2 \Phi(t)dt + \sigma_2 dB_2(t). \end{cases} \tag{1.3}$$

2 Main results

Theorem 2.1. *For every prescribed initial value* $(x(0), y(0), v(0), z(0), \varphi(0), \Phi(0))^T \in \mathbb{R}^4_+ \times \mathbb{R}^2$, *model* (1.3) *exhibits a singular global positive solution* $(x(t), y(t), v(t), z(t), \varphi(t), \Phi(t))$ *for all* $t \geq 0$

with almost certain probability, meaning $(x(t), y(t), v(t), z(t), \varphi(t), \Phi(t))^T \in \mathbb{R}^4_+ \times \mathbb{R}^2$ *holds for all* $t \geq 0$ *almost surely.*

Theorem 2.2. *If* $\mathcal{R}_0^s = \dfrac{\Lambda(\bar{\beta}_1 k e^{\frac{\sigma_1^2}{12\rho_1}} + \bar{\beta}_2 u e^{\frac{\sigma_2^2}{8\rho_2}})}{(a+\alpha)ud} > 1$, *system* (1.3) *has at least one stationary distribution* $\Psi(.)$ *on* $\mathbb{R}^4_+ \times \mathbb{R}^2$.

Theorem 2.3. *If* $\mathcal{R}_0^{ex} < 1$, *then HIV−1 within system* (1.3) *with* $x(0) < x_0$ *will experience exponential decay with a probability of* 1. *To be more precise*

$$\limsup_{t\to\infty} \frac{\ln(\frac{\mathcal{R}_0}{x_0}y + \frac{\bar{\beta}_1}{u}v)}{t} \leq \min_{1\leq i\leq 2}\left\{\frac{\bar{\beta}_i}{\zeta_i}\right\}(\mathcal{R}_0^{ex} - 1) < 0 \quad a.s. \tag{2.1}$$

And

$$\mathcal{R}_0^{ex} = \mathcal{R}_0 + \frac{\mathcal{R}_0 ud}{\min_{1\leq i\leq 2}\left\{\frac{\bar{\beta}_i}{\zeta_i}\right\}\mu_1}(e^{\frac{\sigma_1^2}{\rho_1}} - 2e^{\frac{\sigma_1^2}{4\rho_1}} + 1)^{\frac{1}{2}} + \frac{\bar{\beta}_2\Lambda}{\min_{1\leq i\leq 2}\left\{\frac{\bar{\beta}_i}{\zeta_i}\right\}\mu_1}(e^{\frac{\sigma_2^2}{\rho_2}} - 2e^{\frac{\sigma_2^2}{4\rho_2}} + 1)^{\frac{1}{2}}$$

,

where $\zeta_1 = \dfrac{\bar{\beta}_1}{u}, \zeta_2 = \dfrac{\bar{\beta}_1 k + \bar{\beta}_2 u}{u(a+\alpha)}$

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Using mathematical techniques to find the exact structures of solutions to a class of nonlinear wave equations

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Abstract

In this study, we present a unified mathematical model that describes the general form of travelling wave solutions obtained by applying the functional variable algorithm for nonlinear long wave equation, where this algorithm is based on a set of mathematical transformations and a set of theorems with the help Mathematica software. This model can be applied to several complex problems and a variety of modern examples of applications from nonlinear areas of dynamic systems

Keywords : nonlinear systems, functional variable method, ordinary differential equations

2020 Mathematics Subject Classification: 35G50, 65H20, 34N05

1 Introduction

The investigation of the traveling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear phenomena. Nonlinear wave phenomena appear in various scientific and mathematical fields, such as fluid mechanics, plasma physics, biology, solid state physics, chemical kinematics, chemical physics, geochemistry, and optical fibers. This later includes nonlinear wave dispersion phenomena such as dissipation, diffusion, reaction, and convection, which are very important in nonlinear wave equations.

With the rapid development of nonlinear science, scientists and engineers have been interested in asymptotic analytical techniques for nonlinear problems. Although it is very easy for us now to find solutions for linear systems by using the computer, it is always very difficult to solve nonlinear problems numerically or analytically. This is probably due to the fact that the various methods of numerical simulation apply to iteration techniques to find their numerical solutions to non-linear problems, and almost all iterative methods are sensitive to the initial solutions. Thus, it seems difficult

to obtain consistent results in the case of strong nonlinearity. Other evolution equations in physics are completely integrated, but this unique aspect has the disadvantage of causing an unstable disturbance for the equation, in the sense that it does not allow the refinement of the physical modeling.

However, there are ongoing efforts increased to constructing new approaches for finding exact and new solutions, Where the new exact solutions may help us well to find new phenomena by using a variety of powerful methods, such as inverse scattering method[1], bilinear transformation [2], the tanh-sech method [3], extended tanh method [3], sine–cosine method[4], homogeneous balance method (HBM) [5].The homogeneous balance method is a powerful tool to find solitary wave solutions from nonlinear partial differential equations (NPDEs)[6].

In the context of constructing new approaches for solving the complex nonlinear system, we propose the FVA (functional variable algorithm) that has been developed and applied in our previous works [6-7]. Where our main objective is to study the nonlinear system by searching for the behavior of wave solutions.

In this paper, the FVA apply to a model of the nonlinear system represented by the Benjamin-Bona-Mahony (BBM) equation and propose new theorems for finding solutions and creating a new unified mathematical model for the solutions obtained. Afterward, some applications to get it with the help of symbolic software, like Mathematica [6-7], are examined.

2 Main results

we obtain solitary solutions

$$U_1[\xi]=\left(-\frac{\alpha(-\frac{n}{m+1}+1)\operatorname{sech}^2[\frac{1}{2}\sqrt{\alpha}(-1-\frac{n}{m+1})\xi]}{2(1-c)}\right)^{\frac{-1}{(n+m+1)}} \tag{1}$$

$$U_2[\xi]=\left(\frac{\alpha(-\frac{n}{m+1}+1)\operatorname{csch}^2[\frac{1}{2}\sqrt{\alpha}(-1-\frac{n}{m+1})\xi]}{2(1-c)}\right)^{\frac{-1}{(n+m+1)}} \tag{2}$$

we obtain the periodic solutions

$$U_1[\xi]=\left(-\frac{\alpha(-\frac{n}{m+1}+1)\sec^2[\frac{1}{2}\sqrt{-\alpha}(-1-\frac{n}{m+1})\xi]}{2(1-c)}\right)^{\frac{-1}{(n+m+1)}} \tag{3}$$

$$U_2[\xi]=\left(\frac{\alpha(-\frac{n}{m+1}+1)\csc^2[\frac{1}{2}\sqrt{-\alpha}(-1-\frac{n}{m+1})\xi]}{2(1-c)}\right)^{\frac{-1}{(n+m+1)}} \quad (4)$$

Graphically, by using Mathematica software we show in Figure 1 (a) the form of Equation 2 and (b) Equation 4, respectively.

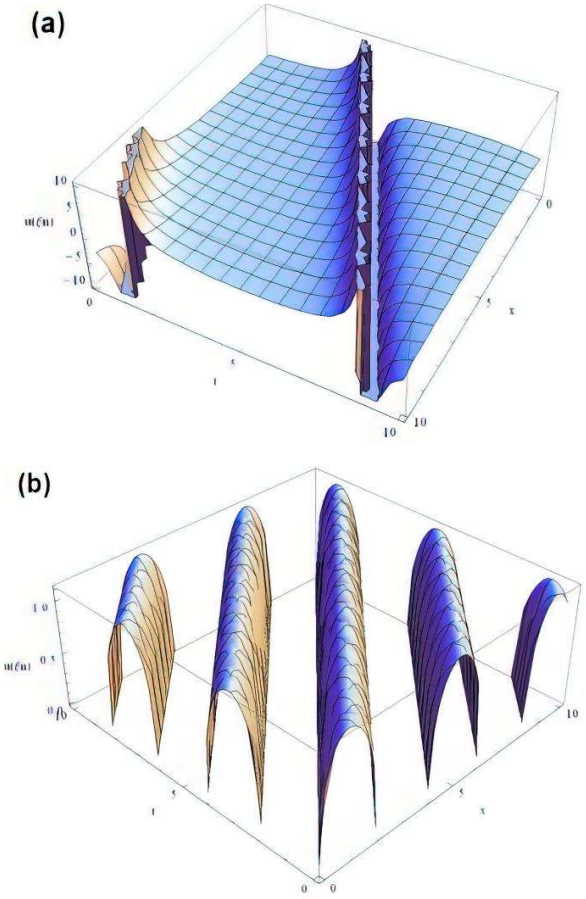

Figure 1 : Plot 3D representation of Equation 2 and (b) Equation 4.

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Customer Behavior in Queueing Models: Analyzing Single-Server System with Working Vacations, Interruptions, and Impatience

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Abstract
We study a $M/M/1$ queueing model with multiple working vacations, vacation interruptions, Bernoulli feedback, balking, and reneging. We assume that arrivals can see the server's state and the queue length before deciding to join or balk. We derive the steady state probabilities and the performance measures of the system. Our model can capture the complex dynamics of queueing systems in various situations and applications.

Keywords: Queueing models, working vacation , balking , reneging.
2020 Mathematics Subject Classification: Primary 60K25, 68M20, 0B22.

1 Introduction

Many practical vacation queueing systems with customers' impatience have been widely applied to many real-life problems as such as call centers, hospital emergency rooms handling critical patients, inventory systems with storage of perishable goods, etc. This subject has attracted the attention of many researchers (see for instance and [5, 4, 2, 3]).
In this work we present an analysis of an infinite buffer single-server Markovian queueing system with Bernoulli feedback, multiple working vacations, vacation interruptions, balking, reneging and retention of reneged customers. In this work, we extend the work of [6] by incorporating the concept of Bernoulli feedback, and reneging. For this system, we investigate the stationary analysis.

The model

In our study, we analyze an $M(\lambda)/M(\mu)/1/FCFS$ queueing system. After serving a customer during regular busy periods, if the system is empty, the server takes a vacation. During vacations, service is provided at a reduced rate μ_0 (where $\mu_0 < \mu_1$), but only the first arriving customer is served during this time. Vacations follow exponential distributions with parameter θ.

Upon completing service during a vacation period, if there are customers in the queue, the vacation is immediately interrupted, and the server switches to the normal busy period. Otherwise, the server remains in vacation mode until the arrival of new customers or the end of the vacation period. If the server returns from a working vacation to find the system empty, another vacation is taken. Otherwise, the server starts providing service to customers and begins a normal busy period immediately. When a customer's service isn't completed before an impatience timer T (exponentially distributed with parameter ξ) expires, they may leave the system. If service is unsatisfactory, customers may return for another service attempt (Bernoulli feedback) with probability β', or they may leave permanently with probability β (where $\beta' + \beta = 1$).
Customers arriving at the system know the current customer count and server state ($J(t)$), where $J(t)$ is either 0 (indicating the server is on a working vacation) or 1 (indicating the server is in the normal busy period). A "balk threshold" $n(i)$ determines whether an arriving customer enters the system, based on the present customer count and system state at arrival time t.

2 Main results

In this section, we establish the steady-state solution of our queueing system. It is clear that the process $\{(N(t), J(t)),\, t \geq 0\}$ is a continuous-time Markov process with state space

$$\Omega = \{(n,i) | n = 0, 1, 2, ..,\ n(i)+1,\ i = 0, 1\}.$$

Let $P_{n,i} = \lim_{t\to+\infty} P\{N(t) = n, J(t) = i\}$ with $(n,i) \in \Omega$; then $\{P_{n,i} : (n,i) \in \Omega\}$ represents the stationary distribution of the process $\{(N(t), J(t)), t \geq 0\}$.

The steady state probabilities

In an infinite-buffer, single-server Markovian queueing system with Bernoulli feedback, observable queues, multiple working vacations, vacation interruptions, balking, and reneging, the steady-state probabilities $P_{n,i}$ are given by:

$$P_{n,0} = \prod_{k=1}^{n} \left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi} \right) P_{0,0},\ \ n = \overline{0, n(0)},$$

$$P_{n(0)+1,0} = \left(\frac{\lambda}{\theta + \beta\mu_0 + (n(0)+1)\xi} \right) \prod_{k=1}^{n(0)} \left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi} \right) P_{0,0},$$

$$P_{n,1} = A + B\left(\frac{\lambda}{\beta\mu_1} \right)^n + D \prod_{k=1}^{n} \left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi} \right),\ \ n = \overline{1, n(0)+1},$$

$$P_{n,1} = \left(\frac{\lambda}{\beta\mu_1} \right)^{n-n(0)-1} \left(A + B\left(\frac{\lambda}{\beta\mu_1} \right)^{n(0)+1} + D \prod_{k=1}^{n(0)+1} \left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi} \right) \right),\ \ n = \overline{n(0)+2, n(1)+1},$$

where

$$D = -\frac{\Phi_1}{\Phi_2} P_{0,0},\ \ \Phi_1 = \theta\lambda + \theta^2 + \theta\beta\mu_0 + n\xi\theta + \theta\xi + \beta\mu_0\lambda,$$

$$\Phi_2 = -\beta\mu_1\theta - \beta^2\mu_1\mu_0 - n\beta\mu_1\xi - \beta\mu_1\xi + \theta\lambda + \theta^2 + 2\theta\beta\mu_0 + 2n\xi\theta + \theta\xi + \beta\mu_0\lambda$$

$$+\beta^2\mu_0^2 + 2n\xi\beta\mu_0 + \xi\beta\mu_0 + n\lambda\xi + n^2\alpha^2\xi^2 + n\alpha^2\xi^2,$$

$$B = \frac{1}{\rho(\rho-1)}(\Gamma_2 - \Gamma_1)P_{0,0}, \;\; A = \left(\frac{\rho\Gamma_1 - \Gamma_2}{\rho - 1}\right)P_{0,0},$$

such that

$$\Gamma_1 = \frac{\Delta_1}{\Delta_2}, \;\; \Gamma_2 = Q_1 - Q_2,$$

$$\Delta_1 = \lambda\left(\theta\beta^2\mu_0^2 + \theta\lambda^2 + 2\lambda\theta^2 + \beta\mu_0\lambda^2 + \lambda\beta^2\mu_0^2 - n\lambda\beta\mu_1\xi + n\lambda^2\xi + 3\lambda\theta\beta\mu_0 + 3n\lambda\xi\theta\right.$$

$$+2n\lambda\xi\beta\mu_0 + \lambda\xi\beta\mu_0 + \lambda\xi\theta - \beta\mu_1\xi\lambda + 2\theta^2\beta\mu_0 + \theta^2\xi + \theta^3 + 2n\xi\theta^2 + n^2\lambda\xi^2$$

$$\left.+n\lambda\xi^2 + 2n\theta\xi\beta\mu_0 + \theta\xi\beta\mu_0 + n^2\theta\xi^2 + n\theta\xi^2\right),$$

$$\Delta_2 = (-\beta\mu_1\theta - \beta^2\mu_1\mu_0 - n\beta\mu_1\xi - \beta\mu_1\xi + \theta\lambda + \theta^2 + 2\theta\beta\mu_0 + 2n\xi\theta + \theta\xi + \beta\mu_0\lambda$$

$$+\beta^2\mu_0^2 + 2n\xi\beta\mu_0 + \xi\beta\mu_0 + n\lambda\xi + n^2\xi^2 + n\xi^2)\left(\lambda + \theta + \xi + \beta\mu_0\right)\beta\mu_1,$$

$$Q_1 = \lambda^2\left(\frac{2\theta\lambda + \theta^2 + \theta\beta\mu_0 + 2\theta\xi + \theta\beta\mu_1 + \lambda^2 + \beta\mu_0\lambda + 2\lambda\xi + \beta\mu_1\lambda + 2\beta\mu_1\xi}{\beta^2\mu_1^2\left(\lambda + \theta + \xi + \beta\mu_0\right)\left(\lambda + \theta + \beta\mu_0 + 2\xi\right)}\right),$$

$$Q_2 = -\frac{\Theta_1}{\Theta_2},$$

$$\Theta_1 = \lambda^2(\theta\lambda + \theta^2 + \theta\beta\mu_0 + n\xi\theta + \theta\xi + \beta\mu_0\lambda),$$

$$\Theta_2 = (-\beta\mu_1\theta - \beta^2\mu_1\mu_0 - n\beta\mu_1\xi - \beta\mu_1\xi + \theta\lambda + \theta^2 + 2\theta\beta\mu_0 + 2n\xi\theta + \theta\xi + \beta\mu_0\lambda + \beta^2\mu_0^2$$

$$+2n\xi\beta\mu_0 + \xi\beta\mu_0 + n\lambda\xi + n^2\xi^2 + n\xi^2)\left(\lambda + \theta + \xi + \beta\mu_0\right)\left(\lambda + \theta + \beta\mu_0 + 2\xi\right).$$

And

$$P_{0,0} = \left(\sum_{n=0}^{n(0)}\prod_{k=1}^{n}\left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi}\right) + \left(\frac{\lambda}{\theta + \beta\mu_0 + (n(0)+1)\xi}\right)\prod_{k=1}^{n(0)}\left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi}\right)\right.$$

$$+\sum_{n=1}^{n(0)+1}\left(\left(\frac{\rho\Gamma_1 - \Gamma_2}{\rho - 1}\right) + \left(\frac{\Gamma_2 - \Gamma_1}{\rho(\rho-1)}\right)\left(\frac{\lambda}{\beta\mu_1}\right)^n - \frac{\Phi_1}{\Phi_2}\prod_{k=1}^{n}\left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi}\right)\right)$$

$$+\sum_{n=n(0)+2}^{n(1)+1}\left(\left(\frac{\lambda}{\beta\mu_1}\right)^{n-n(0)-1}\left(\left(\frac{\rho\Gamma_1 - \Gamma_2}{\rho - 1}\right) + \left(\frac{\Gamma_2 - \Gamma_1}{\rho(\rho-1)}\right)\left(\frac{\lambda}{\beta\mu_1}\right)^{n(0)+1}\right.\right.$$

$$\left.\left.\left.\left.-\frac{\Phi_1}{\Phi_2}\prod_{k=1}^{n(0)+1}\left(\frac{\lambda}{\lambda + \theta + \beta\mu_0 + k\xi}\right)\right)\right)\right)\right)^{-1}.$$

Some performance measures

- The probability that the system is in a normal busy period (P_{nbp}) :

$$P_{nbp} = \mathbb{P}(\text{normal busy vacation}) = \sum_{n=1}^{n(1)+1} P_{n,1}.$$

- The probability that the system is on working vacation period (P_{wv}) :

$$P_{wv} = \mathbb{P}(\text{working vacation period}) = \sum_{n=0}^{n(0)+1} P_{n,0} = 1 - \mathbb{P}(\text{normal busy period})$$

- The average number of customers in the system (L_s) :

$$L_s = \sum_{n=1}^{n(0)+1} nP_{n,0} + \sum_{n=1}^{n(1)+1} nP_{n,1},$$

- The probability of balking (P_{balk}) :

$$P_{balk} = P_{n(0)+1,0} + P_{n(1)+1,1}.$$

- Average reneging rate (R_{ren}) :

$$R_{ren} = \xi \sum_{n=1}^{n(0)+1} nP_{n,0}.$$

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Dynamical Analysis of a New Fractional Symmetry Map: Chaos, Bifurcation, Control

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Abstract
This study describes an innovative discrete-time symmetry system with commensurate fractional order. The non-linear dynamics of this commensurate fractional discrete symmetry system are investigated using various numerical techniques, including the bifurcation diagrams and phase portraits . Finally, non-linear controllers are revealed to stabilize the symmetric fractional order map's states.

Keywords: Dynamical systems, Fractional, Chaos
2020 Mathematics Subject Classification: Primary 74Hxx, 90C32, 74H65.

1 Introduction

In recent years, discrete fractional calculus has received great interest from many researchers due to their memory properties, interest has also been concentrated on fractional difference operators and discrete fractional calculus within the past decade. Thanks to these unique characteristics, fractional order iterated maps have been deeply studied in academic fields including engineering, physics, and economics, among others. Chaotic discrete-time system has become an attractive research subject in nearly all kinds of fields Recently, a few studies have reported the dynamics of the fractional forms of some conventional chaotic maps by applying fractional discrete calculus. The main advantage of this type of calculus is the memory effect incurred in the fractional differentiation. The structure of the work: In Section 1, we define some basic preliminaries to discrete fractional calculus and provide the fractional discrete version of the system. The mathematical model for symmetry map is presented in Section 2. In Section 3, non-linear 3D controllers are revealed to stabilize the symmetric fractional order map's states.

2 Main results

The objective of this paper is to investigate and analyze the chaotic dynamics of a newly proposed commensurate discrete symmetry system as follows:

$$\begin{cases} {}^{c}\Delta_{a}^{\vartheta}x(r) = y(m-1+\vartheta) - x(m-1+\vartheta), \\ {}^{c}\Delta_{a}^{\vartheta}y(r) = \sin(z(m-1+\vartheta)) - y(m-1+\vartheta), \\ {}^{c}\Delta_{a}^{\vartheta}z(r) = \mu x(m-1+\vartheta) + \alpha y(m-1+\vartheta) - \beta\sin(z(m-1+\vartheta)) - z(m-1+\vartheta). \end{cases} \tag{2.1}$$

Where x, y, z denote the state variables, μ, α, β are the bifurcation parameter, and ϑ is fractional orders. The study revealed that the system exhibits intricate and diverse dynamical characteristics. The commensurate fractional-order cases are examined using various methods such as plotting phase portraits, creating bifurcation charts, non-linear controllers are revealed to stabilize the symmetric fractional order map's states. In conclusion, the results indicate that the proposed model produces chaotic behaviour with a higher complexity degree and a broader range of chaotic regions when fractional orders are varied.

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Mathematical modelisation for bovine Babesiosis epidemic.

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Abstract
The main purpose of this talk is to analyze a new dynamical model pertaining to bovine Babesiosis transmission, and investigate its consequent morphology. We present and study various ramifications of our mathematical model for bovine Babesiosis spread, given, firstly, by a temporal system of ordinary differential equations and, finally, by a spatio-temporal system consisting of reaction-diffusion equations. Diffusion terms are incorporated into the model, using specific derivations for both infected ticks and infected bovines. Furthermore, mechanisms for the nearest neighbors' infection are integrated into the model. We determine mathematically the basic reproduction number $\mathcal{R}_0$ via the next-generation matrix. Then, we analyze the stability of the equilibria and the effects of the mobility of infectious agents, being they either ticks or bovines. Finally, model-based analytical-numerical results are obtained and displayed in graphical profiles. The results of the proposed model and the health ramifications are then raised, discussed and validated.

Keywords: Babesiosis disease, Bovine mobility, Reaction-diffusion equations, Spatio-temporal model.
2020 Mathematics Subject Classification: Journal of Theoretical Biology 480 (2019) 192–204.

1 Introduction

Bovine Babesiosis is caused by the sting of ticks, a kind of parasitic infection which attacks the cattle. Babesia bovis and Babesia bigemia are the most known varieties found in tropical and subtropical areas (Friedman and Yakubu, 2014). Intravascular haemolysis, coagulation disorders and release of pharmacologically active substances resulting in vascular malfunction and hypotensive shock are the major pathogenesis of this disease. High fever, de- pression, anorexia, hemoglobinemia and hemoglobinuria are the predominant signs (Demeke et al., 2018).
To control the transmission of Babesiosis, vaccination and antiparasitic treatments are the basic (control) strategies which have been performed (Friedman and Yakubu, 2014). But, due to some problems and side effects, some vaccines and drugs have been eliminated from these strategies (Solorio-Rivera and Rodriguez- Vivas, 1997a; 1997b). All these reasons make this disease interesting to be modeled mathematically in order to know its dynamical behavior. In fact, knowing its behavior can lead to the design of new control strategies.

In this talk, we provide and study a more accurate spatio-temporal model for the bovine Babesiosis transmission by using individual models for the infection of nearest neighbors (Postnikov and Sokolov, 2007) and model derivation (Brauer et al., 2008) of the infected bovines and infected ticks. We study two important factors concerning the diffusion of bovine Babesiosis: the infection of nearest neighbors effect and the mobility of the infectious individuals (bovine and tick).
The corresponding ordinary differential equations describing temporal evaluation of bovines and ticks are given by

$$\begin{cases} \mathbf{S}'_B(t) = -\beta_{TB} B \mathbf{S}_B(t) \mathbf{I}_T(t) + (\mu_B + \alpha_B) \mathbf{R}_B(t) \\ \mathbf{E}'_B(t) = \beta_{TB} B \mathbf{S}_B(t) \mathbf{I}_T(t) - \nu_B \mathbf{E}_B(t) \\ \mathbf{I}'_B(t) = \nu_B \mathbf{E}_B(t) - \lambda_B \mathbf{I}_B(t) \\ \mathbf{R}'_B(t) = \lambda_B \mathbf{I}_B(t) - (\mu_B + \alpha_B) \mathbf{R}_B(t) \\ \mathbf{S}'_T(t) = -\beta_{BT} B \mathbf{S}_T(t) \mathbf{I}_B(t) + p\mu_T (\mathbf{I}_T(t) + \mathbf{E}_T(t)) \\ \mathbf{E}'_T(t) = \beta_{BT} B \mathbf{S}_T(t) \mathbf{I}_B(t) - (p\mu_T + \nu_T) \mathbf{E}_T(t) \\ \mathbf{I}'_T(t) = -p\mu_T \mathbf{I}_T(t) + \nu_T \mathbf{E}_T(t). \end{cases} \tag{1.1}$$

2 Main results

Proposition 2.1. *System (1.1) has the disease-free steady state* $E_0 = (1, 0, 0, 0, 0)$ *whatever the values of the parameters of the system, whereas, only if* $\mathcal{R}_0 > 1$ *there is a unique endemic steady state* $E_1 = (S_B^*, E_B^*, I_B^*, E_T^*, I_T^*)$ *in the interior of* Ω.

Proof. We denote by $E_{i=0,1}$ the steady states points of system (1.1), that is, the equilibrium point where $\mathbf{S}'_B = \mathbf{E}'_B = \mathbf{I}'_B = \mathbf{E}'_T = \mathbf{I}'_T = 0$ for all $t > t_0$. Then, the disease-free steady state $E_0 = (1, 0, 0, 0, 0)$ and the endemic point is given by

$$\begin{aligned} S_B^* &= \frac{ad^2fk + cd^2fk + acdeg + acdek + acdfk}{abceg + abdeg + acdeg + bcdeg + acdek} \\ E_B^* &= \frac{abdeg - ad^2fk}{abceg + abdeg + acdeg + bcdeg + acdek} \\ I_B^* &= \frac{abceg - acdfk}{abceg + abdeg + acdeg + bcdeg + acdek} \\ E_T^* &= \frac{abcegk - acdfk^2}{abceg^2 + abcegk + abcfgk + abdfgk + bcdfgk} \\ I_T^* &= \frac{abceg - acdfk}{abceg^2 + abcegk + abcfgk + abdfgk + bcdfgk} \end{aligned}$$

where

$$\begin{aligned} a = \mu_B + \alpha_B, \quad b = \beta_{TB} B, \quad c = v_B, \quad d = \lambda_B, \\ e = \beta_{BT} B, \quad f = p\mu_T + v_T, \quad g = v_T, \quad k = p\mu_T. \end{aligned} \tag{2.1}$$

Notice that if $\mathcal{R}_0 > 1$, then $(bge - dfk) > 0$. Therefore, since all the parameters of the equations are greater than zero,

$$S_B^*, E_B^*, I_B^*, E_T^*, I_T^* > 0$$

So, there is a (unique) endemic equilibrium point $E_1 = (S_B^*, E_B^*, I_B^*, E_T^*, I_T^*)$ in the interior of Ω.
Now, we will demonstrate that if $\mathcal{R}_0 \leqslant 1$ then the unique equilibrium point in Ω is the disease free equilibrium point $E_0 = (1, 0, 0, 0, 0)$.
We shall distinguish two cases:

- $\mathcal{R}_0 < 1$, in this case: $(bge - dfk) < 0$. So it is easy to check that $E_B^*, I_B^*, E_T^*, I_T^* < 0$. Therefore, this point remains outside the region Ω.
- $\mathcal{R}_0 = 1$, In this situation because $bge - dfk = 0$, we have that $E_B^*, I_B^*, E_T^*, I_T^* = 0$. Therefore, the last four equations of the system (1.1) are checked automatically and, replacing the value of $E_B^*, I_B^*, E_T^*, I_T^*$ in the first equation, we found

$$S_B' = (\mu_B + \alpha_B)(1 - S_B)$$

Thus, it must be $S_B = 1$. But, if it is $S_B = 1$ we have disease-free steady state.

□

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An adaptive step size for chaotic local search algorithm

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Abstract
In this paper, a chaotic strategy based on a 2-D chaotic application is proposed. This method reduces the search space of optimized variables and improves the search precision, which has higher search efficiency. In order to solve the problem between fast convergence and low steady-state, a suitable step size control is proposed. The simulation results show that the new algorithm has faster convergence.

Keywords: chaos; global optimization; evolutionary algorithms; step size control; chaos optimization algorithm.

2020Mathematics Subject Classification:.

1 Introduction

This work aims to study chaotic dynamics produced by a two-dimensional nonlinear system governed by recurrence equations. Chaotic systems are dynamic systems that evolve over time and are both deterministic and exhibit an instability called "sensitivity to initial conditions". These phenomena are therefore not predictable in the long term. Based on these properties of chaos, some new search algorithms called chaotic optimization algorithms (COA) are presented in the literature. (COA) methods can easily escape local minima unlike stochastic optimization algorithms. Therefore this work is devoted to the application of chaos theory to optimization. Its objective was to contribute to the development of a global optimization method based on chaotic sequences, here the application of Gingerbreadman. This method is called COGM.

2 Main results

1- The Principal of Chaos Optimization

2- Design of the Algorithm

3- Step-size control

4- Numerical Results

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Introduction of a model describing two zones with an interface.

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Abstract
In our study, we examine a population of the triatomines structured both in terms of time and space. The demographic and spatial dispersion processes are described using reaction-diffusion equations in a two-dimensional space. By working within appropriate functional spaces, we convert this system of partial differential equations into an abstract differential equation. Our initial goal is to demonstrate that the operator generates an analytic semigroup. Subsequently, we establish the existence and uniqueness of a solution to the corresponding Cauchy problem.

Keywords: Transmission problem; reaction-diffusion; analytical semi-group; sector operator
2020 Mathematics Subject Classification: Primary 34K30, 47D06, 92B05, 35A09, 35C15, 35K57, 35K90.

1 Introduction

In this communication, we investigate the invasion of a village by the domestic species Triatoma Dimidiata. The village is located next to a forest, serving as the natural habitat for these vectors. They enter the village to feed, primarily consuming blood meals from the humans or mammals with whom they coexist. It is during this feeding phase that the primary transmission of Trypanosoma Cruzi from the vector to the host occurs.

2 Main results

We have applied the fixed point theorem to the abstract Cauchy problem, to show the uniqueness of a local solution.

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Some regularity of the solution to nonlinear singular anisotropic elliptic equations with variable exponents

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Abstract
This work is devoted to study the existence and regularity results for non-negative weak solutions of nonlinear singular degenerate elliptic equations with general anisotropic diffusivity with variable exponents. The data are assumed to be merely singular non-linearity term.

Keywords: Anisotropic elliptic equations, Nonlinear elliptic equations, Singular elliptic equations, Degenerate elliptic equations, Weak solutions, Variable exponents.
2020 Mathematics Subject Classification: Primary 35J60,35J75,35J70. Secondary: 35D30,35B65.

1 Introduction

This study is dedicated to exploring the existence and regularity to solutions of nonlinear singular anisotropic elliptic equations with principal part characterized by degenerate coercivity:

$$\begin{gathered} -\sum_{i=1}^{N} D^i\left(\frac{|D^i u|^{p_i(x)-2}D^i u}{(\ln(e+|u|))^{\sigma_i(\cdot)}}\right)+u|u|^{r(\cdot)-1}=\frac{f}{u^{\gamma(\cdot)}}+\mu \text{ in } \Omega, \\ u>0 \quad \text{in } \Omega, \\ u=0 \quad \text{on } \partial\Omega. \end{gathered} \tag{1.1}$$

where Ω is a bounded open domain in $\mathbb{R}^N$ $(N \geq 2)$ with Lipschitz boundary $\partial\Omega$, the variable exponents $p_i : \overline{\Omega} \longrightarrow (1,\infty)$, $\sigma_i : \overline{\Omega} \longrightarrow [0,\infty)$, $i = 1,\ldots,N$, and $r : \overline{\Omega} \longrightarrow (0,\infty)$ are continuous functions.
We suppose that: $\overrightarrow{p}(\cdot) \in (C_+(\overline{\Omega}))^N$,

$$\forall x \in \overline{\Omega},\ p_+(x) < \overline{p}^\star(x). \tag{1.2}$$

with

$$\overline{p}^\star(x)=\begin{cases} \frac{N\overline{p}(x)}{N-\overline{p}(x)}, & \text{for } \overline{p}(x)<N, \\ +\infty, & \text{for } \overline{p}(x)\geq N. \end{cases} \quad \overline{p}(x)=\frac{N}{\sum_{i=1}^{N}\frac{1}{p_i(x)}},$$

and $p_+(x) = \max_{1\le i\le N} p_i(x)$ while γ as in (1.3), $f \geq 0$ belongs to $L^1(\Omega)$, and $\mu \geq 0$ belongs to $L^1(\Omega)$.

This work is motivated by the results of [1], where the author has treated the isotropic case and has studied the existence, regularity, and uniqueness of non-negative weak solutions to the singular elliptic problem involving variable exponents:

$$\begin{cases} -\text{div}\left(\frac{\widehat{a}(x,Du)}{(\ln(e+|u|))^{\sigma(\cdot)}}\right) = \frac{f}{u^{\gamma(\cdot)}} + \mu, & \text{in } \Omega, \\ u > 0, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega, \end{cases}$$

Where $0 \leq \mu \in L^1(\Omega)$, $0 \leq f \in L^{m(\cdot)}(\Omega)$, and $\widehat{a} : \Omega \times \mathbb{R}^N \to \mathbb{R}^N$ is a Leray-Lions type operator satisfying some appropriate assumptions while $\gamma : \overline{\Omega} \to (0,1)$, $m : \overline{\Omega} \to (1,+\infty)$, $\sigma : \overline{\Omega} \longrightarrow [0,\infty)$, and the variable exponents $p : \overline{\Omega} \to (1,+\infty)$ are continuous functions satisfying some appropriate hypotheses, with $\gamma : \overline{\Omega} \longrightarrow (0,1)$ is a continuous function satisfies:

$$0 < \gamma^- := \min_{x\in\overline{\Omega}} \gamma(x) \leq \gamma^+ := \max_{x\in\overline{\Omega}} \gamma(x) < 1, \quad |\nabla\gamma| \in L^\infty(\Omega), \text{ for all } x \in \overline{\Omega}. \tag{1.3}$$

In the anisotropic case when the right hand side, f is assumed to belong to $L^1(\Omega)$ with γ as in (1.3), we will analyse the limit case $f \in L^1(\Omega)$ showing that the lower order term $u|u|^{r(\cdot)-1}$ has still a regularizing effect on the solutions as well as the regularizing given by the singular non-linearity term $\frac{f}{u^{\gamma(\cdot)}}$.
We point out that the regularizing effect of the lower order term and the singular non-linearity term is well known for similar problems. It was pointed out in [1, 2, 4, 5, 6], for elliptic and parabolic problems.

2 Main results

Our aim is to prove the existence of non-negative weak solutions solutions to problem (1.1). Here the notions of solutions we will employ.

Definition 2.1. If $0 < \gamma^+ < 1$ and $\mu \geq 0$ belongs to $L^1(\Omega)$. Then, we say A non-negative function $u \in \mathring{W}^{1,\vec{p}(\cdot)}(\Omega)$ is a weak solution for problem (1.1) if the following conditions are satisfied:

1. $u \in W_0^{1,1}(\Omega)$, $u|u|^{r(\cdot)-1} \in L^1(\Omega)$, $|D^i u|^{p_i(x)-2}D^i u \in L^1(\Omega), \forall i = 1,\dots,N$, $\frac{f}{u^{\gamma(\cdot)}} \in L^1_{loc}(\Omega)$,
2. $$\begin{aligned} &\sum_{i=1}^N \int_\Omega \left(\frac{|D^i u|^{p_i(x)-2}D^i u}{(\ln(e+|u|))^{\sigma_i(\cdot)}}\right)\cdot D^i\phi\, dx + \int_\Omega u|u|^{r(\cdot)-1}\cdot\phi\, dx \\ &= \int_\Omega \frac{f}{u^{\gamma(\cdot)}}\cdot\phi\, dx + \int_\Omega \mu\cdot\phi\, dx \end{aligned} \tag{2.1}$$

for every $\phi \in C_0^\infty(\Omega)$.

In this work we will show the following result.

Theorem 2.1. *Under the assumptions* (1.2)-(1.3). *If* $\mu \in L^1(\Omega)$ *and* $f \in L^1(\Omega)$ *are non-negative functions, the variable exponents* $p_i : \overline{\Omega} \to (1, +\infty)$, $\sigma_i : \overline{\Omega} \to [0, +\infty)$ $i = 1, \cdots, N$, *and* $r : \overline{\Omega} \to (0, +\infty)$ *are continuous functions such that, for all* $x \in \overline{\Omega}$ *and for all* $i = 1, \ldots, N$

$$1 + \frac{\sigma_i(x) + 1 + \gamma(x)}{r(x)} < p_i(x) < 1 + \frac{r(x)}{\sigma_i(x) + 1 + \gamma(x)}, \tag{2.2}$$

$$\max\left(\frac{\sigma_i(x) + 1 + \gamma(x)}{p_i(x) - 1}; (\sigma_i(x) + 1 + \gamma(x))(p_i(x) - 1)\right) < r(x), \tag{2.3}$$

where

$$|\nabla r| \in L^\infty(\Omega), \quad |\nabla \sigma_+| \in L^\infty(\Omega), \quad |\nabla \gamma| \in L^\infty(\Omega),$$

Then, problem (1.1) *has at least one non-negative solution* $u \in \mathring{W}^{1,\vec{q}(\cdot)}(\Omega) \cap L^{r(\cdot)}(\Omega)$ *satisfying* (2.1) *where* $q_i(\cdot)$ *are continuous functions on* $\overline{\Omega}$ *satisfying*

$$1 < q_i(x) < \frac{p_i(x) r(x)}{r(x) + \sigma_i(x) + 1 + \gamma(x)}, \quad \forall x \in \overline{\Omega},\ i = 1, \ldots, N, \tag{2.4}$$

In order to prove these main results, we will work by approximation, truncating the singular term $\frac{f}{u^{\gamma(\cdot)}}$, so that it becomes not singular at the origin. We will get some a prior estimates on the solutions u_n of the approximating problems, which will allow us to pass to the limit and find a solution to problem (1.1). In order to obtain a solution in the sense described above, we will apply the Schauder's fixed point theorem and employ some fresh techniques.

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A numerical solution of Parabolic Quasi Variational inquality with non-linear right hand side using Newton Multigrid method

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Abstract
In this article, we apply three numerical methods to study the $\mathbb{L}^\infty$-convergence of the Newton-Multigrid method for parabolic quasi-variational inequalities with a non-linear right hand side. To discretize the problem, we utilize a finite element method for the operator and Euler scheme for the time. To obtain the system discretization of the problem, we reformulate the parabolic quasi-variational inequality as a Hamilton-Jacobi-Bellman equation. For linearizing the problem on the coarse grid, we employ Newton's method as an external iteration to obtain the Jacobian system. On the smooth grid, we apply the multigrid method as an interior iteration on the Jacobian system. Finally, we provide a proof of the $\mathbb{L}^\infty$-convergence of the Newton-Multigrid method for parabolic quasi-variational inequalities with a nonlinear right hand, with giving an numerical example of this problem.

Keywords: Newton's method, Multigrid method, Parabolic Variational Inequality, Finite element method, Hamilton-Jacobi-Bellman equation
2020 Mathematics Subject Classification:

1 Introduction

Before studying the $\mathbb{L}^\infty$-convergence of the Newton-Multigrid method for parabolic quasi-variational inequalities (QVIp) with a nonlinear right hand side, we will define our problem. Let $\mathbb{A}$ be an elliptic operator, and Ω is an open domain in $\mathbb{R}^d$, where $d \geq 1$, with sufficiently regular bounds Γ. We provide the definition of our proposed problem:
find $\mathtt{u} \in L^2([0,t]; H_0^1(\Omega))$ solution of :

$$\begin{cases} \dfrac{\partial \mathtt{u}}{\partial t} + \mathbb{A}\mathtt{u} \leq f(\mathtt{u}) \quad ; \quad \mathtt{u} - \psi \leq 0, \\ \left(\dfrac{\partial \mathtt{u}}{\partial t} + \mathbb{A}\mathtt{u} - f(\mathtt{u})\right)(\mathtt{u} - \psi) = 0 \ \text{ in } \mathbf{Q}_t, \\ \mathtt{u}(x,t) = \mathtt{u}_0 \qquad in \quad \Omega \\ \mathtt{u}(x,t) = 0 \qquad in \quad \sum. \end{cases} \tag{1.1}$$

where $\mathbf{Q}_t$ defined as: $\mathbf{Q}_t = \Omega \times [0,t]$, $\sum = \Gamma \times [0,t]$, and the obstacle $\psi \in W^{2,\infty}$. such that $\psi \geq 0$. We assumed that the function $f(.)$ is nonlinear, continuous, and Lipschitz.

$$|f(\mathtt{y}) - f(\mathtt{z})| \leq c\,|\mathtt{y} - \mathtt{z}| \quad \forall \mathtt{y}, \mathtt{z} \in \mathbb{R}^d, \quad c > 0.$$

We have divided this research into six parts as follows:
Part one: We formulate our problem and provide the necessary and sufficient conditions for studying the $\mathbb{L}^\infty$-convergence of Newton Multigrid methods.
Part two: To facilitate understanding, we introduce important symbols and hypotheses that will be used in our research. Subsequently, we present the theorem that proves the existence and uniqueness of the solution for the continuous problem.
Part three: To study the discrete problem , we define the space $\mathbb{V}_h$ that

$$\mathbb{V}_h = \{\mathtt{v}_h \in L^2([0,T]; H_0^1(\Omega)) \cap C^1([0,T]; H_0^1(\overline{\Omega})),\ \mathtt{v}_h(.,t) \in P_1\}.$$

with P_1 is a Lagrange polynomial with a degree ≤ 1 .
To discretize, we employ the finite element approximation on the operator $\mathbb{A}$ and use the Euler scheme for time discretization, resulting in the following discrete problem: Find $\mathtt{u}_h^n \in \mathbb{V}_h$ as a solution of:

$$\begin{cases} \langle \mathbb{B}_h \mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n \rangle \quad \geq \quad \langle F^n(\mathtt{u}_h^n), \mathtt{v}_h - \mathtt{u}_h^n \rangle\,, \\ \mathtt{u}_h^n \leq \psi_h, \\ \mathtt{u}_h(x) = \mathtt{u}_h(x,0) \quad in \quad \Omega \\ \mathtt{u}(x,t) = 0. \quad in \quad \Sigma \end{cases} \tag{1.2}$$

Remark 1.1. The problem (1.1) is equivalent to the problem (1.2), see [5][8][9][10].

Part four: To solve the Parabolic Quasi Variational Inequality (PQVI) using Newton-Multigrid methods, we discretize the domain through the finite element method and employ the Euler scheme for time discretization. Subsequently, we reformulate the PQVI as the Hamilton-Jacobi-Bellman (HJB) equation, expressed as follows:

$$\max_{1 \leq i \leq N} \left(\mathbb{B}_{k,i} \mathtt{u}_{k,i}^v - F(\mathtt{u}_{k,i}^v);\ \mathtt{u}_{k,i}^v - \psi_k \right) = 0$$

We employ the iterative procedure introduced by Hoppe (see [22][23][24]) to solve the obtained system. Subsequently, we apply Newton's method to address the nonlinear system by linearizing it to obtain the Jacobian system. Following this, we employ the multi-grid method on the Jacobian system in all iterations.
Part five: To establish the $\mathbb{L}^\infty$-convergence of the Newton-Multigrid method, we rely on the approximation and smoothing suggestions proposed by Huckbach (see[20][21]). In this research, we demonstrate the $\mathbb{L}^\infty$-convergence in the addressed problem based on the techniques presented in Huckbach's work.
Part six: Here, we give a numerical example that confirms the results obtained in the theoretical part

2 Newton-Multigrid method

2.1 Assumpations and symbols

Consider $\mathbb{A}$ as an elliptic operator defined by:

$$\mathbb{A} = -\sum_{1\leq i,j\leq d}^{d} \mathtt{a}_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j} + \sum_{j=1}^{d} \mathtt{a}_j(x)\frac{\partial}{\partial x_j} + \mathtt{a}_0(x). \tag{2.1}$$

The coefficients $\mathtt{a}_{ij}(x),\ \mathtt{a}_j(x),\ \mathtt{a}_0(x) \in L^\infty(\Omega)\cap C^2(\bar{\Omega})$ meet the following conditions:

$$\mathtt{a}_{ij} = \mathtt{a}_{ji}, \quad \mathtt{a}_0 \geq \alpha, \quad \alpha{>}0, \quad x \in \bar{\Omega}, \tag{2.2}$$

and $\mathtt{a}(.,.)$ is associated with the elliptic operator $\mathbb{A}$, where $\mathtt{a}(.,.)$ is a continuous and coercive bilinear form, as given by:

$$\mathtt{a}(\mathtt{u},\mathtt{v}) = \int_\Omega \sum_{1\leq i,j\leq d}^{d} \mathtt{a}_{ij}(x)\frac{\partial \mathtt{u}}{\partial x_i}\frac{\partial \mathtt{v}}{dx_j} + \sum_{j=1}^{d} \mathtt{a}_j(x)\frac{\partial \mathtt{u}}{\partial x_j} + \mathtt{a}_0(x)\mathtt{u}\mathtt{v}\, dx. \tag{2.3}$$

$$\mathtt{a}(\mathtt{v},\mathtt{v}) + \omega\|\mathtt{v}\|^2_{L^2(\Omega)} \geq \gamma\|\mathtt{v}\|^2_{H^1_0(\Omega)}, \quad \gamma{>}0 \quad , \quad \omega{>}0, \quad \forall \mathtt{v} \in H^1_0(\Omega). \tag{2.4}$$

Let $f(\mathtt{u})$ be a non-linear right-hand function given by:

$$f(\mathtt{u}) \in (L^\infty \cap C^1) \quad \text{and} \quad \frac{\partial f}{\partial \mathtt{u}} \geq 0 \qquad \text{in} \qquad \Omega$$

2.2 Continuous problem

After applying Green's formulation, we obtain the weak formulation of our proposed problem. Find $\mathtt{u}$ in $L^2([0,t];H^1_0(\Omega))$, a solution to the following:

$$\begin{cases} \left(\dfrac{\partial \mathtt{u}}{\partial t}, \mathtt{v}-\mathtt{u}\right) + \mathtt{a}(\mathtt{u},\mathtt{v}-\mathtt{u}) \geq (f(\mathtt{u}),\mathtt{v}-\mathtt{u}), \\ \mathtt{u} \leq \psi, \\ \mathtt{u}(x,t) = \mathtt{u}_0 \qquad in \quad \Omega \\ \mathtt{u}(x,t) = 0 \quad in \quad \Sigma \end{cases} \tag{2.5}$$

Certainly, the problem (2.5) has a unique solution, as supported by fixed-point theory and in accordance with the previously established assumptions, see [7][10] [12].

3 Discritization:

3.1 Discretize

To establish a multigrid loop, consider a decreasing sequence $\{h_k\}_{k=0}^{l}$ such that:

$$0 < h_k < h_{k+1} < 1, \quad 0 \leq k \leq m-1,$$

$\{T_h\}$ is a family of uniform triangulations where $\Omega_h = \underset{T \in T_h}{\cup} T_h$, and for all T_h, we have:

$$\Omega_h \subset \Omega_{h+1} \subset \Omega, \quad dist(\Gamma_h, \Gamma) \leq ch_k^2, \quad h_{k+1}h_k \leq c_0.$$

We associate each h_k with the following symbols to facilitate our work.

$$\Omega_{h_k} = \Omega_h \quad , \quad \mathbb{V}_{h_k} = \mathbb{V}_h \quad , \quad \mathbb{A}_{h_k} = \mathbb{A}_h.$$

We consider ϕ_h^i, $i = 1, 2, ..., m(h)$ as the conventional basis for affine functions defined as follows:

$$\phi_h^i(K_h^j) = \delta_{ij},$$

K_h^j denote a vertex of the triangulation T_h. Let $\mathbb{U}_h = R^{mh}$, we consider the restriction operator defined as:

$$r_h : \mathbb{U}_h \longrightarrow \mathbb{V}_h,$$
$$r_h \mathtt{v} = \sum_{i=1}^{m(h)} \mathtt{v}(K_h^i)\phi_h^i(x), \tag{3.1}$$

we provide a definition for the following adjoint operator $r_h^* : \mathbb{V}_h \longrightarrow \mathbb{U}_h$ satisfies :

$$\langle r_h \mathtt{u}, \mathtt{u} \rangle_{L^2} = \langle \mathtt{u}, r_h^* \mathtt{v} \rangle, \forall \mathtt{u} \in \mathbb{U}_h, \mathtt{v} \in \mathbb{V}_h,$$

the norm $\|.\|_\infty$ (on $\mathbb{U}_k$) and the norm $\|.\|_{L^\infty}$ (on $\mathbb{V}_k$) are equivalent, which are indicated by $\|.\|_\infty$.

Lemma 3.1. *[3] There exist λ_1 and λ_2 independent of h where,*

1. $\|r_h(\mathtt{u})\|_\infty = \|\mathtt{u}\|_\infty, \forall \mathtt{u} \in \mathbb{U}_h$.
2. $\lambda_1 \|\mathtt{v}\|_\infty \leq \|r^*(\mathtt{v})\|_\infty \leq \lambda_2 \|\mathtt{v}\|_\infty, \forall \mathtt{v} \in \mathbb{V}_h$.

3.2 Discrete problem

We apply the semi-implicit Euler scheme to discretize problem (2.5) with respect to time. Consequently, we are in seek of a sequence of elements $\mathtt{u}_h^n \in H_0^1(\Omega)$ that approaches $\mathtt{u}(t_n)$, $t_n = n\delta t$, with initial data $\mathtt{u}_0$ and $\dfrac{\partial \mathtt{u}_h}{\partial t} = \frac{\mathtt{u}_h^n - \mathtt{u}_h^{n-1}}{\delta t}$, thus, we have for $n = 1, ..., N$, and $\forall \mathtt{v}_h \in \mathbb{V}_h$

$$\begin{cases} (\dfrac{\mathtt{u}_h^n - \mathtt{u}_h^{n-1}}{\delta t}, \mathtt{v}_h - \mathtt{u}_h^n) + \mathtt{a}(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) \geq (f^n(\mathtt{u}), \mathtt{v}_h - \mathtt{u}_h^n), \\ \mathtt{u}_h^n \leq \psi_h, \end{cases} \tag{3.2}$$

we can write the problem (3.2) as: $\forall \mathtt{v}_h \in \mathbb{V}_h$

$$\begin{cases} \mathtt{a}(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) + \dfrac{1}{\delta t}(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) \geq (f^n(\mathtt{u}_h^n) + \dfrac{\mathtt{u}_h^{n-1}}{\delta t}, \mathtt{v}_h - \mathtt{u}_h^n), \\ \mathtt{u}_h^n \leq \psi_h, \end{cases} \tag{3.3}$$

when we put $\omega = \dfrac{1}{\delta t}$ and $F_h^n(\mathtt{u}) = f_h^n(\mathtt{u}) + \omega \mathtt{u}_h^{n-1}$,
we obtain

$$\begin{cases} \omega(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) + \mathtt{a}(\mathtt{u}_h^n, \mathtt{v}_h - u_h^n) \geq (F_h^n(\mathtt{u}_h^n), \mathtt{v}_h - \mathtt{u}_h^n), \\ \mathtt{u}_h^n \leq \psi_h. \end{cases} \tag{3.4}$$

We can express the problem (3.4) in the following form and find $\mathtt{u}_h^n \in H_0^1(\Omega), \forall \mathtt{v}_h \in H_0^1(\Omega)$,

$$\begin{cases} \mathtt{b}(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) \geq (F_h^n(\mathtt{u}_h^n), \mathtt{v}_h - \mathtt{u}_h^n), \\ \mathtt{u}_h^n \leq \psi_h, \\ \mathtt{u}_h(x) = \mathtt{u}_h(x,t)\mathtt{u}(x,t) = \mathtt{u}_0 \qquad in \quad \Omega_h \\ \mathtt{u}(x,t) = 0 \quad in \quad \Sigma_h \end{cases} \tag{3.5}$$

where

$$\mathtt{b}(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) = \omega(\mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n) + \mathtt{a}(\mathtt{u}_h^n, \mathtt{u}_h - \mathtt{u}_h^n).$$

The bilinear form $\mathtt{b}(.,.)$ exhibits strong coerciveness. Consequently, the formulation (3.5) denotes a coercive and continuous issue concerning elliptic quasi-variational inequalities (see[8]).
Denote by $\mathbb{B}_h$ the outcome obtained by addressing problem (3.5) through a finite element method, resulting in the solution to the subsequent problem: ascertain $\mathtt{u}_h^n \in \mathbb{V}_h$ such that:

$$\begin{cases} \langle \mathbb{B}_h \mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n \rangle \ \geq \ \langle F^n(\mathtt{u}_h^n), \mathtt{v}_h - \mathtt{u}_h^n \rangle, \\ \mathtt{u}_h^n \leq \psi_h, \\ \mathtt{u}_h(x) = \mathtt{u}_h(x,t) \qquad in \quad \Omega_h \\ \mathtt{u}(x,t) = 0. \quad in \quad \Sigma_h \end{cases} \tag{3.6}$$

Where $(\mathbb{B}_h)_{i,j}$ denotes the finite element matrix defined by:

$$(\mathbb{B}_h)_{i,j} = \mathtt{b}(\phi_i, \phi_j) = \mathtt{a}(\phi_i, \phi_j) + \omega(\phi_i, \phi_j)$$

and $\{\phi_i\}$ is basis of $\mathbb{V}_h$, $\quad 1 \leq i, 1 \leq j$.
The discretization matrices $\mathbb{B}_h$ and the generic coefficient matrices $\mathtt{a}(\phi_h^i, \phi_h^j)$ are introduced in a

natural progression, where the customary basic functions are denoted by $\phi_h = 1, 2, \ldots, m(h_h)$. Now that these definitions have been established, we may state the discrete problem as follows: Determine $\mathtt{u}_h^n \in \mathbb{V}_h$, representing the solution for:

$$\begin{cases} \langle \mathbb{B}_h \mathtt{u}_h^n, \mathtt{v}_h - \mathtt{u}_h^n \rangle \ \geq \ \langle F^n(\mathtt{u}_h^n), \mathtt{v}_h - \mathtt{u}_h^n \rangle \,, \\ \mathtt{u}_h^n \leq \psi_h, \\ \mathtt{u}_h(x) = \mathtt{u}_h(x,t) \qquad in \quad \Omega_h \\ \mathtt{u}(x,t) = 0 \quad in \quad \Sigma_h \end{cases} \tag{3.7}$$

The matrix $(\mathbb{B}_h)$ with coefficients $\mathtt{b}(\phi_i, \phi_j)$ is an M-matrix (see([16][17][18]). The problem (3.7) has a unique solution (as is well known), and we obtain the following regularity result:

Theorem 3.1. *[4][5] Let* $\mathtt{u}^\infty$ *and* $\mathtt{u}_h^n$ *be the solutions of (2.5) and (3.7), respectively.*

$$\|\mathtt{u}_h^n - \mathtt{u}^\infty\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t} \right)^n \right] \tag{3.8}$$

where,$\mathtt{u}^\infty$ is the asymptotic solution of the problem (3.5) ,and $\mathtt{u}_h^n$ the discrete solution calculatedat the moment
$T = n\delta t, C > 0$.

3.3 HJB-formulation of discrete problem

Before commencing work in this section, we will use the following symbols:

$$\mathbb{B}_h = \mathbb{B}_k^v, \quad \mathtt{u}_h^n = \mathtt{u}_k^*, \quad \mathbb{U}_h = \mathbb{U}_k, \quad \mathbb{V}_h = \mathbb{V}_k.$$

The problem (3.7) can be expressed as the following HJB equation. Let $\mathtt{u}_k^v$ denote the unique solution of the discrete HJB equation.

$$\max_{1 \leq i \leq N} \left(\mathbb{B}_{k,i} \mathtt{u}_{k,i}^v - F(\mathtt{u}_{k,i}^v); \ \mathtt{u}_{k,i}^v - \psi_k \right) = 0.$$

we pet

$$\mathtt{u}_{k,i}^v - \psi_k = \lambda_{k,i}^v$$

Iterative steps:
Step 1: We are selecting an initial vector $\mathtt{u}_k^0 \in \mathbb{R}^{n_k}$.
Step 2: Let $\mathtt{u}_k^v \in \mathbb{R}^{n_k}, v \geq 0$, and we compute $\mathtt{u}_k^{(v+1)}$, where $\mathtt{u}_k^{(v+1)}$ is a solution of the following equation.

$$\mathbb{B}_k^v \mathtt{u}_k^{v+1} - F_k^v(\mathtt{u}_k^v) = 0, \tag{3.9}$$

where

$$\mathbb{B}_k^v = \begin{cases} \mathbb{B}_{k,i}(\mathtt{u}_k^v) & if \ \mathbb{B}_{k,i} \mathtt{u}_{k,i}^v - F(\mathtt{u}_{k,i}^v) > \lambda_{k,i}^v \\ I_{k,i} & if \ \mathbb{B}_{k,i} \mathtt{u}_{k,i}^v - F(\mathtt{u}_{k,i}^v) \ \leq \ \lambda_{k,i}^v \end{cases} \tag{3.10}$$

$$F_k^v(\mathtt{u}_k^v) = \begin{cases} F_k^v(\mathtt{u}_k^v) & if\ \mathbb{B}_{k,i}\mathtt{u}_{k,i}^v - F(u_{k,i}^v) > \lambda_{k,i}^v \\ \psi_k & if\ \mathbb{B}_{k,i}\mathtt{u}_{k,i}^v - F(\mathtt{u}_{k,i}^v) \leqslant \lambda_{k,i}^v \end{cases} \tag{3.11}$$

Consider $\mathtt{u}_k^*$ as the unique solution to the discrete HJB equation.

$$\max_{1\leq i\leq N}\left(\mathbb{B}_{k,i}\mathtt{u}_{k,i}^* - F(\mathtt{u}_{k,i}^*);\ \mathtt{u}_{k,i}^* - \psi_k\right) = 0$$

The following theorem represents the equivalence between HJB equation and the parabolic quasi variational inequality by using finite elements method, and we will rely in its proof on the work done by Hoppe on the elliptic quasi variational inequality using finite differences method.

Theorem 3.2. *Let $\mathtt{u}_k^v$ be the iterate obtained by the previous iterative scheme, satisfying the HJB-equation above. Additionally, we suppose that $\mathbb{B}_k$ is monotone. Then, the sequence $\mathtt{u}_k^v$, where $(v \geq 0)$, exhibits monotonically decreasing convergence toward the unique solution $\mathtt{u}_k^*$ of (3.6).*

Proof. Let $\mathtt{u}_k^v$ represent an iteration, and $\mathtt{u}_k^*$ denote a solution of the HJB equation. To prove that $\mathtt{u}_k^v$ converges towards $\mathtt{u}_k^*$, it is adequate to show that $(\mathtt{u}_k^v, \mathtt{v} \geq 0)$ consistently decreases towards $\mathtt{u}_k^*$, as outlined below:

$$\mathtt{u}_k^* \leq \mathtt{u}_k^{v+1} \leq \mathtt{u}_k^v.$$

To begin, let us consider $\mathtt{u}_k^0$ satisfying

$$\max(\mathbb{B}_k^0\mathtt{u}_k^0 - F_k, \mathtt{u}_k^0 - \psi_k) \geq 0,$$

for all $v \geq 0$,

$$\max(\mathbb{B}_k^v\mathtt{u}_k^v - F_k, \mathtt{u}_k^v - \psi_k \geq (\mathbb{B}_k^v\mathtt{u}_k^v - F_k) \geq 0.$$

Using (3.9), we get

$$(\mathbb{B}_k^v\mathtt{u}_k^v - \mathbb{B}_k^v u_k^{v+1}) \geq 0,$$

We have $\mathbb{B}_k^v$, which is linear and monotone.

$$\mathbb{B}_k^v(\mathtt{u}_k^v - \mathtt{u}_k^{v+1}) \geq 0,$$

then

$$\begin{aligned}(\mathtt{u}_k^v - \mathtt{u}_k^{v+1}) &\geq 0,\\ \mathtt{u}_k^{v+1} &\leq \mathtt{u}_k^v,\end{aligned} \tag{3.12}$$

We know that $\mathtt{u}_k^*$ is a solution of (3.7).

$$\max(\mathbb{B}_k^v\mathtt{u}_k^* - F_k, \mathtt{u}_k^* - \psi_k) = 0 \geq (\mathbb{B}_k^v\mathtt{u}_k^* - F_k),$$

again, using (3.9)

$$\begin{aligned}\mathbb{B}_k^v\mathtt{u}_k^* - \mathbb{B}_k^v\mathtt{u}_k^{v+1} &\leq 0,\\ \mathbb{B}_k^v\mathtt{u}_k^* &\leq \mathbb{B}_k^v\mathtt{u}_k^{v+1}.\end{aligned}$$

We have $\mathbb{B}_k^v$ is a monotone matrix, i.e., $\mathbb{B}_k^v$ is an invertible matrix ($(\mathbb{B}_k^v)^{-1}$ exists).

$$(\mathbb{B}_k^v)^{-1}\mathbb{B}_k^v\mathtt{u}_k^* \leq (\mathbb{B}_k^v)^{-1}\mathbb{B}_k^v\mathtt{u}_k^{v+1},$$

then,

$$\mathtt{u}_k^* \leq \mathtt{u}_k^{v+1}. \tag{3.13}$$

From (3.12) and (3.13), we obtain

$$\mathtt{u}_k^* \leq \mathtt{u}_k^{v+1} \leq \mathtt{u}_k^v.$$

The uniqueness: To demonstrate uniqueness, we suppose there are two solutions $\mathtt{u}_1^*$ and $\mathtt{u}_2^*$ of (3.7). First, we put $\mathtt{u}_1^* = \mathtt{u}_k^*$ and $\mathtt{u}_2^* = \mathtt{u}_k^{v+1}$

$$\max(\mathbb{B}_k^v \mathtt{u}_1^* - F_k, \mathtt{u}_1^* - \psi_k) = 0 \geq (\mathbb{B}_k^v u_1^* - \mathbb{B}_k^v u_k^{v+1})$$

$$\mathbb{B}_k^v \mathtt{u}_1^* - \mathbb{B}_k^v \mathtt{u}_2^* \leq 0 \qquad \mathbb{B}_k^v \mathtt{u}_1^* \leq \mathbb{B}_k^v \mathtt{u}_2^*.$$

$$\mathtt{u}_1^* - \mathtt{u}_2^* \leq 0, \tag{3.14}$$

secondly, we put $u_2^* = u_k^*$ and $u_1^* = u_k^{v+1}$

$$\max(\mathbb{B}_k^v \mathtt{u}_2^* - F_k, \mathtt{u}_2^* - \psi_k) = 0 \geq (\mathbb{B}_k^v \mathtt{u}_2^* - \mathbb{B}_k^v \mathtt{u}_k^{v+1}).$$

$$\mathbb{B}_k^v \mathtt{u}_2^* - \mathbb{B}_k^v \mathtt{u}_1^* \leq 0 \quad \textit{then} \quad \mathbb{B}_k^v \mathtt{u}_2^* \leq \mathbb{B}_k^v \mathtt{u}_1^*.$$

$$\mathtt{u}_1^* - \mathtt{u}_2^* \geq 0. \tag{3.15}$$

From (3.14) and (3.15) we get

$$\mathtt{u}_1^* = \mathtt{u}_2^*.$$

Then, the solution of (3.7), $(\mathtt{u}_k^v)$, approximates towards the solution of problem (3.6), $\mathtt{u}_k^v \mapsto \mathtt{u}_k^*$. □

3.4 Description of the Newton-multigrid method for PQVIs

In addressing the nonlinear system (3.9), we utilize the Newton-Multigrid method, a hybrid technique that merges Newton's method for nonlinear systems with the Multigrid method for linear systems. The application of the Newton-Multigrid approach involves the following procedural steps:
First step: By utilizing Newton's method, we obtain the nonlinear system, recognizing that f is a Lipschitz function.
Second step: In every linear step of system (3.9), we seek the Jacobian system to derive a linear system connected with the Jacobian matrix.
Third step: We use the Multigrid method to solve the linear system resulting from the Jacobian matrix.
Let $\mathtt{u}_k^v$ be the exact solution of the system (3.9), and $\mathtt{w}_k^v$ an approximation to $\mathtt{u}_k^v$. We define the residual as follows:

$$\mathfrak{R}_k(\mathtt{u}_k^v) = F_k^v(\mathtt{u}_k^v) - \mathbb{B}_k^v \mathtt{w}_k^v, \tag{3.16}$$

Substituting the equation (3.9) in the equation(3.16), we find

$$\mathfrak{R}_k(\mathtt{u}_k^v) = \mathbb{B}_k^v \mathtt{u}_k^v - \mathbb{B}_k^v \mathtt{w}_k^v, \tag{3.17}$$

$$\mathfrak{R}_k(\mathtt{u}_k^v) = \mathbb{B}_k^v(\mathtt{u}_k^v - \mathtt{w}_k^v) = \mathbb{B}_k^v(e_k^v), \tag{3.18}$$

$\mathtt{e}_k^v$ the error, where $\mathtt{e}_k^v = \mathtt{u}_k^v - \mathtt{w}_k^v$. In the context of equation (3.18), determining the error $\mathtt{e}_k^v$ for the linear equation (3.18) on the coarse grid, as commonly done in the multigrid method, is not feasible.

Consequently, we adopt equation (3.18) as a residual equation, given the non-linear characteristics of f.
To facilitate access to the solution, we choose $\mathcal{H}$ as a nonlinear operator, where:

$$\mathcal{H}_k(\mathtt{u}_k^v) = \mathbb{B}_k^v \mathtt{u}_k^v - F_k^v(\mathtt{u}_k^v) = 0. \tag{3.19}$$

The residual in the fine grid can be rewritten as follows:

$$\mathfrak{R}_k(\mathtt{u}_k^v) = -\mathcal{H}_k(\mathtt{u}_k^v). \tag{3.20}$$

To solve (3.19), we employ the Newton iteration method as follows:

$$\mathtt{u}_k^{v+1} = \mathtt{u}_k^v + \frac{\mathfrak{R}_k(\mathtt{u}_k^v)}{\mathcal{J}_k(\mathtt{u}_k^v)}, \tag{3.21}$$

where $\mathcal{J}_k(\mathtt{u}_k^v)$ represents the Jacobian matrix of the nonlinear system, and $\mathcal{J}_k(\mathtt{u}_k^v) = \mathcal{H}_k'(\mathtt{u}_k^v)$.
From equation (3.21), we can derive the following Jacobian linear system for $\mathtt{e}_k^v$.

$$\mathcal{J}_k(\mathtt{u}_k^v)\mathtt{e}_k^v = \mathfrak{R}_k(\mathtt{u}_k^v). \tag{3.22}$$

where $\mathtt{u}_k^{v+1} - \mathtt{u}_k^v = \mathtt{e}_k^v$
The solution to the linear system (3.22) is utilized as an approach to solving the nonlinear system (3.19). Therefore, to find the solution for the nonlinear system (3.19), we employ the multigrid method to solve the associated linear system (3.22).

3.5 Multigrid technique

In addressing the linear system (3.22), we employ the multigrid technique. By selecting an iteration $\mathtt{e}_k^v$, where $v > 0$, within the multigrid method, we derive $\overline{\mathtt{e}}_k^v$ through the application of an iterative method. This iterative process is utilized to solve the system (3.22), utilizing α as the coefficient expression.

$$\overline{\mathtt{e}}_k^v = \mathcal{S}_k^v(\mathtt{e}_k^v), \tag{3.23}$$

Here, $\mathcal{S}_k^v$ stands for the iteration or smoothing operator, and α represents the number of iterations executed. The solution to (3.22) is denoted as $\mathtt{e}^\star$.
The error is defined as $\mathtt{E}_k^v = \overline{\mathtt{e}}_k^v - \mathtt{e}_k^\star$, and the residual is also considered.

$$\mathtt{d}_k^v = \mathfrak{R}_k(\mathtt{u}_k^v) - \mathcal{J}_k(\mathtt{u}_k^v)\overline{\mathtt{e}}_k^v,$$

we can write the equation (3.22) as

$$\mathcal{J}_k(\mathtt{u}_k^v)(\overline{\mathtt{e}}_k^v + \mathtt{E}_k^v) = \mathfrak{R}_k(\mathtt{u}_k^v), \tag{3.24}$$

We derive the subsequent residual equation.

$$\mathcal{J}_k(\mathtt{u}_k^v)\mathtt{E}_k^v = \mathfrak{R}_k(\mathtt{u}_k^v) - \mathcal{J}_k(\mathtt{u}_k^v)\overline{\mathtt{e}}_k^v = \mathtt{d}_k^v,$$

$$\mathcal{J}_k(\mathtt{u}_k^v)\mathtt{E}_k^v = \mathtt{d}_k^v.$$

To fully determine $\mathtt{E}_k^v$, it is necessary to compute $\mathtt{E}_k^v$ at the level $(k-1)$ as the solution to the coarse grid system.

$$\mathcal{J}_{k-1}(\mathtt{e}_{k-1}^v)\mathtt{E}_{k-1}^v = \mathtt{d}_{k-1}^v. \tag{3.25}$$

In the context where $\mathtt{E}_{k-1}^v,\ \mathcal{J}_{k-1}(\mathtt{e}_{k-1}^v),\ \mathtt{d}_{k-1}^v$ represent approximations at the $(k-1)$ of $\mathtt{E}_k^v,\ \mathcal{J}_k(\mathtt{e}_k^v),\ \mathtt{d}_k^v$ respectively.
We have

$$\mathtt{E}_{k-1}^v = \mathtt{R}_k\mathtt{E}_k^v,$$

$$\mathtt{d}_{k-1}^v = \mathtt{R}_k\mathtt{d}_k^v,$$

$$\mathcal{J}_{k-1}^v(\mathtt{e}_{k-1}^v) = \mathcal{R}_k\mathcal{J}_k^v(e_k^v)\mathcal{P}_k,$$

$\mathcal{R}_k$ is the restriction matrix and $\mathcal{P}_k$ the prolongation matrix . Owing to the nested structure, we employ the clearly defined identity operator.

$$\Psi : \mathbb{V}_{k-1} \longrightarrow \mathbb{V}_k.$$

for defining the prolongation and restriction operators, i.e.

$$\mathcal{R}_k = \mathcal{P}_k^t$$

$$\mathcal{P}_k = r_k r_{k-1}^{-1}. \tag{3.26}$$

Note: To solve the linear system (3.22) for the two sequences Ω_k and Ω_{k-1}, we employ the multi-grid technique as an internal iteration. Additionally, to address system (3.19), we utilize Newton's method as an external iteration. This results in the iterative solution of system (3.25), achieved by applying a two-grid iteration repeatedly to the sequence $\Omega_k\{k = 0, ..., m_k\}$ until the iteration process is halted.

4 $\mathbb{L}^\infty$-convergence of multi-grid method

This paragraph explores the assessment of uniform convergence for the multigrid algorithm (inner iteration) and the convergence characteristics of Newton's method (outer iteration). The assumptions employed in these convergence analyses closely resemble those utilized in multigrid methods specifically crafted for addressing nonlinear equations. We now present the main hypotheses:
Hypotheses 1:

$$\exists \mathtt{u}_k^\star \in V_k \quad \textit{such that} \quad \mathcal{H}_k(\mathtt{u}_k^\star)^{-1} = 0$$

Hypotheses 2 (inverse):

$$\mathcal{H}_k^{'}(\mathtt{u}_k^\star)^{-1} \textit{ is exist and } \|\mathcal{H}_k^{'}(\mathtt{u}_k^\star)^{-1}\|_\infty \leq k \quad \textit{with } k > 0$$

Hypotheses 3 (continuous):

$$\forall \epsilon{>}0, \quad \exists \eta{>}0, \quad \|I - \mathcal{H}_k^{'}(\mathtt{u}_k^\star)\mathcal{H}_k^{'}(\mathtt{u}_k)^{-1}\|_\infty \leq \epsilon$$

whenever

$$\|(\mathtt{u}_k - \mathtt{u}_k^\star)\|_\infty \leq \eta.$$

Hypotheses 4: For any $\mathfrak{u}_k$ in the neighborhood of $\mathfrak{u}_k^\star$, there is a linear mapping denoted as $\mathcal{H}_k^{'}(\mathfrak{u}_k)$ such that:
$\forall\epsilon{>}0$ $\exists\eta{>}0$ such that

$$\|\mathcal{H}_k(\mathfrak{u}_k) - \mathcal{H}_k(\mathfrak{u}_k^\star) - \mathcal{H}_k^{'}(\mathfrak{u}_k^\star)\|(\|(\mathfrak{u}_k - \mathfrak{u}_k^\star)\|_\infty \leq \epsilon\|(\mathfrak{u}_k - \mathfrak{u}_k^\star)\|_\infty$$

hold, whenever

$$\|(\mathfrak{u}_k - \mathfrak{u}_k^\star)\|_\infty \leq \eta.$$

4.1 Matrix associated with the MGHJB algorithm

The two-grid method's iteration matrix, which includes α_1 presmoothing and α_2 postsmoothing at level $k-1$, can be represented as follows:

$$\mathbb{TG}_k(\alpha_1,\alpha_2) = \mathcal{S}_k^{\alpha_2}\,(\mathcal{J}_k^{-1} - \mathcal{P}_k\,\mathcal{J}_{k-1}^{-1}\mathcal{R}_k)\mathcal{J}_k\,\mathcal{S}_k^{\alpha_1}, k = 1, 2.. \tag{4.1}$$

Theorem 4.1. *[26][27] The multigrid approach is a linear iterative technique characterized by the iteration matrix $\mathbb{MG}_k$, which is expressed as:*

$$\mathbb{MG}_0 = 0 \quad if \quad k = 0,$$
$$\mathbb{MG}_k = \mathcal{S}_k^{\alpha_2}\left(\mathbb{I}_k - \mathcal{P}_k\left(\mathbb{I}_k - \mathbb{MG}_{k-1}\right)\left(\mathcal{J}_{k-1}\right)^{-1}\mathcal{R}_k\right)\mathcal{J}_k\,\mathcal{S}_k^{\alpha_1} \quad if \quad k = 1, 2, ... \tag{4.2}$$

$$\begin{aligned}\mathbb{MG}_k &= \mathcal{S}_k^{\alpha_2}\left(\mathbb{I}_k - \mathcal{P}_k\left(\mathbb{I}_k - \mathbb{MG}_{k-1}\right)\left(\mathcal{J}_k^{-1}\mathcal{R}_k\right)\mathcal{J}_k\,\mathcal{S}_k^{\alpha_1}\right.\\ &= \mathbb{TG}_k + \mathcal{S}_k^{\alpha_2}\mathcal{P}_k(\mathbb{MG}_{k-1})\mathcal{J}_k^{-1}\mathcal{R}_k\mathcal{J}_k\mathcal{S}_k^{\alpha_1} \quad if \quad k = 1, 2, ...\end{aligned}$$

4.2 Approximation property

The following approximation property is based on the inequality (3.8) in Theorem 3.1 and in Lemma 3.1. The demonstration of the theorem is adapted from the one in[19] , originally presented for the problem of variational inequality.

4.3 The main result

The following theorems gives us the convergence of the multi-grid matrix of the parabolic quasi variational, in its proof we will rely on the work done by Haiour [19] in the elliptic quasi variational inquality.

Theorem 4.2. $\mathbb{X}$ *is the iteration matrix of the multigrid, given by:*

$$\mathbb{X} = \left[\mathcal{J}_k^{-1} - \mathcal{P}_k\,\mathcal{J}_{k-1}^{-1}\mathcal{R}_k\right]. \tag{4.3}$$

Under the previous assumptions, the matrix $\mathbb{X}$ satisfies the following approximation properties:

$$\|\mathbb{X}\|_\infty \leq C\left[h_k^2|log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right]. \tag{4.4}$$

Proof. According to 3.1, let $\mathtt{u} \in H_0^1(\Omega)$, we have

$$\|\mathtt{u}_k^n - \mathtt{u}^\infty\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty.$$

Then

$$\|\mathtt{u}_k^\star - \mathtt{u}^\infty\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty$$

Let $\mathtt{u}_k^\star$ and $\mathtt{u}_{k-1}^\star$ be solutions to the problem (3.6), and we apply 3.1.

$$\begin{aligned}\|\mathtt{u}_k^* - \mathtt{u}_{k-1}^*\|_\infty \leq & \|\mathtt{u}_k^* - \mathtt{u}^\infty\|_\infty + \|\mathtt{u}_{k-1}^* - \mathtt{u}^\infty\|_\infty \\ & \leq C_1 \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty + C_2 \left[h_{k-1}^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty. \\ & \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty\end{aligned}$$

we obtain

$$\|\mathtt{u}_k^* - \mathtt{u}_{k-1}^*\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty \tag{4.5}$$

where $C = (C_1 + C_2)$.
We use Galerkin discretization to obtain:

$$\mathtt{b}(r_k \mathtt{u},\ r_k \mathtt{v}) = \langle (\mathbb{B}_k),\ \mathtt{v} \rangle_{L^2(\Omega)}; \forall \mathtt{u}, \mathtt{v} \in \mathbb{U}_k.$$

So

$$(\mathtt{b}(\mathtt{u},\ \mathtt{v}))' = \langle (\mathcal{J}_k),\ \mathtt{v}_k \rangle_{L^2(\Omega)}; \forall \mathtt{u}, \mathtt{v} \in \mathbb{U}_k.$$

Also we have, $\forall \mathtt{v} \in \mathbb{V}$

$$(\mathtt{b}((r_k)^{-1} (\mathcal{J}_k)^{-1}(F_k),\ \mathtt{v}))' = \left\langle (r_{k-1}^*)^{-1} \nabla F\ ,\ \mathtt{v} \right\rangle_{L^2(\Omega)}$$

Let $\mathtt{u}_k \in \mathbb{V}_k$ and $\mathtt{u}_{k-1} \in \mathbb{V}_{k-1}$

$$\mathtt{b}(\mathtt{u}_k,\ \mathtt{v}) = \left\langle (r_k^*)^{-1} F,\ \mathtt{v} \right\rangle_{L^2(\Omega)},$$

$$\mathtt{b}(\mathtt{u}_{k-1},\ \mathtt{v}) = \left\langle (r_{k-1}^*)^{-1} F,\ \mathtt{v} \right\rangle_{L^2(\Omega)},$$

this implies $\mathtt{u}_k^\star = r_k^{-1} \mathcal{J}_k^{-1} \nabla(F)$ and $\mathtt{u}_{k-1}^\star = r_{k-1}^{-1} \mathcal{J}_{k-1}^{-1} \mathcal{R}_k \nabla(F)$
Using (4.5),(3.26) and 3.1 we get

$$\|r_k^{-1} \mathcal{J}_k^{-1} \nabla(F) - r_{k-1}^{-1} \mathcal{J}_{k-1}^{-1} \mathcal{R}_k \nabla(F)\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty$$

Then

$$\|\mathcal{J}_k^{-1} - r_k r_{k-1}^{-1} \mathcal{J}_{k-1}^{-1} \mathcal{R}_k\|_\infty \|\nabla F\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right] \|\nabla F\|_\infty$$

The proof is now concluded

$$\|\mathcal{J}_k^{-1} - \mathcal{P}_k \mathcal{J}_{k-1}^{-1} \mathcal{R}_k\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right]$$

$$\|\mathbb{X}\|_\infty \leq C \left[h_k^2 |log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right].$$

□

4.4 Smoothing property

To establish a smoothing property, we formed the matrix $\mathcal{J}_k^v = \mathcal{L}_k - \mathcal{N}_k$, and relied on the subsequent assumptions: $\mathcal{L}_k$ is considered regular where

$$\|\mathcal{L}_k^{-1}\,\mathcal{N}_k\|_\infty \leq 1 \qquad \text{for all } k. \tag{4.6}$$

$$\|\mathcal{L}_k^{-1}\|_\infty \leq \frac{C}{h^2}, \quad \text{for all } k, \text{ with } C \text{ independent a } k. \tag{4.7}$$

We utilized a relaxation method with an iteration matrix as a smoother.

$$\mathcal{S}_k = I_k - \omega \mathcal{J}_k^{-1}\mathcal{N}_k, \qquad \omega \in [0;1].$$

Theorem 4.3. *[26][27] Given that the preceding assumptions and notations are met, there exists a constant C, independent of k, such that the following smoothing property holds:*

$$\|\mathcal{J}_k\,\mathcal{S}_k^\alpha\|_\infty \leq \frac{C}{\sqrt{\alpha}\,h_k^2}. \tag{4.8}$$

Following the approximation and smoothing properties, it is essential to establish the subsequent stability bound:

$$\exists C_s \quad \|\mathcal{S}_k^\alpha\|_\infty \leq C_s, \qquad \text{for all k and } \alpha. \tag{4.9}$$

The convergence analysis relies on the following decomposition of the iteration matrix for the two-grid method, where $\alpha_2 = 0$.

$$\begin{aligned}\|\mathbb{TG}_k(\alpha_1,0)\|_\infty &= \|\left[(\mathcal{J}_k)^{-1} - \mathcal{P}_k\,(\mathcal{J}_{k-1})^{-1}\,\mathcal{R}_k\right]\,\mathcal{J}_k\,\mathcal{S}_k^{\alpha_1}\,\|_\infty \\ &\leq \|(\mathcal{J}_k)^{-1} - \mathcal{P}_k\,(\mathcal{J}_{k-1})^{-1}\,\mathcal{R}_k\|_\infty\,\|\mathcal{J}_k\,\mathcal{S}_k^{\alpha_1}\|_\infty.\end{aligned}$$

Now, choosing for a hierarchy comprising more than two grids, we can formulate the matrix $\mathbb{X}$ in (4.3) by recursively utilizing the matrix $\mathbb{MG}_k$ in (4.2) for all levels. Assuming the validity of condition 4.3, convergence outcomes can be easily inferred from the preceding findings.

Theorem 4.4. *[25] Examine a multigrid approach for a given iterative matrix (4.2). Then, based on the earlier assumption, for the specified parameter value $\alpha_1 = \alpha, \alpha_2 = 0, \forall \varepsilon \in [0,1], \quad \exists \alpha^* \leq \alpha$:*

$$\|\mathbb{MG}_k\|_\infty \leq \varepsilon.$$

Proof. Combining the approximation and smoothness properties with (3.5) allows us to utilize identical parameters, as outlined in the subsequent theorem. This theorem constitutes the primary outcome of our study. □

Theorem 4.5. *Given the preceding assumptions and notations, the iterated u_k^v,v>0, for two meshes k and $k-1$, adhere to the following relationship: $\exists C$>0 :*

$$\|\mathtt{u}_k^{v+1} - \mathtt{u}_k^*\|_\infty \leq \frac{C}{\sqrt{\alpha}\,h_k^2}\left[h_k^2|log(h_k)|^2 + \left(\frac{1}{1+\beta\delta t}\right)^n\right]\|\mathtt{u}_k^v - \mathtt{u}_k^*\|_\infty.$$

Proof.

$$\begin{aligned}\|\mathtt{u}_k^{v+1}-\mathtt{u}_k^*\|_\infty &= \|(\mathcal{J}_k\,\mathcal{S}_k^\alpha)(I_k-\mathcal{P}_k(I_k-\mathbb{M}\mathbb{G}_{k-1})(\mathcal{J}_{k-1}^{-1})\mathcal{R}_k(\mathtt{u}_k^v-\mathtt{u}_k^*)\|_\infty\\ &\le \|(\mathcal{J}_k\,\mathcal{S}_k^\alpha)\|_\infty\|(I_k-\mathcal{P}_k(I_k-\mathbb{M}\mathbb{G}_{k-1})(\mathcal{J}_{k-1}^{-1})\mathcal{R}_k\|_\infty\|\mathtt{u}_k^v-\mathtt{u}_k^*\|_\infty\\ &\le \frac{C_1C_2}{\sqrt{\alpha}\,h_k^2}\left[h_k^2|log(h_k)|^2+\left(\frac{1}{1+\beta\delta t}\right)^n\right]\|\mathtt{u}_k^v-\mathtt{u}_k^*\|_\infty\\ &\le \frac{C}{\sqrt{\alpha}\,h_k^2}\left[h_k^2|log(h_k)|^2+\left(\frac{1}{1+\beta\delta t}\right)^n\right]\|\mathtt{u}_k^v-\mathtt{u}_k^*\|_\infty\end{aligned}$$

where $C=C_1\times C_2$. □

5 Numerical exemple

In this numerical example , we find the solution of the problem 5.1 by using multigrid method and Gauss-Siedel method.

$$\begin{cases}\dfrac{\partial u}{\partial t}+Au\le f(u)\ in\ [0,1]\times\Omega\\[2ex] \left\langle\dfrac{\partial u}{\partial t}+Au-f(u)\,;\mathtt{u}-\psi\right\rangle=0,\\[2ex] u(x,t)=0,\ \text{in}\ [0,1]\times\Gamma,\\[2ex] u(x,0)=0,\ \ \text{in}\ \Omega,\end{cases}\tag{5.1}$$

where

$$\Omega=\{(x_1,x_2|x_1+x_2\le 1)\},\quad \Delta t=0.01,\quad A\mathtt{u}=-\Delta\mathtt{u},\quad f(u)=cos2u,\quad \psi=0.$$

For discretization in space we have used PDE toolbox in Matlab (r2018) to generate the mesh, and semi discretization on time, and startiterate $u_k^0=(0,..,0)^t\in R^{1024}$.
We notice from Figure 1 that the multi-grid method(V-cycle ,W-cycle) gives us the solution with the least number of iterations, that is, after 1 iteration, while the Gauss Siedel method after more than 20 iterations. From here we conclude that the multi-grid method gives us the solution to the sets of large linear equations with the least number of iterations.

6 Conclusion

In this work, we have presented the Newton-multigrid method applied to a parabolic quasi-variational problem with nonlinear right hand. We have achieved L^∞ convergence for this inequality. Three numerical methods have been employed: the finite element method combined with Euler's method to derive the discrete problem, Newton's method for obtaining the Jacobian system, and finally, a linear Jacobian multigrid method based on the Jacobian system. In future research, we plan to investigate the regular convergence of the semi-variable inequality using both the fixed-point method and the multigrid method.

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Introduction of a model describing two zones with an interface.

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Abstract
In our study, we examine a population of the triatomines structured both in terms of time and space. The demographic and spatial dispersion processes are described using reaction-diffusion equations in a two-dimensional space. By working within appropriate functional spaces, we convert this system of partial differential equations into an abstract differential equation. Our initial goal is to demonstrate that the operator generates an analytic semigroup. Subsequently, we establish the existence and uniqueness of a solution to the corresponding Cauchy problem.

Keywords: Transmission problem; reaction-diffusion; analytical semi-group; sector operator
2020 Mathematics Subject Classification: Primary 34K30, 47D06, 92B05, 35A09, 35C15, 35K57, 35K90.

1 Introduction

In this communication, we investigate the invasion of a village by the domestic species Triatoma Dimidiata. The village is located next to a forest, serving as the natural habitat for these vectors. They enter the village to feed, primarily consuming blood meals from the humans or mammals with whom they coexist. It is during this feeding phase that the primary transmission of Trypanosoma Cruzi from the vector to the host occurs.

2 Main results

We have applied the fixed point theorem to the abstract Cauchy problem, to show the uniqueness of a local solution.

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The search for the existence and uniqueness of the solution of a nonlinear fractional problem

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Abstract

We investigate a new class of boundary value problems of nonlinear fractional differential equations whose nonlinear f-term depends on a lower-order fractional derivative with separate fractional boundary conditions given by:

$$\begin{cases} {}_{c}D_{0^+}^{\alpha} x(t) = f\left(t, x(t), {}_{c}D_{0^+}^{\beta} x(t)\right) \quad , t\in[0,T], 1 < \alpha \leq 2 \, , 0 < \beta \leq 1 \\ a_1 x(0) + b_1\left({}_{c}D_{0^+}^{\gamma} x(0)\right) = c_1 \\ a_2 x(T) + b_2\left({}_{c}D_{0^+}^{\gamma} x(T)\right) = c_2 \qquad\qquad 0 < \gamma < 1 \end{cases}$$

Where ${}_{c}D_{0^+}^{\alpha}$ is the fractional derivative in the sense of Caputo, $f : [0, T] \times \mathbb{R}^2 \to \mathbb{R}$ is a function continues and a_i, b_i, c_i, $i = 1, 2$ are real constants with: $a_1 \neq 0$ and $T > 0$.

The main objective of this work is the study of the existence and uniqueness of the solutions of a problem with fractional limits separated by application of Banach's fixed point theorem which is also known as: the contracting application theorem, this last is a simple theorem to prove, which guarantees the existence of a fixed point for any contracting map, applies to complete spaces, and has many applications.

And finally, we will present an example of application to illustrate the result of this theorem.

Keywords: fractional differential equation, fractional separated boundary conditions, Banach fixed point theorem,existence.

1 Introduction

The theory of fractional calculus is a subject almost as old as differential calculus and dates back to the times when Leibniz, Gauss, Newton developed the foundations of this type of calculation.

Nonlinear differential equations of fractional order are a natural generalization of ordinary differential equations, they have become an important branch of mathematics because of their applications in several fields of science and engineering such as: physics, chemistry biology,. . . etc.

We will study the existence and uniqueness of solutions for a class of nonlinear differential fractional equations whose term f depends on the lower-order function derived from the unknown function x(t) with fractional separated boundary conditions given by:

$$\begin{cases} {}_{c}D_{0^+}^{\alpha} x(t) = f\left(t, x(t), {}_{c}D_{0^+}^{\beta} x(t)\right), t \in [0,T], 1 < \alpha \leq 2, 0 < \beta \leq 1 \\ a_1 x(0) + b_1 \left({}_{c}D_{0^+}^{\gamma} x(0)\right) = c_1 \\ a_2 x(T) + b_2 \left({}_{c}D_{0^+}^{\gamma} x(T)\right) = c_2 \qquad 0 < \gamma < 1 \end{cases} \quad \ldots\ldots\ldots (1)$$

where ${}_{c}D_{0^+}^{\alpha}$ is the fractional derivative in the sense of Caputo, f : [0, T] × $\mathbb{R}^2 \to \mathbb{R}$ is a continuous function and a_i, b_i, c_i, i = 1, 2 are real constants with: $a_1 \neq 0$ et T > 0.

For this we will apply Banach's fixed point theorem

2 Main results

According to Banach's fixed point theorem, we consider the existence and uniqueness of the boundary problem (1)

Either : I=[0,T] and C(I) the space of all continuous real functions defined on I and defining the space:

$$X=\{x(t): x(t) \in C([0,T]) \text{ and } {}_{c}D_{0^+}^{\beta} x(t) \in C([0,T]),\ 0< \beta < 1.\}$$

Equipped with the standard: $\|x\|_X = \|x\|_X + \|{}_{c}D_{0^+}^{\alpha} x(t)\|_X$

Where : $\|x\|_\infty = \max_{t\in[0,T]}|x(t)|$ and $\|{}_{c}D_{0^+}^{\alpha}\|_\infty = \max_{t\in[0,T]}|{}_{c}D_{0^+}^{\alpha} x(t)|$

(X,‖.‖) is a Banach space.

So the solution of the boundary problem (1) is equivalent to (2).

The operator F : X → X is defined as follows:

$$Fx(t)\int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)} f\left(s, x(s), c_{D_0^{\beta}}\, x(s)\right) ds - \frac{t}{v_1}\left\{ \begin{matrix} a_2 \int_0^T \frac{(T-s)^{\alpha-1}}{\Gamma(\alpha)} f\left(s, x(s), c_{D_0^{\beta}}\, x(s)\right) ds \\ +b_2 \int_0^T \frac{(T-s)^{\alpha-\gamma-1}}{\Gamma(\alpha-\gamma)} f\left(s, x(s), c_{D_0^{\beta}}\, x(s)\right) ds \end{matrix} \right\} + v_2 t + \frac{c_1}{v_1} \quad (3)$$

Thus each solution of the problem at the limits and also solution of the problem at the limits (3).

So we must look for the fixed point of the operator F which is also a solution of the problem (1).

Theorem:

Suppose that: $f : [0, T] \times \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying the condition:

$$|f(t, x_1, y_1) - f(t, x_2, y_2)| \leq m(t)(|x_1 - x_2| + |y_1 - y_2|),$$

For everything: $t \in [0,T]$, $x_i, y_i \in \mathbb{R}$, $i=1,2$ and $m \in L^{1/\tau}([0,T], \mathbb{R}^+)$, $\tau \in (0, \alpha - 1)$ if :

$$\frac{\|m\| T^{\alpha-\tau}}{\Gamma(\alpha)} \left(\frac{1-\tau}{\alpha-\tau}\right)^{1-\tau} \left(1 + \frac{a_2 T}{v_1} + \frac{|a_2| T^{1-\beta}}{|v_1|\Gamma(2-\beta)}\right) + \left(1 + \frac{T^{\beta}}{\Gamma(2-\beta)}\right) \frac{\|m\| |b_2| T^{\alpha-\gamma-\tau+1}}{|v_1|\Gamma(\alpha-\gamma)} \left(\frac{1-\tau}{\alpha-\gamma-\tau}\right)^{1-\tau} + \frac{\|m\| T^{\alpha-\beta-\gamma}}{\Gamma(\alpha-\beta)} \left(\frac{1-\tau}{\alpha-\beta-\gamma}\right)^{1-\tau} < 1$$

Then problem (1) admits a single solution.

Remark: the Riemann–Liouville inequality of order α such that:

$$I_{0^+}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-s)^{\alpha-1} f(s)\, ds$$

Suppose that the continuous function f satisfies:

$$|f(t, x_1, y_1) - f(t, x_2, y_2)| \leq L\,(|x_1 - x_2| + |y_1 - y_2|)$$

For $t \in [0, T]$, $x_i, y_i \in \mathbb{R}$, $i = 1, 2$ and $L > 0$ is a constant, if:

$$\frac{LT^{\alpha}}{\Gamma(\alpha+1)} \left(1 + \frac{|a_2| T^{1-\beta}}{|v_1|\Gamma(2-\beta)}\right) + \frac{LT^{\alpha-\beta}}{\Gamma(\alpha-\beta+1)} + \left(1 + \frac{T^{-\beta}}{\Gamma(2-\beta)}\right) \frac{L|b_2| T^{\alpha-\gamma+1}}{|v_1|\Gamma(\alpha-\gamma+1)} < 1,$$

Then problem (1) admits a single solution.

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Existence Results of Antiplane Contact Problem with Slip-Dependent Friction

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Abstract
This work studies a mathematical problem modeling the antiplane shear deformation of a piezoelectric body in frictionless contact with thermally- electrically conductive foundation. The material is assumed to be thermo- electro-viscoelastic and isotropic and the process is quasistatic. The contact is bilateral and is modelled with a slip-dependent friction law including the electrical and thermal conductivity conditions. A variational formulation of the model in the form of a coupled system for dipsplacements, electric potential, and temperature is derived. Existence and uniqueness of the solution are proved using the results of variational inequalities and a xed point theorem.

Keywords: weak solution, variational formulation, antiplane shear deformation; thermo-electro-viscoelastic material, slip-dependent friction law, xed point; evolution inequality, evolution equation.
2020 Mathematics Subject Classification: Primary 74F05, 74M10, 74M15, 49J40, 74F15.

1 Introduction

This work deals with a new mathematical model which describes the frictional contact between a thermo-piezoelectric cylinder and an obstacle, the so-called foundation. We material is assumed to be isotropic viscoelastic constitutive law and we neglect the inertial term in the equation of motion to obtain a quasistatic approximation of the process. We also restrict attention to the case of antiplane shear deformation, i.e. the displacement is parallel to the generators of the cylinder and is independent of the axial coordinate. In the last years a considerable attention has been paid to the analysis of antiplane shear deformations within the context of elasticity theory (see for example [5, 1]). The novelties in the present paper are connected with the fact that the friction bound is assumed to be historical dependent and the process is quasistatic, leading to an interesting and nonstandard variational problem. Moreover, whereas in [2] the case of linear elastic materials is considered, here we deal with linear thermo-electro-viscovis-elastic materials. Also, notice that the corresponding three-dimensional problem with slip dependent-friction was studied in [5].

The rest of the paper is structured as follows. In Section 2, we describe the model of the frictional contact process between thermo-electro-viscoelastic body and a conductive deformable foundation. In Section 3, we derive the variational formulation. It consists of a variational inequality for the displacement field coupled with a time-dependent variational equation for the electric potential and the heat equation for the temperature. In Section 4 we state our main existence and uniqueness result based on the arguments of evolutionary inequalities for the displacement field and a time-dependent variational aquation for the electric potential field, the classical result of nonlinear first order evolution inequalities for the temperature field and a fixed-point theorem.

2 Mathematical Model

We consider a piezoelectric body $\mathcal{B}$ identified with a region in $\mathbb{R}^3$, it occupies in a fixed and undistorted reference configuration. We assume that $\mathcal{B}$ is a cylinder with generators parallel to the x_3-axis with a cross-section which is a regular region Ω in the x_1x_2-plane, $Ox_1x_2x_3$ being a Cartesian coordinate system. The cylinder is assumed to be sufficiently long so that end effects in the axial direction are negligible. Thus, $\mathcal{B} = \Omega \times (-\infty, +\infty)$. The cylinder is acted upon by body forces of density $\mathbf{f}_0$ and has volume free electric charges of density $\mathbf{q}_0$. It is also constrained mechanically and electrically on the boundary. To describe the boundary conditions, we denote by $\partial\Omega = \Gamma$ the boundary of Ω and we assume a partition of Γ into three open disjoint parts Γ_1, Γ_2, and Γ_3, on the one hand, and a partition of $\Gamma_1 \cup \Gamma_2$ into two open parts Γ_a and Γ_b, on the other hand. We assume that the one-dimensional measures of Γ_1 and Γ_a, denoted meas Γ_1 and meas Γ_a, are positive. The cylinder is clamped on $\Gamma_1 \times (-\infty, +\infty)$ and therefore the displacement field vanishes there. Surface tractions of density $\mathbf{f}_2$ act on $\Gamma_2 \times (-\infty, +\infty)$. We also assume that the electrical potential vanishes on $\Gamma a \times (-\infty, +\infty)$ and a surface electrical charge of density $\mathbf{q}_2$ is prescribed on $\Gamma_b \times (-\infty, +\infty)$. The cylinder is in contact over $\Gamma_3 \times (-\infty, +\infty)$ with a conductive obstacle, the so-called foundation. The contact is frictional and is modeled by Tresca's law. We are interested in the deformation of the cylinder on the time interval $[0, T]$.
We have the classical formulation of the antiplane problem for thermo-electro-viscoelastic materials in frictional contact with a foundation.
Problem $\mathcal{P}$: Find the displacement field $u : \Omega \times (0, T) \to \mathbb{R}$, a temperature field $\theta : \Omega \times (0, T) \to \mathbb{R}_+$ and the electric potential $\varphi : \Omega \times (0, T) \to \mathbb{R}$ such that

$$\operatorname{div}(\zeta \nabla \dot{u} + \mu \nabla u) + \operatorname{div}(e \nabla \varphi) - \operatorname{div}(\theta \mathcal{M}_c) + f_0 = 0 \text{ in } \Omega \times [0, T], \tag{2.1}$$

$$\operatorname{div}(e \nabla u - \alpha \nabla \varphi) - \operatorname{div}(\theta \mathcal{P}) = q_0 \text{ in } \Omega \times [0, T], \tag{2.2}$$

$$\dot{\theta} - \operatorname{div}(K \nabla \theta) = -\mathcal{M}_c \nabla \dot{u} + h(t) \text{ in } \Omega \times [0, T], \tag{2.3}$$

$$u = 0 \quad \text{on } \Gamma_1 \times [0, T], \tag{2.4}$$

$$\zeta \partial_\nu \dot{u} + \mu \partial_\nu u + e \partial_\nu \varphi - \theta \mathcal{M}_c.\nu = f_2 \quad \text{on } \Gamma_2 \times [0, T], \tag{2.5}$$

$$\begin{cases} |\zeta \partial_\nu \dot{u} + \mu \partial_\nu u + e \partial_\nu \varphi - \theta \mathcal{M}_c.\nu| \le g\left(\int_0^t |\dot{u}(s)|\, ds\right), \\ |\zeta \partial_\nu \dot{u} + \mu \partial_\nu u + e \partial_\nu \varphi - \theta \mathcal{M}_c.\nu| < g\left(\int_0^t |\dot{u}(s)|\, ds\right) \Rightarrow \dot{u}(t) = 0, \\ |\zeta \partial_\nu \dot{u} + \mu \partial_\nu u + e \partial_\nu \varphi - \theta \mathcal{M}_c.\nu| = g\left(\int_0^t |\dot{u}(s)|\, ds\right) \Rightarrow \exists \beta \ge 0, \\ \text{such that } \zeta \partial_\nu \dot{u} + \mu \partial_\nu u + e \partial_\nu \varphi - \theta \mathcal{M}_c.\nu = -\beta \dot{u}. \end{cases} \quad \text{on } \Gamma_3 \times [0, T]. \tag{2.6}$$

$$\theta = 0 \qquad \text{on } \Gamma_1 \cup \Gamma_2 \times [0,T]\,, \tag{2.7}$$

$$e\partial_\nu u - \alpha\partial_\nu \varphi = q_2 \quad \text{on } \Gamma_b \times [0,T]\,, \tag{2.8}$$

$$e\partial_\nu u - \alpha\partial_\nu \varphi - \theta\mathcal{P}.\nu = k\left(\varphi - \varphi_F\right) \quad \text{on } \Gamma_3 \times [0,T]\,, \tag{2.9}$$

$$-k_{ij}\frac{\partial\theta}{\partial x_j}n_i = k_e(\theta - \theta_R) \qquad \text{on } \Gamma_3 \times [0,T]\,, \tag{2.10}$$

$$u(0) = u_0, \quad \theta(0) = \theta_0 \qquad \text{in } \Omega. \tag{2.11}$$

3 Variational Formulation

In this section we derive the variational formulation of Problem $\mathcal{P}$ and state our main existence and uniqueness result Theorem 1. By using Green's formula, it is strightforward to drive the following variational formulation of Problem $\mathcal{P}$.

Problem $\mathcal{PV}$: Find a displacement field $u : [0;T] \to V$, an electric potential field $\varphi : [0;T] \to W$ and a temperature field $\theta : [0;T] \to E$ such that

$$\begin{array}{l} a_\zeta(\dot{u}(t), \upsilon - \dot{u}(t)) + a_\mu(u(t), \upsilon - \dot{u}(t) + a_e^*\left(\varphi\left(t\right), \upsilon - \dot{u}\left(t\right)\right) \\ -a_{\mathcal{M}}\left(\theta\left(t\right), \upsilon - \dot{u}\left(t\right)\right) + j(\mathcal{S}_t\left(\dot{u}\left(t\right)\right), \upsilon) - j(\mathcal{S}_t\left(\dot{u}\left(t\right)\right), \dot{u}(t)) \\ \geq \left(f(t), \upsilon - \dot{u}(t)\right)_V \ \forall \upsilon \in V,\ t \in [0,T]\,, \end{array} \tag{3.1}$$

$$a_\alpha\left(\varphi\left(t\right), \psi\right) - a_e\left(u\left(t\right), \psi\right) + a_{\mathcal{P}}\left(\theta\left(t\right), \psi\right) = \left(q\left(t\right), \psi\right)_W \quad \forall \psi \in W,\ t \in [0,T]\,, \tag{3.2}$$

$$\dot{\theta}(t) + K\theta(t) = R\dot{u}(t) + Q(t) \qquad \text{on } E', \tag{3.3}$$

$$u(0) = u_0, \qquad \theta(0) = \theta_0 \qquad \text{on } \Omega. \tag{3.4}$$

4 An Existence and Uniqueness Result

Our main existence and uniqueness result is stated as follows.

Theorem 4.1. *Assume that hypoteses hold. Then there exists a unique solution (u, φ, θ) of Problem $\mathcal{PV}$. Moreover, the solution satisfies*

$$\begin{array}{rcl} u & \in & W^{1,2}(0,T;V);\ \varphi \in W^{1,2}\left(0,T;W\right), \\ \theta & \in & W^{1,2}(0,T;E^{'}) \cap L^2(0,T;E) \cap C(0,T;L^2(\Omega)). \end{array} \tag{4.1}$$

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An Efficient Game Theory Recommender System Based on Machine Learning

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Abstract
The amount of information in the world is increasing more quickly than our ability to process it. Fortunately, some technologies have been developed which can help users alleviate the problem of information overload in order to find what is most valuable to them. Such technology is called Recommender systems. The goal of a RS is to generate valuable recommendations to a group of users for various items or products that might interest them.
This paper presents a Game Theory recommender system based on machine learning to provide healthy food to cancer patients. The user-item ratings are clustered into similar groups based on the user ratings by the game theory and the whole recommendation process is based on machine learning algorithms.

Keywords: Game Theory, Recommender System, Healthcare, Food, Machine Learning, Cancer patient.

1 Introduction and motivation

Nowadays, the world wide web provides access to a vast range of online resources. Although the amount of information available is increasing by the day, people - and consumers in particular - are faced with new challenges: it's too difficult for them to choose what they really want. That is why Recommender systems (RS)s have emerged [1].
RSs are programs which aim at helping users to find and choose items (books, films, restaurants, music, etc.) likely to be of interest to them, from the vast number of choices available on the web or other electronic information sources [2].
Today, a variety of RSs are available to assist users, particularly in the healthcare industry [3]. A health RS provides a user with personalized medical information based on the user's health profile.The major purpose of this paper is to provide healthy and nutritious meal suggestions for cancer patients

2 Proposed approach

In this section, we introduce our approach which consists of a game theory based algorithm for collaborative filtering [4]. Our proposal involves 4 steps. In the first step, the patient profile is built based on his health status and preferences.

The second step consists of creating groups of similar users based on Game theory in order to build similar communities user with respect to the intrinsic relationship among users.
The third step concerns the recommendation process which is based on the collaborative filtering algorithm to generate Top-N food recommendations. The recommendation process uses the K-means clustering algorithm which is the most used algorithm in the literature regarding the recommendation field [7].

3 Main results

To implement the proposed approach, we used the Python language on a PC with 2.4 GHz Intel Core i7 CPU, 4 GB RAM, 1000 GB Hard Disk and 64-bit Windows 7 Integral Edition Operating System.
To evaluate the proposed model, we consider different food datasets uploaded from Kaggle (https://www.kaggle.com/) which are the most widely used dataset. in the literature for testing food RSs. We used metrics to check out the effectiveness of our proposal such as Accuracy, Precision and Recall [6].
Overall, the findings reveal that the proposed food RS approach outperforms the compared algorithms in terms of both quality and processing time [7].

4 Conclusion

This paper presents a RS based on the game theory and Machine learning. The Proposal makes use of the mathematical model proposed by the game theory to form community groups for the collaborative filtering. The suggested approach is thoroughly analyzed using different evaluation criteria and compared to some state-of the-art algorithms.

Overall, the findings reveal that the proposed method outperforms the compared algorithms in terms of both quality and processing time. In future, we intend to introduce swarm intelligence algorithms to make the recommendation more efficient and faster.

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Two regularization methods for a class of inverse fractional pseudo-parabolic equations with Involution perturbation

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Abstract
In this study, we give a theoretical analysis for an inverse problem governed by time-fractional pseudo-parabolic equation with involution. This problem is ill-posed, i.e., the solution (if exists) does not depend on the measurable data. To overcome the instability nature of this problem, we propose two regularization strategies: the first method employs a modified quasi-boundary value method, the second method is a variant of the quasi-reversibility technique. For this purpose, we give the convergence results under an a priori bound assumption and propose a practical a posteriori parameter selection rule.

Keywords: Inverse problems, pseudo-parabolic problems, involution perturbation, modified quasi-boundary-value method, quasireversibility method.
2020 Mathematics Subject Classification: 35R30, 35R25, 47A52, 35R11.

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Two parameters quasi-boundary value method for an abstract degenerate time-fractional backward problem

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Abstract
In this work, for a time degenerate fractional equation we study an inverse problem for restoration of initial condition from the information of final temperature profile. We show that the considered problem is ill-posed in the sense of Hadamard, i.e., small errors in the measurement data may lead to indefinitely large errors in the solutions. This ill-posed problem is regularized by a modified quasi-boundary value method and some convergence estimates for the regularized solution are obtained under both a priori and posteriori bound assumptions on the exact solution. Finally, several numerical experiments are presented to show the accuracy and efficiency of the proposed regularizing strategy.

Keywords: Ill-posed problems, time-fractional degenerate equation, Kilbas-Saigo function, quasi-boundary value method.
2020 Mathematics Subject Classification: 35R25, 35R30, 47A52, 26A33, 35K65.

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A Variant of Iterative Regularization Method for a Fractional Inverse Elliptic Problem of Tricomi-Gellerstedt-Keldysh Type

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Abstract
The present work is devoted to identifying an inaccessible boundary condition for a fractional elliptic problem of Tricomi-Gellerstedt-Keldysh-type. Using the expansion Fourier method, the considered problem can be reformulated as an operator equation of the first kind. To construct a stabilized approximate solution we employ a variant of the iterative method. We also present error estimates between the exact solution and the regularized solution by the a priori and the a posteriori parameter choice rules. Finally, some numerical verifications on the efficiency and accuracy of the proposed algorithm is presented.

Keywords: fractional elliptic equations, Tricomi-Gellerstedt-Keldysh equations, ill-posed problems, inverse problems, a posteriori parameter choice rule, iterative regularization method.
2020 Mathematics Subject Classification: 35R25; 35R30; 65J22; 34K37.

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Studying the Bifurcation Hopf and Stability in Multispiral Generated by New Saturated Function Series

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Abstract
This paper describes Hopf bifurcations of the multispiral chaotic attractor caused by new saturated function series. The basic structure of chaotic attractors can be described by the number of spirals (also known as a multiscroll attractor), which is controlled by the two integer parameters p, q. Being an integer, this value cannot be increased by one without preventing the detection of Hopf bifurcation for $p = q = 2$ spirals.The multispiral attractor generated by saturated function series can be found to bifurcate from a stable state to a chaotic one with this additional hidden parameter. Furthermore, the stability of the our system initial equilibrium point is investigated using the Routh-Hurwitz criteria.

Keywords: Hopf Bifurcation, stability, saturated function series.
2020 Mathematics Subject Classification: Primary 39A21, 39A28, 39A33.

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OPTIMIZATION OF A LINEAR FRACTIONAL FUNCTION OVER AN INTEGER EFFICIENT SET

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Abstract
Over the past few decades, optimization within the efficient set has witnessed significant advancements. In this paper, we introduce a novel exact algorithm designed to maximize the ratio of two linear functions over the integer set of a multi-objective linear program (MOILP), all without the need to explicitly enumerate all efficient solutions. This exact approach is founded upon a straightforward selection technique that enhances the primary linear fractional objective function Φ during each iteration, concurrently reducing the feasible region and eliminating dominated points. New solutions are discovered through the exploration of edges. The optimal value of the function Φ is reached within a finite number of algorithmic steps. To illustrate its efficacy, a numerical example is provided..

Keywords: efficient set, multi-objective linear program, linear fractional programming.
2020 Mathematics Subject Classification: Primary 90C29,90C10,90C32.

In preparing this paper, within we developed an algorithm that optimized linear fractional function ever the efficient set of a MOILP without explicitly having to enumerate all the efficient solutions. Given a Integer Linear Programming problem with Multiple Objective (MOILP):

$$(P_D)\begin{cases} \text{"max"} & Z_i = C_i x, \qquad i \in \{1, ..., p\} \\ \text{s.t.} & x \in D \end{cases} \tag{0.1}$$

Where $C_i \in R^n$, for each $i \in \{1, ..., p\}$, $A \in R^{m\times n}$, $b \in R^m$ and D is a polyhedral set of R^n defined as D=$\{x \in R^n | Ax = b, x \geq 0, \textit{integer}\}$. To avoid the technicality we assume throughout the paper that D is nonempty and bounded.

Our aim in this study is to provide one approach in the discrete case, consists of optimizing $\Phi(x)$ a Linear Fractional function representing the preferences of the decision-maker over the efficient set of (**??**). Formally, the problem under consideration can be defined as:

$$(P_E)\begin{cases} max & \Phi(x) = \dfrac{Ux + \alpha}{Vx + \beta} \\ \text{s.t.} & x \in E(P_D) \end{cases} \tag{0.2}$$

Where α, β are scalars; $p, q \in R^n$.
solving (P_E) is equivalent to solving the following program:

$$(P_{E-relaxed}) \begin{cases} max & \Phi(x) = \dfrac{Ux+\alpha}{Vx+\beta} \\ \text{s.t.} & x \in D \end{cases} \tag{0.3}$$

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Ab initio prediction of half-metallic ferromagnetic materials composed of alkali metals with nitrogen

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ABSTRACT

En utilisant la théorie de la fonctionnelle de la densité (DFT) dans l'approximation du gradient généralisé, nous explorons le comportement magnétique induit par les atomes non magnétiques Ra et N dans le composé binaire RaN dans cinq structures cristallines (cubique à faces centrées (B1), CsCl (B2), ZB (B3), WZ (B4) et NiAs (B81)). Dans toutes les structures considérées, la phase ferromagnétique (FM) est énergétiquement moins favorable que la phase paramagnétique, sauf dans les structures ZB et WZ. Dans les structures (FM) ZB et WZ, nous avons constaté que les orbitales p polarisées en spin du N dans le composé RaN conduisent à un ferromagnétisme demi-métallique (HM-FM) avec un moment magnétique entier de 1 μB par unité de formule. Nous avons calculé des écarts de spin flip (écarts demi-métalliques) de 0,19 et 0,22 eV dans l'approche GGA-PBE, 0,48 eV et 0,36 eV avec l'approche mBJ-GGA-PBE pour les structures ZB et WZ, respectivement. Le composé RaBi dans les structures ZB et WZ pourrait avoir une application potentielle dans les dispositifs spintroniques.

Toutes ces études reposent sur l'approximation du gradient généralisé de Perdew et al. (1996) associée à la théorie de la fonctionnelle de la densité (DFT). La première partie de ce chapitre consiste à déterminer la structure et la phase magnétique les plus stables de ce composé. La seconde partie consiste à étudier les propriétés électroniques et magnétiques dans la phase la plus stable.

Dans cette étude, les calculs ont été effectués en utilisant la méthode des ondes planes augmentées et linéarisées (FP-LAPW) implémentée dans le code WIEN2K. Pour le traitement du potentiel d'échange et de corrélation, nous avons utilisé l'approximation du gradient généralisé (GGA-PBE) telle que développée par Perdew, Burke et Ernzerhof (généralement notée PBE). De plus, pour les propriétés électroniques, nous avons également appliqué l'approximation mBJ GGAPBE, qui combine la partie du potentiel d'échange de TB-mBJ avec la partie du potentiel de corrélation de GGA-PBE. Cette approche récente pour traiter le potentiel d'échange et de corrélation a été introduite récemment.

Key Words: *DFT, FP-LAPW, Half-metallic, Elastic properties.*

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Study of Solutions for Nonlinear Fractional Differential Systems using Caputos q-Derivative

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Abstract
Fractional differential equations are a versatile tool for modeling complex systems, and their study is a rapidly growing field of research, as they are used in a variety of applied areas, including control engineering, physics, biomathematics, chemistry, and materials science [1], [2]. The study of quantum calculus, or infinite calculus, dates back to Jacksons article [3]. Q-calculus has since been used to bridge the gap between mathematics and physics, and the recent interest in EDFs has led to a renewed interest in q-fractional equations. Q-calculus has bridged the gap between mathematics and physics, and q-fractional equations have become increasingly important in recent years due to their applications in a variety of fields, including quantum theory, mechanics, capacitor theory, electrical circuits, particle physics, electroanalytical chemistry, and neurology [1],[2].

Keywords: q-fractional equations, Q-calculus, EDFs.

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Blow-up results for nonlinear wave equations with two dissipative terms

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Abstract
In this work, we prove the non-existence of global solutions to damped wave equation with viscoelastic damping and nonlinear memory term for any space dimension $n > 1$. Our main interest is to study the mixed effect of the term damping and nonlinear memory on blow-up conditions. The problem we consider is parabolic-like from the point of view of the decay of energy at infinity. For this reason we use the test function method to prove our results..

Keywords: Damped wave equation, nonlinear memory, Blow-up, Critical exponent, Test function method.

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Optimal Control Governed by Stationary Marguerre-von Kármán shallow shell Equations

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Abstract
The stationary Marguerre-von Kármán shallow shell equations describe two-dimensional nonlinearly elastic flat shells with boundary conditions akin to von Kármán plates. Initially proposed by Marguerre, von Kármán, and Tsien, these equations were later rigorously justified by Ciarlet and Paumier through formal asymptotic analysis. Our study offers insights into optimal control strategies for complex mechanical systems, leveraging advancements in mathematical analysis and control theory. This paper investigates the optimal control problem governed by these equations, establishing the existence and uniqueness of a non trivial state function, and the existence of the control. Then, by proving the Fréchet differentiability of the state, we establish the necessary optimality conditions for optimal control in terms of cost and prove uniqueness of the control. Furthermore, we define system of optimality.

Keywords: Non-linear elasticity; stationary Marguerre-von kármán's equations; optimal control.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

Introduction

From a mathematical point of view, the theory of optimal control is a continuation of the calculus of variations. This method appeared after the Second World War, responding to practical guidance needs,particularly in the field of aeronautics and flight mechanics. The fields of application are multiple: aerospace, automotive, robotics, aeronautics, internet and communications, the medical sector, etc. The literature on this theory is quite vast, many problems in science and engineering are treated. In particular, applications are made for variety of PDEs, see, e.g., [6].
The advancement in science and technology have brought forward a many of mathematical models. Among these models involving the structural mechanics. Most applications of these structures have been made to plates, and shells; particularly, shallow shell, where the initial shell curvature is assumed to be small. The mathematical formulation of these problems leads to a system of partial differential equations and a set of boundary conditions with complicated geometrical shape like that of many shells. The Marguerre-von Kármán equations has its origin in mechanics, dating to the mid twentieth

century with the works of Marguerre [10] von Kármán and Tsien [11]. These equations represent an extension of von Kármán equations due to von Kármán [12]. Both von Kármán and Marguerre-von Kármán equations have attracted a great deal of interest in mathematics and engineering. They play an important role in applied mathematics and have applications to many fields of solid mechanics. A detailed and mathematical modeling related to these equations can be found in, e.g, Ciarlet [3] and the historical references therein..
Some results are available for optimal control problems governed by the stationary von Kármán's equations,see, e.g., [2]. To the best of our knowledge, there are no results in literature for Marguerre-von Kármán equations.
A natural question arises as: How to extend the results in [2] for von Kármán plates to the more general geometry of Marguerre-von Kármán shallow shells?

Main results

1 von Kármán plate equations /Marguerre-von Kármán shallow shell equations

Let $\omega \in \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\omega$. We consider the von Kármán equations for a thin elastic plate by means the right hand side of the equilibrium equation is considered, where the second side of the first equation depends to $\lambda\Delta$ operator. First we give the reduced cubic form for the system defined by

$$\begin{cases} \Delta^2\zeta = [\varphi,\zeta] - \lambda\Delta\zeta + f & x \in \omega \\ \Delta^2\varphi = -[\zeta,\zeta] & x \in \omega \end{cases} \tag{1.1}$$

Where

$$[\varphi,\xi] = \varphi_{11}\xi_{22} + \varphi_{22}\xi_{11} - 2\varphi_{12}\xi_{21}, \qquad \varphi_{ij} = \frac{\partial^2\varphi}{\partial x_i \partial x_j}, \qquad i,j = 1,2.$$

$\zeta = \zeta(x_1,x_2)$ means the (reduced) deflection of the plate, $\varphi = \varphi(x_1,x_2)$ is the Airy stress function, λ expresses the magnitude of the external forces acting on the boundary of the plate in the direction of the normal vector and f is the resultant of vertical forces acting to the middle surface of the plate. We give the cubic equation for this system by

$$\zeta - \lambda L_2(\zeta) + C(\zeta) = Mf, \quad \text{such} \quad L_2(\zeta) = M(-\Delta\zeta).$$

In this paper, our system may be described as follows: Let $\omega \subset \mathbb{R}^2$ be a bounded domain with smooth boundary $\partial\omega$. We consider the Marguerre-von Kármán of the shallow shell model with Dirichlet boundary conditions

$$\begin{cases} \Delta^2\zeta = [\varphi,\zeta] + [\varphi,\theta] + f & x \in \omega \\ \Delta^2\varphi = -[\zeta,\zeta] - 2[\zeta,\theta] & x \in \omega \end{cases} \tag{1.2}$$

Where

$$\begin{cases} \zeta = \frac{\partial\zeta}{\partial\nu} = 0 & x \in \partial\omega \\ \varphi = \varphi_0 \quad , \frac{\partial\varphi}{\partial\nu} = \varphi_1 & x \in \partial\omega \end{cases} \tag{1.3}$$

Next, we use the reduced cubic form for the Marguerre-von Kármán equations, following the recent definitions with an unknown changes $\tilde{\zeta} = u = \zeta + \theta$ and $\tilde{F} = F + \theta$. Finally, we define the energy given by Ciarlet [3] $\mathcal{I}(v) = \mathcal{J}(u(v)) + i(v)$ to study the optimal control.Moreover, in this paper, after establishing the existence and uniqueness of the state in section §3 we discuss about the existence and uniqueness of optimal control in section §4 and section §6.
Following the idea in [2], we show Frèchet differentiability of the state $x(.)$ in section §5. As a consequence, we prove uniqueness of optimal control in section §6, and define system of optimality in section §7. This is another novelty of the paper.
We formulate these equations into an operational form

$$\zeta - B(\zeta, \theta_0) - B(\zeta + \theta, B(\theta, \theta)) + B(B(\zeta + \theta, \zeta + \theta), \zeta + \theta) = F + B(\theta, \theta_0), \tag{1.4}$$

Where $B(.,.)$ is a bi-linear continuous operator from $(H^2(\omega))^2$ into $H_0^2(\omega)$ and the functions $\theta_0, F \in H_0^2(\omega)$ are determined through the traces (φ_0, φ_1) and the applied body force f, respectively.
xxxxxxxxxxxxxxx

2 Notation and preliminaries

Let us define in Sobolev space $H_0^2(\omega)$ the following inner product

$$(u, v)_\Delta = \int_\omega \Delta u \Delta v dx, ||u||_{2,\omega} = \left(\int_\omega |\Delta u|^2 dx \right)^{\frac{1}{2}}, \forall u, v \in H_0^2(\omega)$$

We denote also $||u||_{0,\omega}$, $||u||_{1,\omega}$ and $||u||_{2,\omega}$ the norms respectively,
of the spaces $L^2(\omega)$, $H_0^1(\omega)$ and $H_0^2(\omega)$.
Assume that $(\varphi_0, \varphi_1) \in H^{\frac{3}{2}}(\partial\omega) \times H^{\frac{1}{2}}(\partial\omega)$, $\theta \in H^2(\omega)$ and $f \in H^{-2}(\omega)$. Then by the classical elliptic theory, there exists a unique function $\theta_0 \in H^2(\omega)$ such that

$$(2.3) \begin{cases} \Delta^2 \theta_0 = 0 & x \in \omega \\ \theta_0 = \varphi_0 \quad , \dfrac{\partial \theta_0}{\partial \nu} = \varphi_1 & x \in \partial\omega \end{cases} \tag{2.1}$$

and there exists a unique function F such that

$$\begin{cases} \Delta^2 F = f & x \in \omega \\ F \in H_0^2(\omega) \end{cases} \tag{2.2}$$

We assume that $\tilde{\zeta} = \zeta + \theta$, $\tilde{\zeta} \in H^2(\omega)$ and $\tilde{F} = F + \theta$, $\tilde{F} \in H_0^2(\omega)$

$$M = \Delta^{-2}, \tilde{F} = M\tilde{f} \in H_0^2(\omega),$$

M is the inverse of Δ^2 with homogenous Dirichlet boundary condition in ω.

Theorem 2.1. *The wellknown cubic opertor are B, L_2, L_1 and C. First, we let the bilinear continuous operator*

$$B : H^2(\omega) \times H^2(\omega) \longrightarrow H_0^2(\omega)$$

B be a bilinear operator, unique solution of $\Delta^2 B(\xi,\eta)=[\xi,\eta]$.
Then define the operator $C: H_0^2(\omega)\longrightarrow H_0^2(\omega)$, written by $C(\eta)=B(B(\eta,\eta),\eta)$ is a cubic operator. Assuming that $(\varphi_0,\varphi_1)\in H^{\frac{3}{2}}(\partial\omega)\times H^{\frac{1}{2}}(\partial\omega)$, we let $\theta_0\in H^2(\omega)$ satisfying $\Delta^2\theta_0=0$, in ω, $\theta_0=\varphi_0$ and $\partial_\nu\theta_0=\varphi_1$ in $\gamma=\partial\omega$, and we define the linear operator $L_2: H_0^2(\omega)\longrightarrow H_0^2(\omega)$ by letting $\lambda L_2(\eta)=B(\theta_0,\eta)$, and the linear operator $L_1: H_0^2(\omega)\longrightarrow H_0^2(\omega)$ given by $L_1(\eta)=B(\chi,\eta)=B\big(B(\theta,\eta),\theta\big)$. Then, assume that $f\in H^{-2}(\omega)$ and $\tilde{F}\in H_0^2(\omega)$ be the unique solution of $\Delta^2\tilde{F}=\tilde{f}=f+\Delta^2\theta$ in ω.

Proof. (**Theorem 6**,[5]). □

Let $\chi\in H^2(\omega)$ is the unique solution of the boundary value problem

$$\begin{array}{lll}\Delta^2\chi=0 & \text{and} & \chi=\varphi_0,\dfrac{\partial\chi}{\partial\nu}=\varphi_1 \quad \text{in} \quad \partial\omega,\\ \text{Where,} & \chi=B(\theta,\theta), & \\ \text{Therefore,} & \varphi=\chi-[\tilde{\zeta},\tilde{\zeta}]. & \end{array}$$

Hence, the canonical Marguerre-von Kármán equations are reduced to a cubic operator equation, such that the pair $(\tilde{\zeta},\lambda)\in(H_0^2(\omega),\mathbb{R})$ satisfies the continuous operator equation:

$$\tilde{\zeta}-\lambda L_2(\tilde{\zeta})-L_1(\tilde{\zeta})+C(\tilde{\zeta})-\tilde{F}=0 \tag{2.3}$$

Then, the canonical Marguerre-von Kármán equations are reduced to a cubic operator equation, such that the pair $(\tilde{\zeta},\lambda)\in(H_0^2(\omega),\mathbb{R})$ satisfies the variational equation:

$$\left\{\begin{array}{ll}\textbf{Find}\ \ (\tilde{\zeta},\lambda)\in(H_0^2(\omega),\mathbb{R}) & \textbf{such that},\\ \big(\tilde{\zeta}-\lambda L_2(\tilde{\zeta})-L_1(\tilde{\zeta})+C(\tilde{\zeta})-\tilde{F},\eta\big)_\Delta=0 & \forall\eta\in H_0^2(\omega)\end{array}\right. \tag{2.4}$$

Then $\forall\eta\in H_0^2(\omega)$, $\tilde{\zeta}$ it is a weak solution of problem 2.4.

We gather in the next theorem some useful properties of the operators involved in the reduced Marguerre-von Karman equation.

Theorem 2.2. *The following implication holds for the Monge-Ampere equation:*

1. $$\zeta\in H_0^2(\omega)\ \text{and}\ [\zeta,\zeta]=2\det(\partial_{\alpha\beta}\zeta)=0\Longrightarrow\zeta=0.$$
2. *Let* $(\zeta,\eta)_\Delta:=\displaystyle\int_\omega\Delta\zeta\Delta\eta d\omega$. *Then the operators B and C satisfy*
$$(B(\zeta,\eta),\chi)_\Delta=(B(\zeta,\chi),\eta)_\Delta,$$
$$\forall(\zeta,\eta,\chi)\in H^2(\omega)\times H_0^2(\omega)\times H_0^2(\omega),$$
3. *The nonlinear operators B and C are sequentially compact and hence, a fortiori, continuous.*

4. The operator L_2 is linear, compact, and symmetric.

Proof. (Theorem 7, [5]) □

Theorem 2.3. *We assume that data have the following regularities:*

$$(\varphi_0, \varphi_1) \in H^{\frac{3}{2}}(\partial\omega) \times H^{\frac{1}{2}}(\partial\omega), \quad \theta \in H^2(\omega) \quad \text{and} \quad f \in H^{-2}(\omega)$$

and the operators B, C, L_2, and L_1 define the functional

$$\forall v \in H_0^2(\omega), \mathcal{I}(v) := \mathcal{J}(u(v)) + j(v) \tag{2.5}$$

$$\mathcal{I} := \frac{1}{4}(C(u), u)_\Delta + \frac{1}{2}((I - \lambda L_2 - L_1)u, u)_\Delta + (Mf + \theta, u)_\Delta \tag{2.6}$$

Then, solving the reduced Marguerre-Von Karman equation is equivalent to finding all the stationary points of the functional $\mathcal{I}$, that is, those ζ satisfy

$$\zeta \in H_0^2(\omega) \quad \text{and} \quad \mathcal{I}'(\zeta) = 0$$

There exists at least one minimizer for the energy functional $\mathcal{I}$:

$$\zeta \in H_0^2(\omega) \quad \text{and} \quad \mathcal{I}(\zeta) = \inf_{u \in H_0^2(\omega)} \mathcal{I}(u) \tag{2.7}$$

Therefore, any such minimizer $\zeta \in H_0^2(\omega)$ is a solution of the reduced Marguerre-Von Karman equation, to which there corresponds a solution of the canonical Marguerre-Von Karman equations.

Proof. (Theorem 8, [5]) □

We identify these functions 2.5, 2.6 and the equation 2.4, we assume that $u = u(v)$, and $v = f$, we should have

$$\mathcal{I}(u(v)) = \frac{1}{4}\big(C(u(v), u(v)\big)_\Delta + \frac{1}{2}\big((I - \lambda L_2 - L_1)u(v), u(v)\big)_\Delta \tag{2.8}$$

$$j(v) = \big(\tilde{F}, u(v)\big)_\Delta =< \Delta^2 \tilde{F}, u(v) >=< \tilde{f}, u(v) >= \big(Mv + \theta, u(v)\big)_\Delta \tag{2.9}$$

Where

$$< ., . > \quad \text{define a scalar product in} \quad L^2(\omega). \tag{2.10}$$

The linearized problem for the canonical Marguerre-von Kármán equations consists in finding the pair $(v, \lambda) \in (H_0^2(\omega), \mathbb{R})$ such that

$$v = \lambda L_2(v),$$
$$\Delta^2 v = \lambda[v, \theta_0] \text{ in } \omega, \; v = \frac{\partial v}{\partial \nu} = 0 \text{ in } \gamma = \partial\omega,$$

Regualrity and spectral theory

Theorem 2.4. *Assuming that $f \in L^2(\omega)$, $\theta \in H^2(\omega)$, and $\theta_0 \in H^4(\omega)$. Then all solutions of* 1.2, 1.3 *have the regularity*

$$(\zeta, \phi) \in (H_0^2(\omega) \cap H^4(\omega))^2 \tag{2.11}$$

Where

$$\phi = \varphi - \theta_0.$$

Proof. (**Theorem 2.4**, [8]) □

Corollary 1. *Assuming that $f \in L^2(\omega)$, $(\varphi_0, \varphi_1) \in H^{\frac{7}{2}}(\partial\omega) \times H^{\frac{5}{2}}(\partial\omega)$. Then all solutions of the Marguerre-von Kármán* (2), (2.1) *have the regularity*

$$(\zeta, \phi) \in (H_0^2(\omega) \cap H^4(\omega)) \times H^4(\omega)$$

Proof. (**Corollary 2.5**, [8]) □

Since the linear operators L_2 and L_1 are compact, self-adjoint and positive definite, we conclude that L_2 and L_1 have an infinite number of distinct eigenvalues such that

$$0 < \lambda_1 < \lambda_2 < \lambda_3 < ... < \lambda_k < ... \to +\infty \text{ for } L_2,$$
$$\text{and } 0 < \alpha_1 < \alpha_2 < \alpha_3 < ... < \alpha_k < ... \to +\infty \text{ for } L_1.$$

where

$$\lambda_1 = \inf_{u \in H_0^2(\omega), ||u||_{2,\omega} \neq 0} \frac{||u||_{2,\omega}^2}{(L_2 u, u)_\Delta}, \quad \mu_1 = \frac{1}{\lambda_1} = \sup_{u \in H_0^2(\omega), ||u||_{2,\omega} = 1} (L_2 u, u)_\Delta \tag{2.12}$$

then

$$\alpha_1 = \inf_{u \in H_0^2(\omega), ||u||_{2,\omega} \neq 0} \frac{||u||_{2,\omega}^2}{(L_1 u, u)_\Delta}, \quad \beta_1 = \frac{1}{\alpha_1} = \sup_{u \in H_0^2(\omega), ||u||_{2,\omega} = 1} (L_1 u, u)_\Delta \tag{2.13}$$

The spectral theory of the operator L_2 is proved by P.G..Ciarlet, Rabier, now we are going to prove this results for L_1. First the compacity is satisfied by the compacity of the operator B, then we verify that L_1 is self-adjoint,

$\forall (\eta_1, \eta_2) \in H_0^2(\omega)$, $(L_1\eta_1, \eta_2)_\Delta := (B(\chi, \eta_1), \eta_2)_\Delta = (B(\chi, \eta_2), \eta_1)_\Delta = (L_1\eta_2, \eta_1)_\Delta$ ...(**theorem1 (b)**).

Therefore, L_1 is positive definite

$$\forall \eta \in H_0^2(\omega)\ (L_1\eta, \eta)_\Delta := \big(B(\chi, \eta), \eta\big)_\Delta = \Big(B\big(B(\theta, \eta), \theta\big), \eta\Big)_\Delta,$$

$$(L_1\eta, \eta)_\Delta = \Big(B\big(\theta, B(\theta, \eta)\big), \eta\Big)_\Delta \qquad \forall \Big(\theta, B\big(\theta, \eta\big), \eta\Big) \in H^2(\omega) \times H_0^2(\omega) \times H_0^2(\omega) ...(\textbf{B is symetric}),$$

$$\text{Hence,} \qquad (L_1\eta, \eta)_\Delta = \big(B(\theta, \eta), B(\theta, \eta)\big)_\Delta = ||B(\theta, \eta)||_{2,\omega}^2 \geq 0 \quad ...(\textbf{theorem1 (b)}).$$

3 Existence and Uniqueness of a Solution of the State Problem

Theorem 3.1. *Let*

$$||Mv+\theta||_{2,\omega} < ||B||^{-1}, \tag{3.1}$$

$$\lambda < \lambda^{\star}\big(||Mv+\theta||_{2,\omega}\big). \tag{3.2}$$

Where $\lambda^{\star}\big(||Mv+\theta||_{2,\omega}\big) = \frac{\lambda_1}{\alpha_1} + \lambda_1(1-||B||^{\frac{2}{3}}||Mv+\theta||^{\frac{2}{3}}_{2,\omega})$, *Then there exists a unique solution* $u(v) \in H_0^2(\omega)$ *of the equation (1.2). Moreover* $u(v)$ *satisfies the estimate*

$$||u(v)||_{2,\omega} \leq \tilde{K}_1(\lambda)^{-1}||Mv+\theta||_{2,\omega}, \tag{3.3}$$

where

$$\tilde{K}_1(\lambda) = 1 - \big(\frac{\lambda}{\lambda_1} + \frac{1}{\alpha_1}\big) \tag{3.4}$$

4 The Cost function and the minimization problem

Let the functional

$$\forall v \in H_0^2(\omega), \mathcal{I}(v) := \mathcal{J}(u(v)) + j(v), \tag{4.1}$$

$\mathcal{I}(u) := \frac{1}{4}(C(u),u)_{\Delta} + \frac{1}{2}((I-\lambda L_2 - L_1)u,u)_{\Delta} + (Mv+\theta,u)_{\Delta}$.

Theorem 4.1. *solving the reduced Marguerre-Von Karman equation is equivalent to finding all the stationnary points of the functional* $\mathcal{I}$, *that is, those* ζ *satisfy*

$$\zeta \in H_0^2(\omega) \text{ and } \mathcal{I}'(\zeta) = 0$$

Then, there exists at least one minimizer for the energy functional $\mathcal{I}$*:*

$$\zeta \in H_0^2(\omega) \text{ and } \mathcal{I}(\zeta) = \inf_{u \in H_0^2(\omega)} \mathcal{I}(u) \tag{4.2}$$

5 Existence of the optimal control

Assuming that $G(v) := ||v+\Delta^2\theta||_{0,\omega} - \frac{\alpha}{c}||B||^{-1}\tilde{K}_1(\lambda)^{\frac{3}{2}}$, $\alpha \in]0,1[$.

Let us define the admissible set of controls

$$\mathcal{U}_{ad} = \Big\{ v \in L^2(\omega) : G(v) \leq 0 \Big\} = G^{-1}(]-\infty,0]) \ ,$$

, when $\lambda < \lambda_1(1-\frac{1}{\alpha_1})$.

Where $\mathcal{U}_{ad}$ is a closed and convex set.

Theorem 5.1. *Let* $\mathcal{J} : H_0^2(\omega) \longrightarrow \mathbb{R}$, $j : L^2(\omega) \longrightarrow \mathbb{R}$ *be weakly lower semicontinuous. Then there exists a solution* $u \in \mathcal{U}_{ad}$ *of problem*

$$\mathcal{I}(u) = \min_{v \in \mathcal{U}_{ad}} \mathcal{I}(v) \tag{5.1}$$

6 Uniqueness of the Control

Assuming that $\mathcal{J}$ and j are *Frèchet* differentiable and satisfying conditions

$$\left\langle \mathcal{J}'(u_1) - \mathcal{J}'(u_2), u_1 - u_2 \right\rangle_* \geq m||u_1 - u_2||_{2,\omega}, \forall m > 0, \forall u_1, u_2 \in H_0^2(\omega). \tag{6.1}$$

$$\left\langle j'(v_1) - j'(v_2), v_1 - v_2 \right\rangle_0 \geq N||v_1 - v_2||_{0,\omega}, \forall N > 0, \forall v_1, v_2 \in L^2(\omega). \tag{6.2}$$

If $v \in \mathcal{U}_{ad}$ is a solution of problem (5.1), then $\left\langle \mathcal{I}'(v), w - v \right\rangle_0 \geq 0$, $\forall w \in \mathcal{U}_{ad}$.

$$||\mathcal{J}'(x)||_* \leq a_0||x||_{2,\omega} + a_1, \forall u \in H_0^2(\omega); a_0, a_1 > 0, \tag{6.3}$$

where $||.||_* = ||.||_{\mathcal{L}(H_0^2(\omega), H_0^2(\omega))}$,

Theorem 6.1. *Let the functionals $\mathcal{J}$, j be lower bounded, weakly lower semi-continuous with Frèchet derivatives satisfying the conditions (6.1),(6.2) and (6.3), if*

$$\left[-m + (a_0\alpha||B||^{-1}\tilde{k}_1(\lambda)^{\frac{1}{2}} + a_1).\frac{6\alpha}{1-\alpha^2}\tilde{k}_1(\lambda)^{-\frac{1}{2}}||B||\right].$$
$$c^2[\tilde{K}_1(\lambda)(1-\alpha^2)]^{-2} < N$$

where $\tilde{k}_1(\lambda) = 1 - (\frac{\lambda}{\lambda_1} + \frac{1}{\alpha_1})$, then there exists a unique solution u of the problem (4.2).

7 Necessary Condition of Optimality

$$\left\langle p, v - u \right\rangle + \left\langle R_2 j'(u), v - u \right\rangle \geq 0, \forall v \in \mathcal{U}_{ad}, \tag{7.1}$$

$$[I - \lambda L_2 - L_1 + C'(x(u))]p = R_1\mathcal{J}'(x(u)), \tag{7.2}$$

$$x(u) - \lambda L_2 x(u) - L_1 x(u) + C(x(u)) = Mu + \theta, \tag{7.3}$$

The system (7.1),(7.2) and (7.3) is the optimality system of the problem (5.1). The problem (7.2) is the adjoint problem to the problem (7.3)

8 Conclusion

This study is concerned with optimal control problem for Marguerre-von Kármán equations. First, we establish the existence and uniqueness of non trivial solutions. Then we define the admissible set of control to study the existence and by the *Fèchet* differentiability of the state we get the uniqueness of the control. At the end by Riesz theorem, and the adjoint problem we establish the system of optimality to define the necessary condition of optimality. Note that, in the case $\theta \equiv 0$ in $\bar{\omega}$, we recover the optimal control problem for von Kármán equations.

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Limit cycles of perturbed polynomial differential systems by averaging method

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Abstract
We study the existence and stability of T-periodic solutions of T-periodic systems in R^n of the form $dx/dt = \varepsilon g(t, x, \varepsilon)$ (1)
where $\varepsilon > 0$ is a parameter sufficiently small and the function g is T-periodic with respect to the variable t, by using the averaging theory.we present some applications of the averaging theory of order higher than one. Thus using the averaging theory of second order we study:
- the periodic solutions of the 3-dimensional Hopf bifurcation via averaging theory of second order,
- the periodic solutions for differential systems in R^3 and R^4,
- the 4-dimensional zero-Hopf bifurcation via averaging theory
- the zero-Hopf perodic orbits for a Rossler differential systems
We analyze the stability of the periodic solutions.
The averaged function play a key role in this study.

(see the references below).

Keywords: averagng theory, limit cycle, zero-Hopf bifurcation, polynomial differential systems,stability.

2020 Mathematics Subject Classification: Primary 34C25 , 34C29 , 34C07, 37G15.

1 Introduction

A limit cycle is a periodic orbit isolated in the set of periodic solutions.
In general to obtain analytically periodic solutions of a differential system is a very difficult problem, many times a problem impossible to solve.
As we shall see when we can apply the averaging theory, this difficult problem for the differential systems (1) is reduced to find the zeros of a nonlinear function of dimension at most n, i.e. now the problem has the same difficulty of the problem of finding the singular or equilibrium points of a differential system.
An important problem for studying the periodic solutions of the differential systems of the form

$\frac{dx}{dt} = F(t,x)$, or $\frac{dx}{dt} = F(x)$, (2)
using the averaging theory is to transform them in systems written in the normal systems of the form of the averaging theory , i.e. as a system (1).
Note that systems (2), in general,are not periodic in the independent variable t and
do not have any small parameter !. So we must find changes of variables which allow to write
the differential systems (2) into the form (1) .
This problem is with relation with the sixteenth Hilbert problem which is not yet solved.
There are many applications of averaging method, in celestial mechanics, physics, biology,...

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The dynamic behavior of a hydrogen-powered internal combustion engine

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Abstract
The study examines the dynamic behavior of a hydrogen-powered internal combustion engine using numerical techniques in the Open Modelica environment. The investigation explores the influence of engine speed on crucial parameters, such as mass flow, fuel cycle, temperature, pressure, and rate of heat release (HRR). The results showcase a nuanced interplay of these factors as the engine transitions from 1700 to 2000 to 2500 RPM, providing valuable insights into the complex combustion processes within the hydrogen-powered system.
Keywords: Direct-injection, Spark-ignition, Combustion, Heat release ratio, Engine Speed, Hydrogen.

1 Introduction

There is a major health and environmental risk associated with the toxic exhaust emissions from road transport cars, especially those with gasoline and diesel engines [1]. Through investigations on the control of the undesired emissions including HC, CO, NOx, and PM, the researchers are currently looking into the development of a finer alternative and cleaner fuels with larger performances.[2],[3]One of the cleanest fuels for gasoline engines is compressed natural gas (CNG), although it has the disadvantage of burning slowly, especially for tiny engines[4]. However, due to its extremely quick burning velocity and other properties, prior research has demonstrated that adding hydrogen to CNG fuel improved engine performance, brake thermal efficiency, brake specific fuel consumption decrease, and exhaust emissions reduction.[5]–[6]Consequently, up to 80% and 45%, respectively, less CO and hydrocarbons can be released when gasoline is replaced with HCNG fuel [7] The majority of the time, CNG fuel is used in biofuel-type vehicles, which can alternate between running on gasoline and another fuel on occasion. However, gasoline is typically the primary fuel in these vehicles and can be switched to CNG mode. When the system is converted from gasoline to compressed natural gas (CNG), it performs worse in terms of reduced brake power and volumetric efficiency. This is because the system uses port injection for gasoline [8]. Direct injection has proven to be more beneficial than port injection for achieving better engine compression ratios, reduced energy loss, and increased fuel efficiency while using CNG fuel, according to earlier studies [9]. The engine performs better because of a faster flame speed because of the high response rate of hydrogen in the fuel composition, but this also results in increased fuel consumption. Engine performance is increased by 8% and exhaust carbonic emissions are increased by 50% when the hydrogen proportion in the fuel is increased to 30%. Additionally, the amount of NOx emissions from engines is reduced when EGR is used.[5], [10]–[14] These days, scientists believe that switching from port injection (PI) engines to direct-injection (DI) engines is the best way to increase engine economy because DI engines not only consume less gasoline but also perform better and produce less exhaust fumes[15]–[17] Additional research has demonstrated that gasoline may be converted to CNG direct injection by implementing sophisticated cylinder head modifications and adding measurement

components to the injection. When compared to port injection CNG, this switch to direct injection on CNG has demonstrated improvements in brake power of up to 9% and thermal efficiency of up to 7%[8]. For optimal ignition performance, the fuel injection timing must be somewhat delayed in relation to the ignition timing, according to simulation results for a DI natural gas engine [16]. In this research, we use Open Modelica to obtain results related to the effect of changing speed in hydrogen internal combustion engines on the heat release ratio and pressure, as well as the engine power

2 Main results

Increasing the speed in a hydrogen internal combustion engine results in heightened pressure, greater engine Temperature, and an increased heat release rate due to several interconnected factors within the combustion process. When we transition from 1700 to 2000 to 2500 revolutions per minute (RPM), the engine undergoes a series of changes that impact these variables.

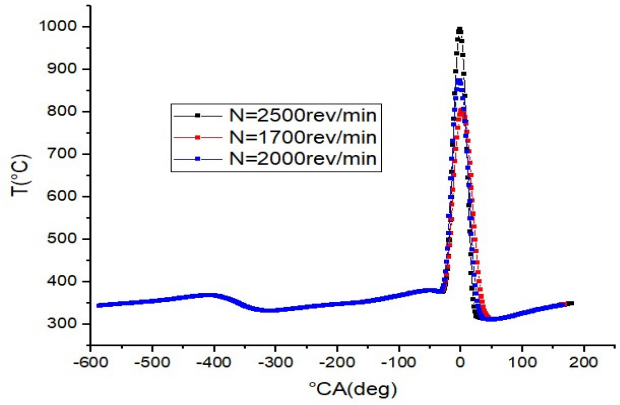

Fig 1. Variation of Temperature as a function of crankshaft rotation angle

The effects of increasing speed (RPM) in a hydrogen internal combustion engine on temperature (T)

- Increased RPM means more frequent combustion cycles: As the engine speed increases, the time available for each combustion cycle decreases since the crankshaft completes more revolutions per minute. This results in a shorter combustion duration.

- Shorter combustion duration leads to higher temperature: The compression and combustion processes occur more rapidly, leading to less time for heat dissipation. Consequently, the temperature within the combustion chamber increases due to the compression and combustion of the hydrogen-air mixture.

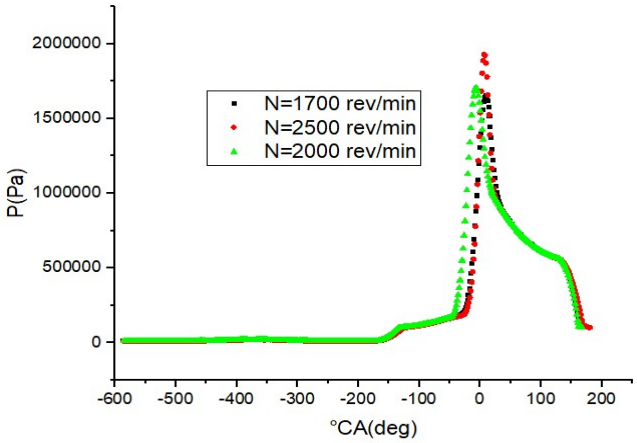

Fig 2. Variation of the pressure in the cylinder as a function of crankshaft rotation angle

The effects of increasing speed (RPM) in a hydrogen internal combustion engine on pressure (P):

-Faster piston movement increases pressure: Higher RPM implies faster movement of the pistons within

the engine. The piston's rapid motion during the compression stroke compresses the hydrogen-air mixture more quickly, resulting in higher pressures within the combustion chamber.

- Increased pressure during combustion: The faster combustion process, triggered by the quicker movement of the piston, leads to increased pressure during the power stroke. This is due to the force generated by the rapid expansion of the high-temperature gases.

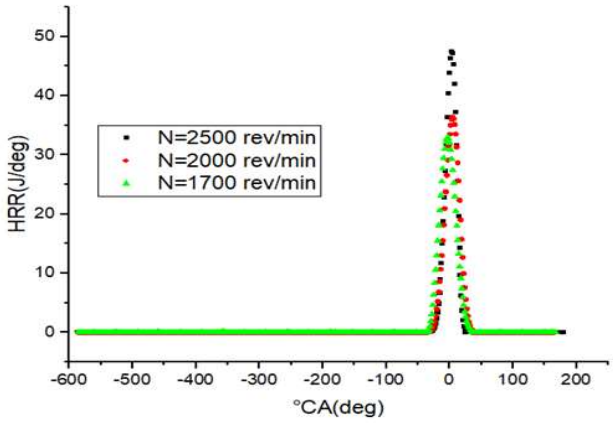

Fig 3. Variation of HRR as a function of crankshaft rotation angle

The effects of increasing speed (RPM) in a hydrogen internal combustion engine on rate of heat release (HRR).

- Shorter combustion duration intensifies heat release rate: The heat release rate is a measure of the rate at which energy is released during combustion. With higher RPM, the combustion duration shortens, causing a more rapid release of energy.

- Quicker combustion results in a more intense HRR: The increased frequency of combustion events, combined with the shorter duration of each cycle, contributes to a higher overall heat release rate. This is a result of more hydrogen molecules undergoing combustion in a given time frame.

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Boundedness and Behavior for a Differential-Algebraic System with Hybrid Functional Response

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Abstract
The purpose of this work is to investigate the dynamics of a bio-economic predator-prey system that only uses harvesting prey and a hybrid Holling II and Bedding-DeAngelis response function. Since the system earns money, it has an algebraic equation. To interpret some of the main results, we provide a comprehensive mathematical analysis of the proposed model. The boundedness and positivity of the model's solutions were examined. Numerous qualitative theories of dynamical systems, including local stability.

Keywords: Algebraic differential equations, Equilibrium point, Predator-prey system.
2020 Mathematics Subject Classification: Primary: 34D20; 34C23. Secondary: 37N25.

1 Introduction

There is a lot of interest in understanding and designing bio-economic models for biodiversity for the long-term benefit of humanity. Researchers are working to develop certain potentially beneficial results in order to ensure the ecosystem's long-term viability and prosperity.
The investigation of such dynamics has been the focus of a lot of research. The dynamical behavior of a type of predator-prey ecosystem was explored using numerous differential equations and an algebraic equation [1, 2]. They discovered important discoveries like interior equilibrium stability, Hopf bifurcation, limit cycle, singularity driven bifurcation and control, and so on. However, only the prey population is harvested in all of the models studied. With the assumption that the isolated predator species has a natural mortality, the interaction between predator and prey was explored using several functional response such as Holling I, Holling II, Holling III [1], and Beddington-DeAngelis [3] .

2 Main results

2.1 Local stability analysis

For the analysis of the local stability of X_e, we have the following proposition

Proposition 2.1.

For the positive equilibrium point $\bar{X}_e$ of the system (2.1), we have
(i) If $T_1^2(v) \geq 4T_2(v)$ and $T_2(v) > 0$, then when $T_1(v) > 0$, $\bar{X}_e$ is locally asymptotically stable node. When $T_1(v) < 0$, $\bar{X}_e$ is unstable node.
(ii) If $T_2(v) < 0$,then, $\bar{X}_e$ is an unstable saddle point.
(iii) If $T_1^2(v) < 4T_2(v)$, then, when $T_1(v) > 0$, $\bar{X}_e$ is locally asymptotically focus. When $T_1(v) < 0$, $\bar{X}_e$ is unstable focus.

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A new family of globally convergent conjugate gradient methods

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Abstract
We know many conjugate gradient algorithms (CG) for solving unconstrained optimization problems. In this paper, based on the three famous Liu-Storey (LS), Fletcher-Reeves (FR) and Polak-Ribiere-Polyak (PRP) conjugate gradient methods, a new hybrid CG method is proposed. Furthermore, the search direction satisfies the sufficient descent condition independent of the line search. Likewise, we prove, under the strong Wolfe line search, the global convergence of the new method. In this respect, numerical experiments are performed and reported, which show that the proposed method is efficient and promising.

Keywords: Unconstrained optimization, hybrid method, sufficient descent, convex combination.
2020 Mathematics Subject Classification: Primary 65K05, 90C25, 90C26, 90C27, 90C30.

1 Introduction

Unconstrained optimization is a branch of optimization in which we minimize an objective function that depends on real variables with the total absence of restrictions on their values of those variables. Thus, we consider the general unconstrained optimization problems, as follows:

$$\min\{f(x), x \in \mathbb{R}^n\}, \tag{1.1}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the continuously differentiable function and its first derivative is represented by $g(x) = \nabla f(x)$. Though several robust optimization algorithms with rapid convergence have shown to be available to solve the above nonlinear optimization model, many researchers still refer to the conjugate gradient algorithm (CG) as it uses low memory and good convergence properties. Besides, this method was first established by Hestenes and Stiefel [1] to solve unconstrained linear optimization problems. Then, in 1964, Fletcher and Reeves [2] extended the form of the conjugate gradient method to solve unconstrained nonlinear minimization problems. As a consequence, the results of the expansion inspired researchers to suggest a new conjugate gradient method with good computational performance and, at the same time, good convergence properties.

Generally, the iterates of the CG methods are usually determined through the following recursive computational scheme:

$$x_{k+1} = x_k + \alpha_k d_k. \quad k = 0, 1, \dots n, \tag{1.2}$$

and

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -g_k + \beta_k d_{k-1}, & \text{if } k \geq 1, \end{cases} \tag{1.3}$$

where x_k is the current iteration, g_k is the gradient of f at the point x_k, d_k is the search direction, $\beta_k \in \mathbb{R}$ is the conjugate parameter which characterizes different versions of the CG methods and $\alpha_k > 0$ is the step size many line search techniques can obtain. In the midst of which, exact line search, weak Wolfe line search, or strong Wolfe line search, but we use, in our research, strong Wolfe line search, which is defined by the following conditions:

$$f(x_k + \alpha_k d_k) - f(x_k) \leq \delta \alpha_k g_k^T d_k, \tag{1.4}$$

$$|g_{k+1}^T d_k| \leq -\sigma g_k^T d_k, \tag{1.5}$$

where scalars δ and σ satisfy $0 < \delta \leq \sigma < 1$.

As we said, the CG algorithms differ by choosing the coefficient β_k. The most common standard CG methods are the Hestenes-Stiefel (HS) method [1], the Fletcher-Reeves (FR) method [2], the Liu-Storey (LS) method [3] and the Polak-Ribiére-Polyak (PRP) method [4, 5], respectively, which are defined as:

$$\beta_k^{HS} = \frac{g_k^T y_{k-1}}{y_{k-1}^T d_{k-1}} \qquad \beta_k^{FR} = \frac{||g_k||^2}{||g_{k-1}||^2}$$

$$\beta_k^{LS} = \frac{g_k^T y_{k-1}}{-d_{k-1}^T g_{k-1}} \qquad \beta_k^{PRP} = \frac{g_k^T y_{k-1}}{||g_{k-1}||^2},$$

where $y_{k-1} = g_k - g_{k-1}$ and $||.||$ stands for the Euclidean norm. If f is a strictly convex quadratic function, all the above methods are equivalent, while they behave differently for general non-quadratic functions.

One of the essential classes of CG methods is the hybrid conjugate gradient algorithms. Further, the hybrid schemes have better computational performances and more robust convergence properties than conventional CG methods as they take advantage of the two parameters used to build them. For this reason, many researchers cared about hybrid or mixed conjugate gradient methods. Djordjevic' [6], proposed the following hybrid method: $\beta_k^{hyb} = \theta_k \beta_k^{FR} + (1 - \theta_k)\beta_k^{HS}$, Andrei [7], proposed the following hybrid method: $\beta_k^c = \theta_k \beta_k^{DY} + (1 - \theta_k)\beta_k^{HS}$. In addition, Sabrina and al [8] proposed a new hybrid CG method based on combination of FR, PRP and DY conjugate gradient algorithms in which

$$\beta_k^{hyb} = \delta_k \beta_k^{FR} + \gamma_k \beta_k^{PRP} + (1 - \delta_k - \gamma_k)\beta_k^{DY},$$

where

$$\gamma_k = -\frac{g_k^T g_{k+1}||g_k||^2 + \delta_k(y_k^T s_k - ||g_k||^2)||g_{k+1}||^2}{(g_{k+1}^T y_k)(y_k^T s_k) - ||g_{k+1}||^2||g_k||^2}, \quad \text{where, } 0 < \delta_k < 1.$$

Inspired by this research, we propose in this study a new hybrid CG method based on combination of LS, FR and PRP conjugate gradient algorithms for solving unconstrained optimization problems.

2 Main results

In this paper, we propose another combination of LS, FR and PRP conjugate gradient algorithms. We use the following conjugate gradient parameter:

$$\beta_k^{New} = \delta_k \beta_k^{LS} + \gamma_k \beta_k^{FR} + (1 - \delta_k - \gamma_k)\beta_k^{PRP}. \tag{2.1}$$

As a consequence, the direction d_k is given by:

$$d_k = \begin{cases} -g_k, & \text{if } k = 0, \\ -g_k + \beta_k^{New} d_{k-1}, & \text{if } k \geq 1, \end{cases} \tag{2.2}$$

Where $0 \leq \delta_k, \gamma_k \leq 1$ are named the hybridization parameters and they are given by the following form:

$$\delta_k = \frac{\left(g_{k+1}^T y_k(||g_k||^2 - d_k^T y_k) - \gamma_k(g_{k+1}^T g_k)(d_k^T y_k)\right)}{(||g_k||^2 + d_k^T g_k)(-g_{k+1}^T y_k)(d_k^T y_k)} d_k^T g_k, \qquad \text{where } \ 0 < \gamma_k < 1. \tag{2.3}$$

The following Theorem proved that the search direction d_k generated by the selected method satisfies the sufficient descent condition.

Theorem 2.1. *Let the sequences $\{g_k\}$ and $\{d_k\}$ be generated by the new method. Then, the search direction d_k satisfies the sufficient descent condition:*

$$g_k^T d_k \leq -K \ ||g_k||^2, \quad \forall k \geq 0, \tag{2.4}$$

where $K > 0$.

Now, we give the next theorem which establishes the global convergence of our proposed method with strong Wolfe line searches.

Theorem 2.2. *Consider the iterative method, defined by the new method and let all conditions of Theorem (2.1). Then*

$$\lim_{k \to \infty} \inf ||g_k|| = 0. \tag{2.5}$$

Numerical Results:
The results are given in two Figures which are represented the performance profiles of our new hybrid method versus FR, PRP and LS based on the CPU time and number of iterations, respectively. They shown that the performance profiles of our method was higher than the other CG methods on the testing problems.

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Decay Rate Result For The Evolution Viscoelastic Problem With Nonlinear Dynamics Source

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Abstract
By using the technique of energy decay and some properties of convex functions we establish a general decay solution for the evolution viscoelastic problem with nonlinear dynamics source in the right-hand side

$$\begin{cases} u_{tt} + \Delta^2 u - \int_0^t g(t-s)\,\Delta^2 u(s)\,ds = |u|^{\gamma-2}\,u \ln|u|, & \text{in } \Omega \times (0,\infty), \\ u = 0, & \text{on } \partial\Omega \times (0,\infty), \\ \mathcal{B}(u) - \mathcal{B}\left(\int_0^t g(t-s)\,u(s)\,ds\right) = 0, & \text{on } \partial\Omega \times (0,\infty), \\ u(0) = u_0,\ u_t(0) = u_1, & x \in \Omega, \end{cases} \tag{0.1}$$

where $\gamma > 2$, Ω is a bounded domain of $\mathbb{R}^2$, and a relaxation function g satisfies $g'(t) \leq -\xi(t)\,H(g(t))$, where H is an increasing and convex function near the origin and $\xi(t)$ is a positive nonincreasing function, from which the usual exponential and the polynomial decay are only special case of our result.
The differential operator $\mathcal{B}$ is denoted as follows:

$$\mathcal{B}u = \Delta u + (1-\nu)\,Bu,$$

where

$$Bu = 2\tau_1\tau_2 \frac{\partial^2 u}{\partial x_1 \partial x_2} - \tau_1^2 \frac{\partial^2 u}{\partial x_1^2} - \tau_2^2 \frac{\partial^2 u}{\partial x_2^2}. \tag{0.2}$$

and Poisson coefficient is equal to ν with $\nu \in \left(0, \frac{1}{2}\right)$.

Keywords: Viscoelasticity; Convex function; Logarithmic nonlinearity; Relaxation function;
2020 Mathematics Subject Classification: 35L55, 35A01, 93D20, 93D15, 74D10.

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Well-posedness and stabilization of a hyperbolic equation by the internal dissipation with fractional derivative

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Abstract
In this work, by using the semigroup theory of linear operators we estabish the well-posedness of a hyperbolic equation (**??**) with an internal damping of fractional derivative type. we use a general Theorem of Arendt-Batty in [**?**] and Lyubich-Vu in [**?**] to show the strong stability of the C_0-semigroup $e^{t\mathcal{A}}$ associated to the wave system (**??**) in the absence of the compactness of the resolvent of $\mathcal{A}$.

Keywords: hyperbolic equation, stabilization, semigroup theory, internal dissipation with derivative fractional .
2020 Mathematics Subject Classification: Primary 93D15, 35B40, 47D03, 74D05.

1 Introduction

In this work we establish the existence, uniqueness and the strong stability of the system of the solution (**??**) under the internal dissipation with derivative fractional in an open bounded domain Ω of $\mathbb{R}^n$ with boundary Γ of class C^2. The system (**??**) is given by :

$$\begin{cases} y_{tt}(x,t) - \Delta y(x,t) - \Delta y_{tt}(x,t) + \gamma \partial_t^{\alpha,\eta} y = 0 & \text{in } \Omega \times (0,+\infty) \\ y(x,t) = 0, & \text{on } \Gamma \times (0,+\infty) \\ y(x,0) = y_0(x), \quad y_t(x,0) = y_1(x) & \text{on } \Omega \end{cases} \tag{1.1}$$

the notation $\partial_t^{\alpha,\eta}$ stands for the generalized Caputo's fractional derivative of order α, $0 < \alpha < 1$, with respect to the time variable (see [**?**] and [**?**]). It is defined as follows

$$\partial_t^{\alpha,\eta} z(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} e^{-\eta(t-s)} \frac{dz}{ds}(s)\, ds, \quad \eta \geq 0.$$

Very recently, in [**?**], Ammari et al., studied the wave equation with internal fractional damping. The system considered is as follows:

$$\begin{cases} u_{tt}(x,t) - \Delta u(x,t) + \gamma \partial_t^{\alpha,\eta} u(x,t) = 0 & \text{in } \Omega \times (0,+\infty), \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,+\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) & \text{on } \Omega. \end{cases}$$

The authors proved that the energy decays polynomially as $t^{-2/(1-\alpha)}$.
In [?] Berbiche studies the boundedness and asymptotic properties of solutions as $t \longrightarrow \infty$ of the following linear wave equation with memory:

$$\begin{cases} u_{tt}(x,t) - \Delta u(x,t) - \Delta u_{tt}(x,t) + k * u_t = 0 & \text{in } \Omega \times (0,+\infty), \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,+\infty), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) & \text{on } \Omega. \end{cases}$$

2 Main results

Lemma 2.1.
The energy functional associated with problem **(??)** *is given by*

$$E(t) = \frac{1}{2}\|y_t\|_2^2 + \frac{1}{2}\|\nabla y_t\|_2^2 + \frac{1}{2}\|\nabla y\|_2^2 + \frac{\zeta}{2}\int_\Omega \int_{-\infty}^{+\infty} |\phi(x,\xi,t)|^2 \, d\xi \, dx. \tag{2.1}$$

and satisfies

$$E'(t) = -\zeta \int_\Omega \int_{-\infty}^{+\infty} (\xi^2 + \eta)|\phi(x,\xi,t)|^2 \, d\xi \, dx \leq 0. \tag{2.2}$$

Theorem 2.1.
The semigroup $S_{\mathcal{A}}(t)_{t\geq 0}$ is strongly stable in $\mathcal{H}$; i.e, for all $Y_0 \in \mathcal{H}$, the solution of (??) satisfies

$$\lim_{t\to\infty} \|e^{t\mathcal{A}} Y_0\|_{\mathcal{H}} = 0.$$

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Some Fixed Point Theorem of Integral Type Under C-class Functions in b-Metric Spaces

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Abstract In this presentation, we establish the existence and uniqueness of the fixed point for a self-map of integral type using the notion of C-class functions in a complete b-metric space, after giving definition of C-class functions covering a large class of contractive conditions by Ansari. And provide a definition of the B-metric space, which is considered a generalization of the metric space.

Keywords: fixed point, b-metric space, C-class function, self mappings

1 Introduction

Fixed point theory is one of the most important topics used in solving many problems in mathematics. The first significant result in this theory is Banach's contraction, which has been generalized in various ways by many authors. One notable generalization was made by Branciari, who was the first to study the existence of fixed points for contractive mappings of the integral type. Later, in 2014, Ansari further extended Banach's contraction principle by introducing the concept of C-class functions. In this presentation, we will demonstrate the fixed point theorem of the integral type using C-class functions in B-metric spaces.

2 Main results

Theorem 2.1. *Let (X, d) be a complete b-metric space and suppose mapping $U : X \to X$ satisfying:*

$$\int_0^{sd(Ux,Uy)} \varphi(t)\, dt \leq m \int_0^{M(x,y)} \varphi(t)\, dt, \text{ for all } x, y \in X$$

where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ which is non-negative, summable on each compact subset of $\mathbb{R}^+$, and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)\, dt > 0$, and

$$M(x, y) = \max\left\{ d(x, y), d(x, Ux), d(y, Uy), \frac{d(x, Uy) + d(y, Ux)}{2s} \right\}.$$

Then U has a unique fixed point in X.

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Abstract

We study a system of coupled differential equations with mixed nonlocal boundary conditions. We transform this system to an abstract Cauchy problem, and we obtain the existence and uniqueness solution of this problem with the use of an infinitely differentiable semigroup that has a singularity at zero.

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A Novel Approch for Solving Fredholm Fractional integro-différential equation using Mann's iteration

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Abstract
In this work, the concept of Mann's iteration method is introduced for solving Fredholm fractional integro-differential equation.
The derivative is considered in the Caputo sense.
to verify the reliability and efficiency of the method, illustrative examples are given.

Keywords: Mann's iteration method , Fredholm fractional integro-differential equation, Caputo fractional derivative.

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Multi-valued Fractional Differential Equation

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Abstract
In this paper, we study the well-posedness (in the sense of existence and uniqueness of a solution) of a sweeping process involving fractional order derivative of Caputo type in Hilbert spaces. The convex constraint set $C(t)$ is assumed to satisfy a Hölder continuous variation. The notion of a solution of this fractional sweeping process is proposed, the existence and uniqueness of such solution are proved and a stability result is provided, as well. More precisely, we present new arguments for the proof of existence of the fractional sweeping process. It is still based on the ideas of the catching-up algorithm of J.J. Moreau.

Keywords: Variational analysis, Fractional differential inclusion, Moreau's sweeping process, Cauchy problem.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

The sweeping processes was introduced and deeply studied by J. J. Moreau in the from of the discontinuous di?erential inclusion

$$\begin{cases} \dfrac{du}{dt}(t) \in -N_{C(t)}\big(u(t)\big) & \text{for a.e. } t \in [0,T], \\ u(0) = u_0 \in C(0), \end{cases} \tag{1.1}$$

where $T > 0$ and $N_{C(t)}(\cdot)$ denotes here the normal cone of $C(t)$ in the sense of convex analysis. Whereas the case of Lipschitz (resp. Absolutely continuous) $t \mapsto C(t)$ with respect to the Hausdorff metric, always leads to the existence and uniqueness of a Lipschitz (resp. an absolutely continuous) solution $u(t)$ for any initial condition, the case where $t \mapsto C(t)$ is a convex valued function of

$\alpha-$*Hölder continuous* variation doesn't ensure solvability of (1.1) in the class of Lipschitz (absolutely) continuous functions. That is why the extended concept of the derivative (called the Caputo fractional concept) is required in (1.1) when the map $t \mapsto C(t)$ is an α-Hölder continuous function, in which case (1.1) is formulated in term of α-order time fractional derivative of u in the sense of Caputo as

$$\begin{cases} {}^{C}D_0^{\alpha}u(t) \in -N_{C(t)}\big(u(t)\big) & \text{for a.e. } t \in [0,T], \\ u(0) = u_0 \in C(0). \end{cases}$$

Even in this α-Hölder continuous variation of the moving set $C(\cdot)$, we prove that $u(\cdot)$ inherits the regularity of $C(\cdot)$ (i.e., the solution $u(\cdot)$ will be α-Hölder continuous).

2 Main results

Let $\alpha \in (0,1)$. In this talk, we analyze and discuss the well-posedness of the following abstract Moreau's sweeping process with time fractional order derivative of Caputo type:
Find $u : [0,T] \to H$ such that

$$\begin{cases} {}^{C}D_0^{\alpha}u(t) \in -N_{C(t)}\big(u(t)\big) & \text{for a.e. } t \in [0,T], \\ u(0) = u_0 \in C(0), \end{cases} \tag{2.1}$$

where ${}^{C}D_0^{\alpha}u(t)$ stands for the α-order time fractional derivative of u in the sense of Caputo.
Our aim is to show that for a Hölder continuous moving set $C(t)$ there exists a unique solution to (2.1). To prove the solvability of problem (2.1), we now impose the following assumptions on the data of the problem.
$H(C)$: The set-valued mapping $C\colon [0,T] \to 2^H$ is such that $C(t)$ is nonempty, closed, and convex for all $t \in [0,T]$, and varies in α-Hölder continuous way, namely, there exists a real constant $L_C > 0$ (possibly dependent of α, i.e., $L_C = L_{C,\alpha}$) such that

$$C(t) \subset C(s) + L_C|t-s|^{\alpha}B_H[0,1], \quad \forall\, t,s \in [0,T].$$

The main result of this this talk is stated by the following theorem.

Theorem 2.1. *Assume that hypotheses $H(C)$ hold. Then fractional sweeping process, problem* (2.1)*, has a unique α-Hölder continuous solution.*

We now turn to the dependence of the solution with respect to the initial data.

Proposition 2.1. *[Dependence on initial data] Assume that the assumptions of Theorem 2.1 holds. For each $u_0 \in C(0)$, denote by $u(\cdot)$ the unique solution of the fractional sweeping process* (2.1)*. Then, the map $\psi : u_0 \longrightarrow u(\cdot)$ from $C(0)$ to the space $\mathcal{C}([0,T],H)$ endowed with the supremum norm (of uniform convergence) is Lipschitz.*

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Chaotic dynamic systems for symmetric encryption

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Abstract
Symmetric systems, also known as conventional systems, is based on reversible mathematical functions. In symmetric encryption, only one key is used to encrypt and decrypt data. When two parties communicate and use symmetric encryption to encrypt their data, they both use the same key for encryption and decryption. This is why symmetric encryption is also known as shared key encryption. logistic maps offer us good properties such us non-linearity and sensitivity to initial conditions. In this work, using chaotic logistic sequences, we will see how to realize and create a common secret key to use for the encryption and decryption of digital images.

Keywords: Symmetric encryption, share keys, Logistic maps, BB84 protocol, digital images.
2020 Mathematics Subject Classification: 08A70, 11C08, 34H10.

1 Introduction

Encryption ensures that even if sensitive information falls into the wrong hands, it cannot be understood by unauthorized persons. There are two types of encryption: asymmetric encryption and symmetric encryption. The difference between the two lies in the keys used for encryption and decryption. In asymmetric encryption, also known as public key encryption, there are two keys, one used for encryption and the other for decryption. In the case of symmetric encryption, only one key is used to encrypt and decrypt the encrypted data. When two parties communicate and use symmetric encryption to encrypt their data, they both use the same key for encryption and decryption. This is why symmetric encryption is also known as shared key encryption. It is therefore important that this key is kept secret from unauthorized persons. This is also the reason why symmetric encryption is also called secret key encryption. The security of symmetric encryption lies in the fact that the key remains secret for this we use the quantum protocol called $BB84$ to share some parameters secretly and we create a new matrix their element are the terms of some logistic maps, were theit initial conditions and parameters are same that are exchanged using $BB84protocol$. this new matrix will be considered the key of encryptino and decryption of digial images.

2 Logistic Maps

In mathematics, a logistic suite is a simple real suite, but whose recurrence is not linear. Its recurrence relationship is

$$x_{n+1} = \mu x_n(1 - x_n), \quad x_0 \in [0; 1].$$

Depending on the value of the parameter $\mu(in[0; 4]$ to ensure that the values of x remain in $[0; 1])$, it generates either a convergent sequence, a oscillating sequence, or a chaotic sequence.

Around $\mu = 3.57$, chaos sets in. No oscillation is yet found and small variations in the initial conditions lead to radically different results (snnsitivity to initial conditions).

3 Generate a new key

In this section we propose a new construction of an encryption key in the form of a square matrix their components are the terms of three logistic maps.

1. First we must secretly share some parameters of three logistic maps, using a famous known protocol colled The $BB84$ quantum protocol.

2. - Let (μ_1, x_0^1), (μ_2, x_0^2) and (μ_3, x_0^3) three couples of parameters shared parameters, where, $\{\mu_i : 3 < \mu_i < 4$, for $i \in \{1, 2, 3\}\}$ and $\{x_0^i : i \in \{1, 2, 3\}\}$ is the initial terms of sequences defiend as in (**??**), these couples will be exchanged between the two interlocutors through the quantum channal using the $BB84$ protocol.
   - After having exchanged these three couples of parameters, we introduce the logistic map defined in the previous section for getting the n terms of each sequence :

$$\{x_1^1, x_2^1, \cdots, x_n^1\} \quad , \quad \{x_1^2, x_2^2, \cdots, x_n^2, \cdots, x^2_{\frac{n^2-n}{2}}\} \quad and \quad \{x_1^3, x_2^3, \cdots, x_n^3, \cdots, x^3_{\frac{n^2-n}{2}}\}$$

   - We create the matrix

$$\mathcal{K} = \begin{pmatrix} x_1^1 & x_1^2 & x_2^2 & \cdots & \cdots & x_{n-2}^2 & x_{n-1}^2 \\ x_1^3 & x_2^1 & x_n^2 & \cdots & \cdots & x_{2n-4}^2 & x_{2n-3}^2 \\ x_2^3 & x_3^3 & x_3^1 & \cdots & \cdots & x_{3n-7}^2 & x_{3n-6}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x^3_{\frac{n^2-5n+4}{2}} & x^3_{\frac{n^2-5n+6}{2}} & x^3_{\frac{n^2-5n+8}{2}} & \cdots & \cdots & x_{n-1}^1 & x^2_{\frac{n^2-n}{2}} \\ x^3_{\frac{n^2-3n+2}{2}} & x^3_{\frac{n^2-3n+4}{2}} & x^3_{\frac{n^2-3n+6}{2}} & \cdots & \cdots & x^3_{\frac{n^2-n}{2}} & x_n^1 \end{pmatrix}$$

4 Application

Our application is in the encryption of digital images. We propose to use a symmetric algorithm to encrypt and decrypt an image using the common key shared in the previous section, for this we follow the following steps:

- Step 1: We convert the digital image wanted to encrypt into a matrix $\mathcal{M}$ of the same order n.
- Step 2: The sender encrypt $\mathcal{M}$ using the formula: $\mathcal{K} \times \mathcal{M} \times \mathcal{K}^{-1} = \mathcal{M}_c$=the cipher matrix.
- Step 3: The receiver decrypt $\mathcal{M}_c$ using the formula: $\mathcal{K}^{-1} \times \mathcal{M}_c \times \mathcal{K} = \mathcal{M}$=the initial matrix.

Proof: $\mathcal{K}^{-1} \times \mathcal{M}_c \times \mathcal{K} = \mathcal{M} = \mathcal{K}^{-1} \times (\mathcal{K} \times \mathcal{M} \times \mathcal{K}^{-1}) \times \mathcal{K} = \mathcal{M} = \mathcal{I} \times \mathcal{M} \times \mathcal{I} = \mathcal{M}$.

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Regularity for solutions of elliptic $p(x)-$Laplacian type equations with lower order terms and hardy potential

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Abstract
This paper aims to investigate a type of nonlinear elliptic equation that contains a Hardy potential, lower order term and $L^{m(\cdot)}$ datum in the setting of Sobolev spaces with variable exponent. Despite the presence of a Hardy potential, the existence of solutions and regularizing effect of some lower order terms in $p(\cdot)-$Laplacian type elliptic equations are proved.

Keywords: Variable exponents; Hardy potential; $L^{m(\cdot)}$ data; Regularizing effect; Lower order terms.
2020 Mathematics Subject Classification: 35J70; 35J60; 35B65.

1 Introduction

Let Ω be a bounded open subset of $\mathbb{R}^N$, $N > 2$, with a Lipschitz boundary $\partial\Omega$ and $p, s, m \in C(\overline{\Omega})$, $p^+ = \max_{x\in\overline{\Omega}} p(x), p^- = \min_{x\in\overline{\Omega}} p(x)$, and $p^*(x) = \frac{p(x)N}{N-p(x)}$, such that

$$1 < p^- \leq p(x) \leq p^+ < N, \;\; 1 < m(x) < \frac{N}{p(x)}, \;\; |\nabla m|, |\nabla s| \in L^\infty(\Omega), \;\; s^+ > s(x) > p^*(x). \tag{1.1}$$

In this paper, we are interested by the study of the following nonlinear elliptic equation

$$\begin{cases} -\Delta_{p(x)}u + \nu|u|^{s(x)-2}u = \mu\dfrac{u^{p(x)-1}}{|x|^{p(x)}} + f, & \text{in } \Omega, \\ u = 0, & \text{on } \partial\Omega. \end{cases} \tag{1.2}$$

where $\nu > 0$, $\mu \geq 0$ and the operator $\Delta_{p(x)}u$ is defined by $\Delta_{p(x)}u = \operatorname{div}\left(|\nabla u|^{p(x)-2}\nabla u\right)$.

2 Main results

Definition 2.1. Let $f \in L^m(\Omega)$, with $m \geq 1$.

(i) A measurable function $u \in W_0^{1,p(.)}(\Omega)$ is a weak solution to problem (0.1), if

$$|\nabla u|^{p(x)-1} \in L^1(\Omega), \quad \nu|u|^{s(x)-2}u \in L^1(\Omega), \quad \mu\frac{u^{p(x)-1}}{|x|^{p(x)}} \in L^1(\Omega), \tag{2.1}$$

and for every $\varphi \in W_0^{1,p(.)}(\Omega) \cap L^\infty(\Omega)$

$$\int_\Omega |\nabla u|^{p(x)-2}\nabla u.\nabla\varphi dx + \nu\int_\Omega |u|^{s(x)-2}u\varphi dx = \int_\Omega \left(\mu\frac{u^{p(x)-1}}{|x|^{p(x)}} + f\right)\varphi dx, \tag{2.2}$$

(ii) We say that $u \in W_0^{1,1}(\Omega)$ is a distributional solution to problem (0.1) if the conditions (2.1) hold and u satisfies the equality (2.2) for all $\varphi \in C_c^1(\Omega)$.

Theorem 2.1. *Let us assume that* (1.1) *holds true. Furthermore suppose that* $f \in L^{m(.)}(\Omega)$, *with*

$$\frac{s(x)}{s(x)-1} \leq m(x) < \frac{N(s(x)-p(x))}{p(x)(s(x)-1)}, \quad \forall x \in \overline{\Omega}. \tag{2.3}$$

Then there exists a weak solution u *to problem* (0.1) *in the sense of Definition 2.1, such that* $u \in W_0^{1,p(.)}(\Omega) \cap L^{m(.)(s(.)-1)}(\Omega)$.

Theorem 2.2. *Let us assume that* (1.1) *holds true. Let* $f \in L^{m(.)}(\Omega)$, *with* $1 < m(x) < \frac{s(x)}{s(x)-1}$, $\forall x \in \overline{\Omega}$. *If* $p(x)$ *and* $m(x)$ *satisfy one of the following assumptions*

$$1 + \frac{N-1}{N+1} < p(x) < N \quad \text{and} \quad 1 < m(x) < \frac{s(x)}{s(x)-1} \ \ \forall x \in \overline{\Omega}, \tag{2.4}$$

$$1 < p(x) < 1 + \frac{N-1}{N+1} \quad \text{and} \quad \frac{s(x)}{(s(x)-1)p(x)} < m(x) < \frac{s(x)}{s(x)-1}, \ \ \forall x \in \overline{\Omega}. \tag{2.5}$$

Then, there exists a distributional solution u *to problem* (0.1) *in the sense of Definition 2.1, such that* $u \in W_0^{1,q(.)}(\Omega) \cap L^{m(.)(s(.)-1)}$, *with* $q(x) = \frac{m(x)(s(x)-1)}{s(x)}p(x)$, $\forall x \in \overline{\Omega}$.

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Numerical simulation for Parabolic Trough Collectors (PTC)

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Abstract
Parabolic Trough Collectors (PTSC) using are currently the best option for their wide application in industries operating within temperatures ranging from 150 to 400 degrees Celsius.This work proposes a mathematical model of the (PTSC) in one dimensional (1-D) based on the thermal analysis of PTSC simulated under the Saharan climate conditions of the Algeria. A model has been proposed for estimating solar radiation, The Gauss-Seidel method is used and simulated with Matlab software. The mathematical models show agreement with previous experimental studies, From it we can conduct a numerical study was conducted to enhance the thermal performance of the Parabolic Trough Collectors By changing the diameter and length of the absorbent tube.

Keywords: solar energy, Parabolic Trough Collectors (PTSC), simulation.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

Recently solar thermal energy has emerged as an alternative solution to reduce the problem caused by non-renewable energy, and this energy emitted by the sun can be harnessed to produce electricity through concentrated solar power plants (CSP), and since the eighties solar numerical modeling has received increasing attention in order to examine thermal performance. In this research,the thermal performance of the Parabolic basin in the weather conditions of the Ouargla region as well as the effect of heat transfer fluids and PTC dimensions on thermal performance were examined.

2 Main results

showing a sensitivity to mass flow rate variations with temperatures.
temprature de sortie du leau Un maximum de 90 C .
There is a direct proportionality between the length of the tube and the temperature of the fluid.

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Septic Kolmogorov system with non algebraic stable limit cycle

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Abstract
There are many natural phenomena which can be modeled the Kolmogorov systems such as mathematical ecology and population dynamics, chemical reactions, hydronamics, economics, etc. In this work, we study a class of Septic Kolomgorov systems of the form :

$$\begin{cases} x' = (x+p)\,P_6\,(x,y) \\ y' = (y+q)\,Q_6\,(x,y) \end{cases} \tag{2}$$

where

$$\begin{aligned} P_6\,(x,y) &= \begin{pmatrix} x\,(q+y)\left(ax^4+ay^4+2bxy^3+2bx^3y\right) \\ +cx(x+p)(q+y)^2+\left(x^2+y^2\right)\left(-4qy^3+x^4-3y^4\right) \end{pmatrix} \\ Q_6\,(x,y) &= \begin{pmatrix} y\,(p+x)\left(ax^4+ay^4+2bxy^3+2bx^3y\right) \\ +cy(q+y)(x+p)^2+\left(x^2+y^2\right)\left(4px^3-y^4+3x^4\right) \end{pmatrix} \end{aligned}$$

and p, q, a, b, c are real numbers. We show that our system exhibiting an explicit expression of first integral. Moreover, according to certain conditions on the parameters the system admits a non algebraic limit cycle which can be explicitly given. We give an example to illustrate this proved result.

Keywords: kolmogorov systems, first integral, limit cycle.
2020 Mathematics Subject Classification: 34A05 ; 34C05 ; 34C07 ; 34C25.

1 Introduction

In biology, the first model with a limit cycle is the one introduced in 1936 by the Russian mathematician Andrei Kolmogorov (1903-1987) which bears his name and has the following form:

$$\begin{cases} x' = x f\,(x,y) \\ y' = y g\,(x,y) \end{cases} \tag{1}$$

where f and g are functions of class C^1 on an open set of $\mathbb{R}^2$ in $\mathbb{R}$.
More specifically, in the following we introduce a Kolmogorov models whish widely used in ecology to describe the interaction between two populations, and a limit cycle of system (1) is an isolated periodic orbit and it is said to be algebraic if it is contained in the zero set of an algebraic curve, otherwise it is called non-algebraic, and it also corresponds to an equilibrium state of the system.There are works which are interested in the study of limit cycles for Kolmogorov type systems, see for example ([17], [18]), but very few concern limit cycles expressed in an explicit way. The only known limit cycles were algebraic, see for example the work of A. Bendjeddou, A. Berbache and R. Cheurfa ,S. Benyoucef and A. Bendjeddou ([2], [3]),and R. Boukoucha, A. Bendjeddou [4].With respect to algebraic limit cycles. One of the most important topics in qualitative theory of planar dynamical systems is related to the second part of the unsolved Hilbert 16th problem which consisted to study the maximum number of limit cycles and their relative distributions of the real planar polynomial system of degree n. There is an extensive literature on that subject, most of it deals essentially with detection, number and stability of limit cycles, see for instance ([12], [17]). We say that a polynomial differential system is integrable if it admits a first integral. We note that the determination of a first integral for a given differential system is not an easy task. The importance of the existence of first integral for a differential system is that it completely determines its phase portrait see ([2], [7]). Many different methods have been used for proving the existence and nonexistence of limit cycles in simply connected region, for instance see ([3], [4]). In recent years, existence and nonexistence of limit cycle for some class of Kolmogorov system has been studied, see for instance ([4], [5], [14]). In the literature, we can find also another interesting but even more difficult problem is to give an explicit expression of a limit cycle. In 1995, Odani [20], showed that the limit cycle appearing in Van der Pol's equation is not algebraic but without expressing it explicitly. Since 2006, other mathematical researchers have published articles where they give the explicit expression of the non-algebraic limit cycles whose existence they have proven for polynomial systems ([1], [6], [7], [13], [15]). In this work, we are interested in studying the integrability and the existence of limit cycles for a septic Kolmogorov systems of the form:

$$\begin{cases} x' = (x+p)\,P_6\,(x,y) \\ y' = (y+q)\,Q_6\,(x,y) \end{cases} \tag{2}$$

where

$$\begin{aligned} P_6\,(x,y) &= \begin{pmatrix} x\,(q+y)\,(ax^4+ay^4+2bxy^3+2bx^3y) \\ +cx(x+p)(q+y)^2+(x^2+y^2)\,(-4qy^3+x^4-3y^4) \end{pmatrix} \\ Q_6\,(x,y) &= \begin{pmatrix} y\,(p+x)\,(ax^4+ay^4+2bxy^3+2bx^3y) \\ +cy(q+y)(x+p)^2+(x^2+y^2)\,(4px^3-y^4+3x^4) \end{pmatrix} \end{aligned}$$

and p, q, a, b, c are real numbers. We show that our system exhibiting an explicit expression of first integral. Moreover, according to certain conditions on the parameters the system admits a non algebraic limit cycle which can be explicitly given. We give an examples to illustrate this proved result.

2 The main result

As a main result, we shall prove the following theorem.

Theorem 2.1. *Consider Kolmogorov differential system* (2). *The System* (2) *has the first integral given by the expression*

$$I(x,y)=\frac{(x^4+y^4)\exp\left(-\int_0^{\arctan\frac{y}{x}}F(s)\,ds\right)}{(x+p)(y+q)}-\int_0^{\arctan\frac{y}{x}}\frac{G(s)}{\exp\left(\int_0^s F(w)\,dw\right)}ds \tag{3}$$

Where

$$\begin{aligned}F(s) &= \frac{(3a+a\cos 4s+4b\sin 2s)}{3+\cos 4s}\\ G(s) &= c\end{aligned}$$

3 Proof of the main result

Beginning by discussing the singular points of system (2).We have

$$xy'-yx'=\left(x^2+y^2\right)\left(x^4+y^4\right)(4pq+3py+3qx+2xy)$$

and any singular point must satisfy

$$\begin{cases}(x+p)P_6(x,y)=0\\(y+q)Q_6(x,y)=0\end{cases}$$

so the singular points are $(0,0)$, $(-p,-q)$ and any point of the form $\left(x,-q\frac{4p+3x}{3p+2x}\right)$ where x is solution of equation $P_6\left(x,-q\frac{4p+3x}{3p+2x}\right)=0$.

- If $(-81cp^2+1024q^4+768apq^3)=0$ then $\left(0,-\frac{4}{3}q\right)$ is a singular point.
- If $(-81cq^2-1024p^4+768ap^3q)=0$ then $\left(-\frac{4}{3}p,0\right)$ is a singular point.
- If $p=q=0$, the unique singular point is $(0,0)$.

In order to prove theorem, it must to transform the system to polar coordinates (r,θ) where $x=r\cos\theta$, $y=r\sin\theta$, then system (2) becomes

$$\begin{cases}r'=f_1(\theta)r^7+f_2(\theta)r^6+f_3(\theta)r^5+f_4(\theta)r^4+f_5(\theta)r^3+f_6(\theta)r^2+f_7r\\ \theta'=f_8(\theta)r^6+f_9(\theta)r^5+f_{10}(\theta)r^4\end{cases} \tag{4}$$

Where

$$\begin{aligned}
&f_1(\theta) = 4b + 18\cos 2\theta - 2\cos 6\theta - 4b\cos 4\theta + 5a\sin 2\theta + a\sin 6\theta,\\
&f_2(\theta) = (6p + 6aq + 4bp)\cos\theta + (6ap - 6q + 4bq)\sin\theta + (5p + aq - 4bp)\cos 3\theta\\
&+ (5q - ap + 4bq)\sin 3\theta + (3q + ap)\sin 5\theta + (aq - 3p)\cos 5\theta,\\
&f_3(\theta) = \left(\begin{array}{l} c\cos^2\theta\sin^2\theta + apq\cos^4\theta + apq\sin^4\theta\\ +(-4pq + 2bpq)\cos\theta\sin^3\theta + (4pq + 2bpq)\cos^3\theta\sin\theta \end{array}\right),\\
&f_4(\theta) = cp\cos\theta + cq\sin\theta - cp\cos 3\theta + cq\sin 3\theta,\\
&f_5(\theta) = cp^2 + cq^2 + (-cp^2 + cq^2)\cos 2\theta + 4cpq\sin 2\theta,\\
&f_6(\theta) = 2cp^2q\sin\theta + 2cpq^2\cos\theta,\\
&f_7(\theta) = cp^2q^2,\\
&f_8(\theta) = 5\sin 2\theta + \sin 6\theta,\\
&f_9(\theta) = 18q\cos\theta + 18p\sin\theta + 3q\cos 3\theta + 3q\cos 5\theta - 3p\sin 3\theta + 3p\sin 5\theta,\\
&f_{10}(\theta) = pq\left(3 + \cos 4\theta\right).
\end{aligned}$$

The differential system (4) where

$$r^4\left(f_8(\theta)r^2 + f_9(\theta)r + f_{10}(\theta)\right) \neq 0,$$

can be written as the equivalent differential equation

$$\frac{dr}{d\theta} = \frac{f_1(\theta)r^6 + f_2(\theta)r^5 + f_3(\theta)r^4 + f_4(\theta)r^3 + f_5(\theta)r^2 + f_6(\theta)r + f_7(\theta)}{r^3\left(f_8(\theta)r^2 + f_9(\theta)r + f_{10}(\theta)\right)}. \tag{5}$$

We use the transformation of variable $\rho = \frac{\left((r\cos\theta)^4 + (r\sin\theta)^4\right)}{((r\cos\theta)+p)((r\sin\theta)+q)}$ the equation (5) becomes a linear differential equation

$$\frac{d\rho}{d\theta} = F(\theta)\rho + G(\theta), \tag{6}$$

where $F(\theta) = \frac{(3a + a\cos 4\theta + 4b\sin 2\theta)}{\cos 4\theta + 3}$ and $G(\theta) = c$.
The general solution of this linear equation is

$$\rho(\theta) = \exp\left(\int_0^\theta F(s)\,ds\right)\left(h + \int_0^\theta G(s)\exp\left(-\int_0^s F(w)\,dw\right)ds\right). \tag{7}$$

Where $h \in \mathbb{R}$, and the functions $F(s)$, $G(s)$ are defined in theorem.
Consequently, the implicit solution of equation (7) is given by

$$H(r,\theta) - \exp\left(\int_0^\theta F(s)\,ds\right)\left(h + \int_0^\theta G(s)\exp\left(-\int_0^s F(w)\,dw\right)ds\right) = 0 \tag{8}$$

Where

$$H(r,\theta) = \frac{(r\cos\theta)^4 + (r\sin\theta)^4}{((r\cos\theta) + p)((r\sin\theta) + q)}$$

By passing to Cartesian coordinates, we deduce the first integral is

$$I(x,y) = \frac{(x^4 + y^4)}{(x+p)(y+q)\exp\left(a\arctan\frac{y}{x} + b\arctan\left(\frac{y}{x}\right)^2\right)} - \int_0^{\arctan\frac{y}{x}} c\exp\left(-as - b\arctan(\tan s)^2\right)ds.$$

Hence the theorem is proved.

4 Applications

The following example are given to illustrate our result.

4.1 Septic Kolmogorov system with non algebraic stable limit cycle

Let $a=-1$ and $b=c=p=q=1$, then system (2) becomes

$$\begin{cases} x'=(x+1)\begin{pmatrix} x\left(1+y\right)\left(-x^4-y^4+2xy^3+2x^3y\right) \\ +x(x+1)(1+y)^2+\left(x^2+y^2\right)\left(-4y^3+x^4-3y^4\right) \end{pmatrix} \\ y'=(y+1)\begin{pmatrix} y\left(1+x\right)\left(-x^4-y^4+2xy^3+2x^3y\right) \\ +y(1+y)(x+1)^2+\left(x^2+y^2\right)\left(4x^3-y^4+3x^4\right) \end{pmatrix} \end{cases} \tag{9}$$

The system (9) has the first integral

$$I\left(x,y\right)=\frac{\left(x^4+y^4\right)\exp\left(-\int\limits_0^{\arctan\frac{y}{x}}F\left(s\right)ds\right)}{\left(x+1\right)\left(y+1\right)}-\int\limits_0^{\arctan\frac{y}{x}}\frac{G\left(s\right)}{\exp\left(\int\limits_0^s F\left(w\right)dw\right)}ds.$$

Where

$$\begin{aligned} F\left(s\right) &= \frac{\left(-3-\cos 4s+4\sin 2s\right)}{3+\cos 4s}. \\ G\left(s\right) &= 1. \end{aligned}$$

The expression of limit cycle is implicitly given by

$$\frac{\left(r\cos\theta\right)^4+\left(r\sin\theta\right)^4}{\left(1+r\cos\theta\right)\left(1+r\sin\theta\right)}=\exp\left(-\theta+\arctan\left(\tan\left(\theta\right)\right)^2\right)\left(\rho_0^*+\int_0^\theta\exp\left(s-\arctan\left(\tan^2 s\right)\right)\,ds\right)$$

where

$$\rho_0^*=e^{-2\pi}\frac{\int_0^{2\pi}\exp\left(-\arctan\left(\tan^2 s\right)+s\right)\,ds}{1-e^{-2\pi}}=0.623\,29$$

Clearly, the system has one limit cycle as it is shown on the phase portrait of Poincaré disc in figure1.

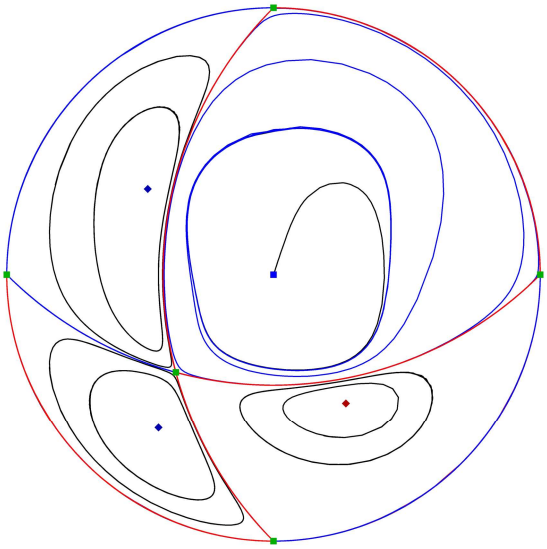

Figure 1: Phase potrait of system (9) on the Poincare disc.

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M^{2}DS'24-National Conference:
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Achieving anti-synchronization of two distinct chaotic systems with unknown parameters using feedback control

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Abstract
In this study, we introduce the anti-synchronization of two distinct chaotic systems with unknown parameters. An adaptive control strategy, rooted in the Lyapunov stability theorem, is developed to facilitate anti-synchronization between these chaotic systems.
This approach allows for achieving anti-synchronization in nearly all chaotic systems.
While the models in the research have unknown parameters, the method is equally applicable and significant for systems with known parameters.
To accurately verify the effectiveness of the design scheme and support the theoretical results, an illustrative example is presented with numerical simulations.
The proposed example demonstrates anti-synchronization between two different chaotic systems: the Lu system and the Zeraoulia system. .

Keywords: chaotic systems, Anti-synchronization, feedback control , stability
2020 Mathematics Subject Classification: Primary 34H10, 37C75, 93D21, 93D05.

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Shell-model view of the ^{132}Sn region through particle-hole representation

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Abstract
Theoretical Nuclear structure far from stability plays a crucial role in the progress of understanding the spectroscopic properties of nuclei. Within the shell model framework, a new effective interaction is developed by making use of a low momentum potential approach, to study the nuclei around ^{132}Sn core, with neutron holes. The purpose of this talk is to discuss widely the energy levels of isotopic Z=50 and isotonic N=82 chains as well as their electric $B(E2)$ transitions and magnetic moments μ compared to the experimental data.

Keywords: nuclear structure, shell-model, effective interaction

2020 Mathematics Subject Classification: Numerical implementation of nuclear structure physics

1 Introduction

The nuclear structure of nuclei around ^{132}Sn constitutes an interesting experimental and theoretical research. Within the shell-model framework [1], we have developed a new effective interaction by using low momentum potential approach [2]. The spectroscopic properties such as: energy levels and electromagnetic transitions of nuclei from this mass region are calculated and compared to the experimental data [3]. The numerical calculations are made using Antoine shell-model code, based on Lanczos Algorithm for diagonalisation [4].

2 Main results

The results of energy levels compared to the experimental data [3] show a significant reliability of the TBME's of the effective interaction developed in the ^{132}Sn mass region. This is confirmed further by $B(E2)$ transitions and magnetic moments μ of different nuclei.

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Presented by OUAHABI Rabiaa

The advantages of fixed-time synchronization for chaotic systems and its comparison with conventional synchronization methods

Abstract

The main goal of this work is to study a comparison between fixed-time synchronization of two different chaotic systems and conventional synchronization methods. To achieve this goal, we introduce some important lemmas and relied on the basic theories, such as Lyapunov stability theory and fixed-time theory, to develop effective fixed-time control laws.

Compared with traditional synchronization, the benefits of the proposed fixed-time synchronization scheme is the simplicity of designed control, in addition to the possibility of achieving synchronization between almost all the various chaotic systems in a verry short time .

To accurately verify the effectiveness of the design scheme and support the theoretical results, an illustrative example with numerical simulation is presented.

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On an Unreliable Markovian retrial G-queue

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Abstract
We examine a Markovian retrial G-queue where the customer currently being served is removed. The server is susceptible to both active and passive breakdowns. To evaluate system performance and server reliability, we utilize the generating function method. Furthermore, we assess the influence of different parameters on metrics such as the average number of customers in the queue and the operational states of the server through numerical analysis.

Keywords: Retrial Queues, Negative Arrivals., reliability
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

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Selfish behavior in a Markovian Queue with Two Vacations.

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Abstract We examine how customers behave in an equilibrium state within a Markovian queue featuring two distinct vacation. Customers make joining decisions based on the information they have upon arrival. We study two levels of information: a queue that is nearly observable and one that's entirely observable. We assess equilibrium thresholds and find the system stationary behavior. Using numerical analysis, we demonstrate how certain system parameters influence equilibrium threshold strategies and overall social benefit.

Keywords: queueing systems, vacations, threshold, strategies, social benefit.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

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Synchronization of SIR model using active control

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Abstract
In epidemiology, the SIR (Susceptible-Infectious-Recovered) model is pivotal for understanding disease spread. Recent research focuses on enhancing SIR predictions through synchronization using active control methods. This involves aligning multiple models to reflect interconnected populations and employing interventions dynamically. Benefits include improved prediction accuracy and effective outbreak mitigation. Synchronization via active control offers insights into complex interactions and aids in real-time response planning. Further exploration promises refined techniques for better epidemic management, bolstering our ability to combat infectious diseases.

Keywords: SIR model, synchronization, active control.
2020 Mathematics Subject Classification: 92D30 , 37N35, 9310.

1 Introduction

In the realm of epidemiology, the Susceptible-Infectious-Recovered (SIR) model stands as a cornerstone for comprehending the dynamics of infectious disease transmission within populations. Developed nearly a century ago, the SIR model has provided invaluable insights into the spread of diseases ranging from the common cold to more severe pandemics such as influenza and COVID-19. However, as the global landscape of public health challenges evolves in complexity and scale, there arises a pressing need for advanced methodologies to enhance the predictive capabilities of the SIR model.
One promising avenue of exploration in recent research endeavors is the synchronization of multiple SIR models through active control mechanisms. Traditionally, the SIR model operates on the assumption of isolated populations with fixed parameters governing disease dynamics. Yet, in reality, populations are interconnected through various social, geographical, and economic networks, leading to complex interactions that influence disease transmission patterns. By synchronizing multiple instances of the SIR model, researchers aim to capture these intricate interdependencies and simulate more realistic scenarios of disease spread.
Central to this approach is the concept of active control, wherein intervention strategies are dynamically employed to adjust model parameters in response to changing conditions. These interventions may include public health measures such as vaccination campaigns, social distancing policies, or

targeted quarantine protocols. By incorporating active control mechanisms into the SIR framework, researchers can not only improve the accuracy of disease forecasts but also devise more effective strategies for mitigating outbreaks and minimizing their societal impact.
This introduction sets the stage for delving into the intricacies of synchronizing SIR models via active control, highlighting its potential implications for epidemic understanding, response planning, and public health policymaking in an increasingly interconnected world.

2 Main results

$$\begin{cases} \dot{S} &= \sigma - \beta_0(1+\varepsilon\sin(\frac{2\pi}{T}t))SI - \mu S \\ \dot{I} &= \beta_0(1+\varepsilon\sin(\frac{2\pi}{T}t))SI - (\gamma+\mu)I \\ \dot{R} &= \gamma I - \mu R. \end{cases} \tag{2.1}$$

on pose $S = x_1, I = x_2, R = x_3$ donc le système devient:

$$\begin{cases} \dot{x_1} &= \sigma - \beta_0(1+\varepsilon\sin(\frac{2\pi}{T}t))x_1x_2 - \mu x_1 \\ \dot{x_2} &= \beta_0(1+\varepsilon\sin(\frac{2\pi}{T}t))x_1x_2 - (\gamma+\mu)x_2 \\ \dot{x_3} &= \gamma x_2 - \mu x_3. \end{cases} \tag{2.2}$$

Où(x_1, x_2, x_3) sont les variables d'etats et $\sigma, \mu, \gamma, \beta_0, \varepsilon$ sont les paramètres tel que $\sigma = \mu = 0.01, \gamma = 50, \beta_0 = 1510, \varepsilon \in [0.0960; 0.1073]$. Le système (**??**) est non autonome, faisant un changement de variable pour le rendre autonome. En posant

$$x_4 = \frac{2\pi}{T}t$$

ce qui implique

$$\dot{x_4} = \frac{2\pi}{T}$$

Donc le nouveau système est

$$\begin{cases} \dot{x_1} &= \sigma - \beta_0(1+\varepsilon\sin(x_4))x_1x_2 - \mu x_1 \\ \dot{x_2} &= \beta_0(1+\varepsilon\sin(x_4))x_1x_2 - (\gamma+\mu)x_2 \\ \dot{x_3} &= \gamma x_2 - \mu x_3 \\ \dot{x_4} &= \frac{2\pi}{T}. \end{cases} \tag{2.3}$$

Le système (**??**) est le système maître et le système esclave est

$$\begin{cases} \dot{y_1} &= \sigma - \beta_0(1+\varepsilon\sin(y_4))y_1y_2 - \mu y_1 + u_1 \\ \dot{y_2} &= \beta_0(1+\varepsilon\sin(y_4))y_1y_2 - (\gamma+\mu)y_2 + u_2 \\ \dot{y_3} &= \gamma y_2 - \mu y_3 + u_3 \\ \dot{y_4} &= \frac{2\pi}{T} + u_4. \end{cases} \tag{2.4}$$

Ou $u_i, i = 1...4$ sont les fonctions du contrôl actifs qui seront déterminées ultérièrement.Définissons les états erreurs pour chaque variable d'état : L'erreur de la synchronisation complète entre les

systèmes (??) et (??)

$$\begin{cases} \dot{e_1} &= -\mu e_1 - \beta_0(1+\varepsilon\sin(y_4))y_1y_2 - (1+\varepsilon\sin(x_4))x_1x_2) + u_1 \\ \dot{e_2} &= (\gamma+\mu)e_2 + \beta_0(1+\varepsilon\sin(y_4))y_1y_2 - (1+\varepsilon\sin(x_4))x_1x_2) + u_2 \\ \dot{e_3} &= \gamma e_2 - \mu e_3 + u_3 \\ \dot{e_4} &= u_4. \end{cases} \tag{2.5}$$

Redéfinissons les fonctions de contrôle actifs comme :

$$\begin{cases} u_1 &= +\beta_0(1+\varepsilon\sin(y_4))y_1y_2 - (1+\varepsilon\sin(x_4))x_1x_2) + v_1 \\ u_2 &= -\beta_0(1+\varepsilon\sin(y_4))y_1y_2 - (1+\varepsilon\sin(x_4))x_1x_2) + v_2 \\ u_3 &= -\gamma e_2 + v_3 \\ u_4 &= -e_4 + v_4. \end{cases} \tag{2.6}$$

Les dynimiques du système erreur devient

$$\begin{cases} \dot{e_1} &= -\mu e_1 + v_1 \\ \dot{e_2} &= -(\gamma+\mu)e_2 + v_2 \\ \dot{e_3} &= -\mu e_3 + v_3 \\ \dot{e_4} &= -e_4 + v_4. \end{cases} \tag{2.7}$$

$$[\dot{e_1}\ \dot{e_2}\ \dot{e_3}\ \dot{e_4}]^T = B[e_1\ e_2\ e_3\ e_4]^T \tag{2.8}$$

Dans la méthode du contrôl actif, on choisit une matrice constante A qui contrôle les dynamiques de l'erreur telle que :

$$[v_1\ v_2\ v_3\ v_4]^T = A[e_1\ e_2\ e_3\ e_4]^T \tag{2.9}$$

tel que

$$A = \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix} \tag{2.10}$$

Donc le système(??) devient :

$$\begin{pmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{e_4} \end{pmatrix} = \begin{pmatrix} -\mu & 0 & 0 & 0 \\ 0 & -(\gamma+\mu) & 0 & 0 \\ 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} + \begin{pmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} \tag{2.11}$$

d'où

$$\begin{pmatrix} \dot{e_1} \\ \dot{e_2} \\ \dot{e_3} \\ \dot{e_4} \end{pmatrix} = \begin{pmatrix} -\mu + k_{11} & +k_{12} & +k_{13} & k_{14} \\ k_{21} & -(\gamma+\mu) + k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & -\mu + k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & -1 + k_{44} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix} \tag{2.12}$$

Plusieurs choix de la matrice A pouront satisfaire les dynamiques de l'erreur, mais on pourrai utiliser la matrice A qui satisfait le critére de Routh-Hurwitz pour la stabilite du système d'erreur autour de zéro. Donc

$$A = \begin{pmatrix} -0.99 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.99 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.13}$$

Participants are encouraged to enroll and submit between two to three three pages abstract written in English, in accordance with the Latex template. The abstract should include the communication title and addresses of the authors as well as the keywords, main findings, and references.

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BIFURCATION AND CHAOS OF HIGHER DIMENSIONAL COSINE MAPS

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Abstract
In this paper, we present such bifurcation curves and bifurcation diagrams with two parameters obtained using different analytic and numerical techniques. These techniques are used on specific electronic circuits with mathematical models. In particular, we consider a communication system. We study the equation resulting from this system in discrete time, and we generalize the study in dimensions 2, 3, and N. These equations depend on a cosine squared function. The main result is that we present the bifurcation curves of pitchfork, fold, flip and Neimark-Hopf types and bifurcation diagrams, and we study the main properties of the fractal bifurcation called "box-within-a-box". As a main tool, we use critical curves, which have been proven to be a good tool for the analysis of the global properties of symmetry and periodicity for discrete dynamical systems.

Keywords: Circuit RLC; Dynamical System; Bifurcation; Attractor; Box-within-a-box Structure.
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

Today, mathematical theory of the dynamical systems is a fundamental base of natural science, for example in communications, biology or economics... see [13] and [23]. In this paper, we choose a communication system linked with an electrical circuit RLC as shown in figure 7 which describes a communication exchange between two stations, transmitter and receiver, see [9]. The objective of the present paper is to present an analytical study of the N-dimensional cosine system in discrete time. We shall present some global bifurcations, see [5], and we shall show the main properties of the fractal bifurcation organization called in the literature as box-within-a-box see [6]. The main tool that we are going to use to investigate such bifurcation will be critical curves, a powerful tool proved for the global properties dynamics analysis independent from the dimension of the map which defines the discrete dynamical system.

Bifurcation is a qualitative or topological structure change in system behavior caused by a change in a parameter when a small change in the parameter values of a system leads to a sudden qualitative change in its behavior. This change could be due to the appearance or disappearance of new singularities [2], or it could be due to a change in the nature of singularities. Bifurcations for fixed points and period two cycles can be classified as saddle-node, period doubling, transcritical, or bifurcations curves. Understanding the mechanisms that lead to chaos can be gained by studying the sequence of bifurcations [12].
We consider the following system

$$X_{n+1} = T(X_n, a, b) = \begin{cases} X_{n+1}^{(1)} = & X_n^{(2)} \\ \vdots & \vdots \\ X_{n+1}^{(N-1)} = & X_n^{(N)} \\ \\ X_{n+1}^{(N)} = & a\cos^2(x_n^{(1)} + b) \end{cases} \tag{1.1}$$

where $a \geq 0$ and b are real parameters. We can obtain the N-dimensional system.
In this paper, we present two-parameter bifurcation diagrams of such circuits and systems, obtained when two parameters, we study fold and Neimark-Hopf bifurcation properties of a one, two and three-dimensional cosine map with two parameters. The objective is to locate the bifurcation curves and their evolution in the parametric plane (a, b). The equations of some bifurcation curves are given analytically; cusp points and behaviour of fixed and critical points on these bifurcation curves are determined. The other is related to the study of the coupled cosine system in the phase space (x, y). We study the strange attractor and Lyapunov exponent in all cases. The focus of this paper is on analytical analysis and computationally intensive calculations to get accurate results.
The RLC circuit is the electrical circuit consisting of a resistor of resistance R, a coil of inductance L, a capacitor of capacitance C and a voltage source arranged in series. If the charge, the capacity C is charged by the output of the transformation discrete chaotic. The transformation discrete chaotic is defined by $F(x_n, a, b) = a\cos^2(x_n + b)$, where $a \geq 0$ and b are real parameters.

2 Study of the System of Dimension One

Let the one-dimensional system

$$T : x_{n+1} = a\cos^2(x_n + b) \tag{2.1}$$

where $a \geq 0$ and $b \in [0, \pi]$. and the initial condition x_0 for each integer n. Note that for this system the bounded chaotic orbits can be obtained, due to the cosine function in the system (2.1). This property will play a key role in applications, i.e., all orbits are bounded for any parameter values under any initial condition, see [4].
In figures 1 and 2 give an example of the bifurcation diagram of a system (2.1), when the parameter a varies and the parameter $b = \frac{\pi}{4}$ fixed, we can observe the bifurcation in a fixed point from an attractive and then to a cycle of order 4 attractive and then to a cycle of order 8 and then to a cycle of order 16 attractive and then to a cycle of order $2^i, i = 1, 2, ...$

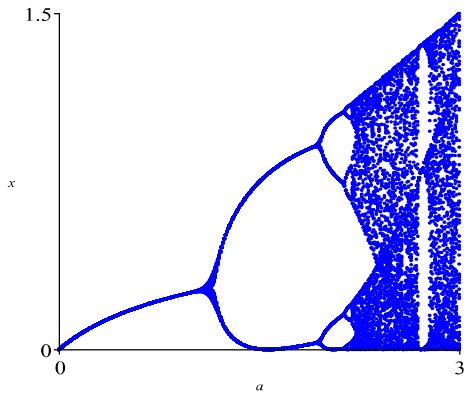

Figure 1: Bifurcation diagram of system (2.1), $a \in [0, 3]$ and $b = \frac{\pi}{4}$.

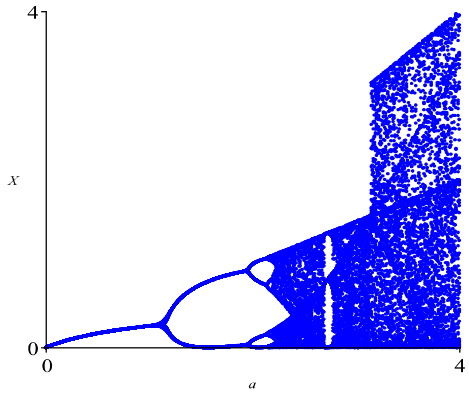

Figure 2: Bifurcation diagram of system (2.1), $a \in [0, 4]$ and $b = \frac{\pi}{4}$.

2.1 Bifurcation Curves

In this subsection we present the equations for existences of the cusp points on the bifurcation curves of the types fold and flip of the system (2.1).
The characteristic equation of fixed point is given by

$$a\cos^2(x+b) - x = 0$$

The eigenvalue of the fixed point is given by :

$$\lambda = -a\sin(2x+2b)$$

For a cycle of period k of T with $\Lambda = (a, b)$.
In one-dimensional, we have $X = x, T(X, a, b) \equiv F(x, a, b)$.
The construction of $\Lambda^j_{(k)_0}$ for $S = +1$ and Λ^j_k for $S = -1$.
Can be described with:

$$\begin{cases} F^k(x,a,b) - x = 0 \\ \frac{\delta F^k}{\delta x}(x,a,b) - S = 0 \\ S = \pm 1. \end{cases} \tag{2.2}$$

Therefore bifurcation curves (fold and flip) of the cycle of one-order (a fixed point) satisfy the following system of equations

$$\begin{cases} a\cos^2(x+2)-x=0 \\ a\sin(2x+2b)=S=\pm 1 \end{cases} \tag{2.3}$$

If $S=+1$ and by x eliminated in the system (2.3) we obtain the equation of the curve $\Lambda^1_{(1)_0}$ in the plane (a,b):

$$a\cos^2(\frac{1}{2}\arcsin(\frac{1}{a}))-\frac{1}{2}\arcsin(\frac{1}{a})+b-h\pi=0, h\in\mathbb{Z} \tag{2.4}$$

or

$$a\cos^2(\frac{1}{2}(\arcsin(\pi-\frac{1}{a})))-\frac{1}{2}(\pi-\arcsin(\frac{1}{a}))+b-h\pi=0, h\in\mathbb{Z} \tag{2.5}$$

If $S=-1$ and by x eliminated in the system (2.3) we obtain the equation of the curve Λ^1_1 in the plane (a,b):

$$a\cos^2(\frac{1}{2}\arcsin(\frac{-1}{a}))-\frac{1}{2}\arcsin(\frac{-1}{a})+b-h\pi=0, h\in\mathbb{Z} \tag{2.6}$$

or

$$a\cos^2(\frac{1}{2}(\pi-\arcsin(\frac{-1}{a})))-\frac{1}{2}(\pi-\arcsin(\frac{-1}{a}))+b-h\pi=0, h\in\mathbb{Z} \tag{2.7}$$

The conditions of existence of a cusp points for the fold Bifurcation given by:

$$\begin{cases} F(x,a,b)-x=0 \\ \frac{\delta^2F}{\delta x^2}(x,a,b)=0 \\ S=\frac{\delta F}{\delta x}=+1 \end{cases} \tag{2.8}$$

The cusp points given by

$$\begin{cases} a\cos^2(\frac{1}{2}\arcsin(\frac{1}{a}))-\frac{1}{2}\arcsin(\frac{1}{a})+b=0, h\in\mathbb{Z} \\ 2a\cos(2x+2b)=0 \end{cases} \tag{2.9}$$

Therefore

$$\begin{cases} a=1, b=\frac{(4k+1)}{4}\pi-\frac{1}{2} \quad with \quad x=\frac{1}{2}, k\in\mathbb{Z} \\ a=-1, b=\frac{(4k+3)}{4}\pi-\frac{1}{2} \quad with \quad x=\frac{-1}{2}, k\in\mathbb{Z} \end{cases} \tag{2.10}$$

With transformation T a second time, we obtain the recurrence giving the cycle of the second period:

$$x_{n+2}=a\cos^2(a\cos^2(x_n+b)+b) \tag{2.11}$$

The cycle of period 2 are solutions of following system:

$$\begin{cases} x_{n+2}-x_n=0 \\ \frac{\delta F^2}{\delta x}(x,a,b)-S=0 \\ S=\pm 1 \end{cases} \tag{2.12}$$

Therefore

$$\begin{cases} a\cos^2(a\cos^2(x+b)+b)-x=0 \\ a^2\sin(2x+2b)\sin(2a\cos^2(x+b)+2b)=\pm 1 \end{cases} \tag{2.13}$$

2.2 The Fractal "box-within-a-box" Bifurcation Structure

The fractal bifurcation organization called in the literature in a non-formal way box-within-a-box, is the simplest case configuration generated by the one-dimensional quadratic map [12]. Note that in this case, the cycles with the same period are different from each other due to the type of cyclic transfer (permutation) under the iteration. Myrberg defines these permutations see [22] using a binary code constituted by sequence signs $[+,-]$. the critical points given by: $\mid \frac{\delta F}{\delta x} \mid = 0$.
We obtain the critical points $C_{-1}^{(k)}$ whose coordinates are given by:

$$x = \frac{k\pi}{2} - b \quad with \quad k = 0, 1, 2, ...$$

The consequences $C_0^{(k)}$ are:

$$C_0^{(k)} = T(C_{-1}^{(k)}) = \begin{cases} x = 0 & \text{with} \quad k \quad \text{odd} \\ x = a & \text{with} \quad k \quad \text{even} \end{cases} \tag{2.14}$$

where denotes $C_{0,1}$ and $C_{0,2}$ the points are confused for k odd and k even:

$$C_{1,j} = T(C_{0,j}) = \begin{cases} x = a\cos^2(b), j = 1 \\ x = a\cos^2(a+b), j = 2 \end{cases} \tag{2.15}$$

In figure 4 the points $C_{0,1}$ and $C_{0,2}$ are the extreme points of the recurrence T in the plane (x_n, x_{n+1}). The border of sequence are given by the curve $\Omega_{(1)_0}$ and by first value of the parameters a and b, where $(a,b) = (a_{1,j}^*, b_{1,j}^*)$ with $j = 1, 2$. The values a and b is a basic nonclassical bifurcation related to the merging of the unstable fixed point q_1 with the rank-two critical point $C_{1,j}$, $j = 1, 2$ with multiplier (or eigenvalue) $S > 1$, and shows the fixed points. They are the points of intersection of the curve of the function (2.1) with the line $X_{n+1} = X_n$. These curves show periodic points q_i and cusp points $C_{i,j}$. We change the value of the parameters in each case.
In the direction of a and b increasing, which corresponds to the confusion of the critical point $C_{1,j}$, $j = 1, 2$ to repulsive fixed point q_1 with multiplier (or eigenvalue) $S > 1$.

$$C_{1,j} = q_1 = \begin{cases} x = a\cos^2(a\cos^2(b)+b) - a\cos^2(b) = 0, j = 1 \\ x = a\cos^2(a\cos^2(a+b)+b) - a\cos^2(a+b) = 0, j = 2 \end{cases} \tag{2.16}$$

We represent in Fig. 3 the bifurcation curves of first-order and second-order see [20]. The curve $\Lambda_{(1)_0}^1$ is obtained with $h = 0$ for the equation (2.3) and (2.4). The curve Λ_1^1 obtained with $h = 0$ for the equations (2.4) and (2.5), and for the equations (2.6) and (2.7). The curve $\Lambda_{(2)_0}^1$, is confused with the curve Λ_1^1. The cusp points are noted C_1^1 and C_2^1.

3 Study a two-dimensional System

This section studies a two-dimensional system for $N = 2$. An interesting property is that this system presents the existence of bounded chaotic orbits, which can play an important role in secure transmissions. The map 2.1 shows a decoupled variable. The form of this two-dimensional system is:

$$T(x_n, y_n) := \begin{cases} x_{n+1} = y_n = f(y_n) \\ y_{n+1} = a\cos^2(x_n + b) = g(x_n), \end{cases} \tag{3.1}$$

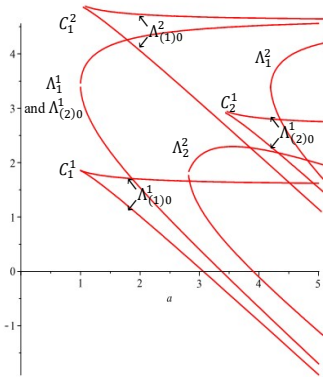

Figure 3: Bifurcation curves of the cycles of order 1 and 2.

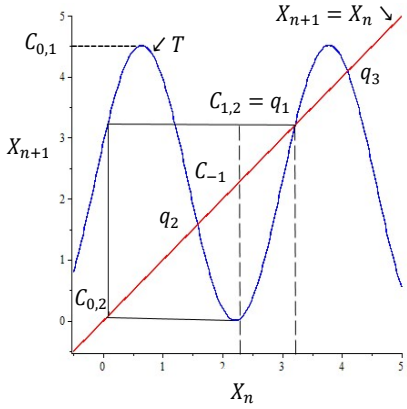

Figure 4: Recurrence of (2.1), $a = 4.51$ and $b = 2.5$.

where a and b are real parameters.
Looking at previous studies, the main number of results are for the one-dimensional case. Here, we are able to present results for this two-dimensional system in the plane of parameters (a, b).
We are going to study equation (3.1 in the state space $X = (x, y)$, the Jacobian matrix of equations (3.1) at the point X is

$$J(X) = \begin{bmatrix} 0 & 1 \\ \frac{\delta F}{\delta x} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a\cos(2x + 2b) & 0 \end{bmatrix} \tag{3.2}$$

As it is typical, in dynamic models with adaptive adjustment, the fixed point loses stability as one or both of the adjustment speeds are increased, and more attractors appear around the unstable fixed point see [18]. These results are obtained through a study on the local stability of the fixed point, obtained by a numerical method based on Newton's algorithm solution of the characteristic equation for the localization, in the plane, of the eigenvalues of the Jacobian matrix. Our analysis is based on the two eigenvalues λ_i, $i = 1, 2$ they are calculated using $J(X)$ the Jacobian matrix.

The transformation T is then described with the following expression:

$$\begin{cases} x_n = F^p(x_{n-2p}) = F^p(x_{n-(2p+1)}) \\ y_n = F^p(y_{n-2p}) = F^p(y_{n-(2p+1)}) \quad \text{where} \quad F^p = \underbrace{F \circ F \circ ... \circ F}_{p \text{ times}} \end{cases} \tag{3.3}$$

Regarding the cycles of T of odd period $k = 2p + 1$ then described with the following expression:

$$\begin{cases} x_\Delta = F^p(y_\Delta) \\ y_\Delta = F^{p+1}(x_\Delta) \end{cases} \tag{3.4}$$

The Jacobian matrix is given by

$$J(X) = \begin{bmatrix} 0 & A_y \\ B_x & 0 \end{bmatrix}$$

where $A_y = \frac{\delta F^p}{\delta y}(y)|_{(y=y_\Delta)}$ and $B_x = \frac{\delta F^{p+1}}{\delta x}(x)|_{x=x_\Delta}$.
The Characteristic equation of the eigenvalues is $\lambda^2 - A_y B_x = 0$ where A_y and B_x are real. We have two solutions $\lambda_1, \lambda_2 \in \mathbb{C}$ with $|\lambda_1| = |\lambda_2|$. Therefore the cycles of odd period $k = 2p + 1$ are either nodes or saddles.
The points of the cycles of even period $k = 2p$ can be described by

$$\begin{cases} x_\Delta = F^p(x_\Delta) \\ y_\Delta = F^p(y_\Delta) \end{cases} \tag{3.5}$$

The Jacobian matrix is

$$J^{2p}(X) = \begin{bmatrix} C_x & 0 \\ 0 & C_y \end{bmatrix}$$

where $C_x = \frac{\delta F^p}{\delta x}(x)|_{x=x_\Delta}$ and $C_y = \frac{\delta F^p}{\delta y}(y)|_{(y=y_\Delta)}$.
The eigenvalues are real $\lambda_1 = C_x$ $\lambda_2 = C_y$ The cycles of even period $k = 2p$ can be col or node.

3.1 Bifurcation Curves

In this subsection we present the equations for existences of the cusp points on the bifurcation curves of the Neimark-Hopf types of the system (3.1) and Lyapunov exponents.
The bifurcation curves of cycle of k period of T is given by:

$$\begin{cases} f^k(x, y, a, b) - x = 0 \\ g^k(x, y, a, b) - y = 0 \\ G = \begin{bmatrix} \frac{\delta f^k}{\delta x} - S & \frac{\delta f^k}{\delta y} \\ \frac{\delta g^k}{\delta x} & \frac{\delta g^k}{\delta y} - S \end{bmatrix} = 0 \\ S = \pm 1 \end{cases} \tag{3.6}$$

Therefore the system (3.6) then described with the following

$$\begin{cases} y - x = 0 \\ a\cos^2(x + b) - y = 0 \\ S^2 + a\sin(2x + 2b) = 0 \\ S = \pm 1. \end{cases} \tag{3.7}$$

We obtain the eigenvalues of system (3.7) $S_1 = -S_2 = 1$. In this case, the curve corresponds to the bifurcation node saddle.
The equations of bifurcation curve are:

$$a\cos^2\left(\frac{1}{2}\arcsin(\frac{-1}{a})\right) - \frac{1}{2}\arcsin(\frac{-1}{a}) + b - h\pi = 0 \quad \text{where} \quad h \in \mathbb{Z} \tag{3.8}$$

or

$$a\cos^2\left(\frac{1}{2}(\pi - \arcsin(\frac{-1}{a}))\right) - \frac{1}{2}\left(\pi - \arcsin(\frac{-1}{a})\right) + b - h\pi = 0 \quad \text{where} \quad h \in \mathbb{Z} \tag{3.9}$$

The Fig. 5 represents the bifurcation curve $\Lambda^1_{(1)_0}$ and Λ^1_1 with $h = 0$ for the equations (3.8) and (3.9), with fixed point of the transformation T.

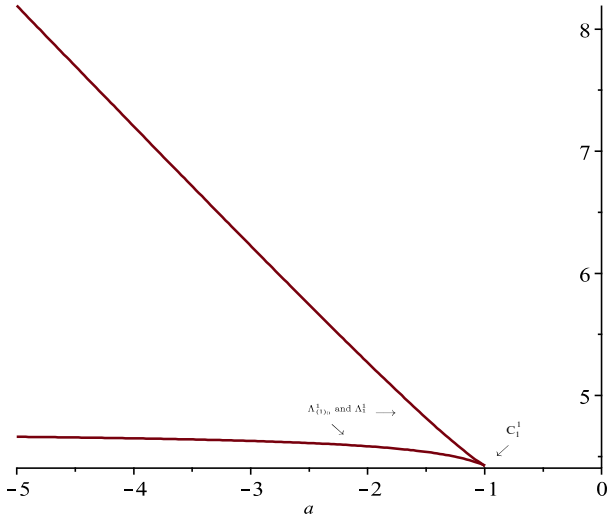

Figure 5: Bifurcation curves $\Lambda^1_{(1)_0}$ and Λ^1_1.

The cuspidal points satisfy the following system:

$$\begin{cases} f^k(x,y,a,b) - x = 0 \\ g^k(x,y,a,b) - y = 0 \\ \det\begin{bmatrix} \frac{\delta g^k}{\delta x} & \frac{\delta g^k}{\delta y} \\ \frac{\delta J^k}{\delta x} - \frac{\delta N^k}{\delta x} & \frac{\delta J^k}{\delta y} - \frac{\delta N^k}{\delta y} \end{bmatrix} = 0 \\ S = +1 \end{cases} \tag{3.10}$$

where $J^k(x) = a\sin(2x + 2b)$ is the determinant of the Jacobian matrix and N^k is the trace ($N^k = 0$ here). For the recurrence T, the cuspidal points are solution of the following system:

$$\begin{cases} y - x = 0 \\ a\cos^2(x + b) - y = 0 \\ \sqrt{-a\sin(2x + 2b)} = 1 \\ \det\begin{bmatrix} -a\sin(2x + 2b) & -1 \\ 2a\cos(2x + 2b) & 0 \end{bmatrix} = 0 \end{cases} \tag{3.11}$$

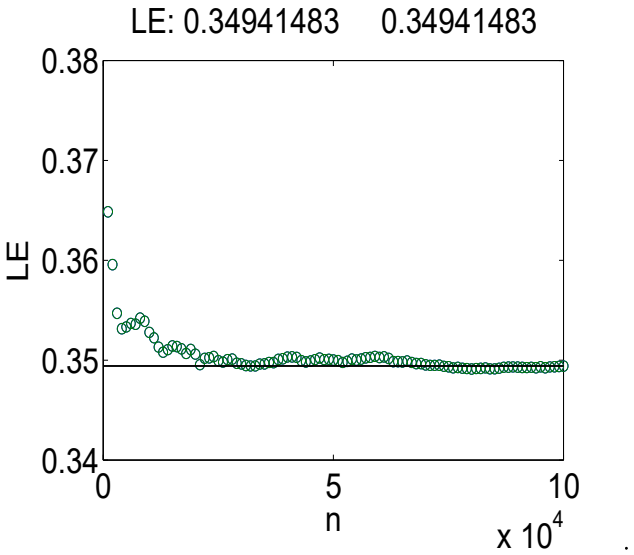

Figure 6: Lyapunov exponents diagram of (3.1) for $a = 3.54$, $a = 3.4$ and $x_0 = y_0 = 0.1.$, $\lambda_1 = \lambda_2 = 0.35$.

The coordinates are:

$$\begin{cases} a = +1, \quad b = \frac{3k+2}{4}\pi - \frac{1}{2}, \quad x = y = \frac{1}{2} \text{ where } k \in \mathbb{Z} \\ a = -1, \quad b = \frac{3k+1}{4}\pi + \frac{1}{2}, \quad x = y = -\frac{1}{2} \text{ where } k \in \mathbb{Z} \end{cases} \tag{3.12}$$

The Neimark-Hopf bifurcation is the birth of a closed invariant curve from a fixed point in dynamical systems with discrete time (iterated maps), when the fixed point changes stability via a pair of complex eigenvalues with unit modulus see [16]. The bifurcation can be supercritical or subcritical, resulting in a stable or unstable (within an invariant two-dimensional manifold) closed invariant curve, respectively. When it happens in the Poincare map of a limit cycle, the bifurcation generates an invariant two-dimensional torus in the corresponding ODE see [18].
The Neimark-Hopf bifurcation satisfies the following equations:

$$\begin{cases} f^k(x, y, a, b) - x = 0 \\ g^k(x, y, a, b) - y = 0 \\ \det \begin{bmatrix} \frac{\delta f^k}{\delta x} - \rho e^{i\theta} & \frac{\delta f^k}{\delta y} \\ \frac{\delta g^k}{\delta x} & \frac{\delta g^k}{\delta y} - \rho e^{i\theta} \end{bmatrix} = 0 \\ \rho = 1 \end{cases} \tag{3.13}$$

Therefore

$$\begin{cases} y - x = 0 \\ a\cos^2(x + b) - y = 0 \\ \sqrt{-a\sin(2x + 2b)} = 1 \\ (\rho e^{i\theta})^2 + a\sin(2x + 2b) = 0 \\ \rho = 1 \end{cases} \tag{3.14}$$

We denote by Γ_1^1 the Neimark-Hopf bifurcation therefore

$$a\cos^2\left(\frac{1}{2}\arcsin(\frac{1}{a})\right)-\frac{1}{2}\arcsin(\frac{1}{a})+b-h\pi=0 \text{ where } h\in\mathbb{Z} \tag{3.15}$$

or

$$a\cos^2\left(\frac{1}{2}(\pi-\arcsin(\frac{1}{a}))\right)-\frac{1}{2}\left(\pi-\arcsin(\frac{1}{a})\right)+b-h\pi=0 \text{ where } h\in\mathbb{Z} \tag{3.16}$$

The cycles of 4 periods of T satisfy:

$$\begin{cases} x_{n+4}=a\cos^2(b+a\cos^2(x_n+b)) \\ y_{n+4}=a\cos^2(b+a\cos^2(y_n+b)) \end{cases}$$

The system of equations giving the bifurcations of a cycle of 4 periods is:

$$\begin{cases} a\cos^2(b+a\cos^2(x+b))-x=0 \\ a\cos^2(b+a\cos^2(y+b))-y=0 \\ S=\pm 1 \end{cases} \tag{3.17}$$

Electric RLC Circuit

The output voltage of the chaotic transformation is e_n. The circuit see [19] of interest is shown in Fig. 7, including cosine-wave sources. We start with the series connection, writing Kirchoff's law see [6] for the loop in terms of the charge $v(t)$ on the capacitor and the current $I=\frac{dv(t)}{dt}$ in the loop.

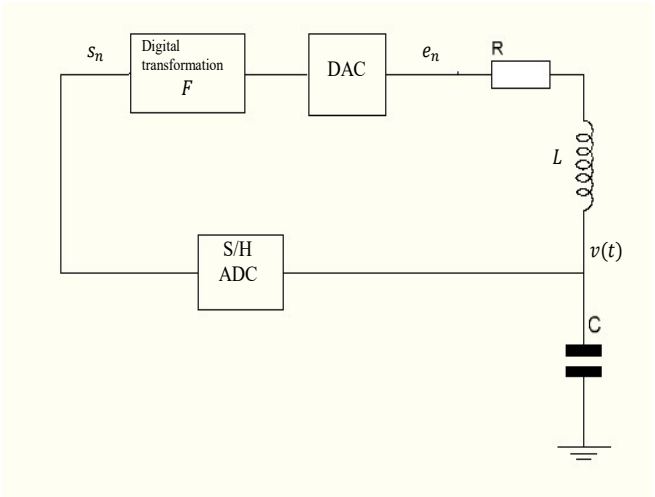

Figure 7: The circuit.

We obtain

$$LC\frac{d^2v(t)}{dt^2}+RC\frac{dv(t)}{dt}+v(t)=e_n$$

$$\frac{d^2v(t)}{dt^2} + 2\alpha\theta_0\frac{dv(t)}{dt} + \theta_0 v(t) = \frac{e_n}{LC}, \tag{3.18}$$

where $\theta_0 = \frac{1}{\sqrt{LC}}$ and $\alpha = \frac{R}{2L\theta_0}$.
We obtain three cases of solution $v(t)$, where $nT < t \leq (n+1)T$.
For $\alpha > 1$ The solution to equation (3.18) is

$$v(t) = e_n + A_n \exp^{-(\alpha\theta_0+\theta_1)(t-nT)} + A'_n \exp^{-(\alpha\theta_0-\theta_1)(t-nT)}.$$

For $\alpha = 1$ The solution to equation (3.18) is

$$v(t) = e_n + (A_n + A'_n(t-nT)) \exp^{-\theta_0(t-nT)}.$$

For $\alpha < 1$ The solution to equation (3.18) is

$$v(t) = e_n + A_n \exp^{-\alpha\theta_0(t-nT)} \cos(\theta_1(t-nT)) + A'_n \exp^{-\alpha\theta_0(t-nT)} \sin(\theta_1(t-nT)),$$

where $\theta_1 = \sqrt{\alpha^2 - 1}$ and $(A_n, A'_n) \in \mathbb{R}^2$.
Note that $v(t) = l_n(t-nT)$ and $\Lambda_n = [A_n A'_n]^T$ and $\Lambda_n = k_n(\Lambda_{n-1})$.
The complete model of the system is described with the following equations

$$\begin{cases} v_{n+1} = l_n(t) \\ e_{n+1} = F(l_n(\delta_s)), \end{cases} \tag{3.19}$$

where $0 \leq \delta_s \leq T$.

4 Main results

In this paper, we have researched the complex dynamic behaviors of the communication system (2.1). By the use of fixed points, periodic orbits, and bifurcation theory we have been able to analyze the dynamics of the discrete dynamical system (2.1) in one, two, three, and higher-dimensional. We have presented some numerical results. We have studied this model of chaotic generator with the RLC circuit and we choose in (3.19) the function $F(x) = a\cos^2(x+b)$.
Participants are encouraged to enroll and submit between two to three three pages abstract written in English, in accordance with the Latex template. The abstract should include the communication title and addresses of the authors as well as the keywords, main findings, and references.

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Mathematical modeling and numerical simulation for complex dynamic system using intelligent techniques

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Abstract
Modeling complex systems requires an analysis of the interactions associated with analytical data and structure selection. We are interested in finding complex models between the operating parameters of a blast furnace defined essentially by temperature variations. Both supervised and unsupervised learning algorithms will be studied and implemented numerically, and Granger-type dynamic models will be used to define cause-effect dynamics. An adaptive approach to existing algorithms will be introduced to take account of local variations in control process parameters. The validation and limits of use will be carried out using real data collected from typical steelmaking processes.

Keywords: Mathematical modeling, dynamic system, data analysis, nonlinear regression.
2020 Mathematics Subject Classification: 37A50, 60E05, 62J99.

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Semi-parametric regression under dependent truncation

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Abstract
The product-limit estimator calculated on data subject to random left- truncation relies on the testable assumption of quasi-independence between the failure time and the truncation time. This work focuses on introducing and studying the almost sure (a.s.) convergence of a new semi-parametric estimator of the regression function. The real response random variable (r.v.) is supposed subject to left-truncation by another r.v. while the covariate is a random vector $X \in \mathbb{R}^d$. Our model is based on the assumption of quasi-independence between the time of interest and the truncation time such that the dependence is modeled by a parametric family of copulas using recent results based on generalizations of the graphical copula estimator with truncated variables.

Keywords: copula, dependent truncation, regression function, Kendall's tau.
2020 Mathematics Subject Classification: 62G07, 62G20, 62R10, 62N01.

1 Introduction

A crucial point in a variety of statistical problems is the study of a relation between two random variables, a variable of interest Y and a covariate X. This can be done via estimating the function $m_\Phi(x) = E(\Phi(Y)|X = x)$, where $\Phi(.)$ is an arbitrary measurable function on the real line and it is assumed that $E|\Phi(Y)| < \infty$. The introduction of $\Phi(.)$ allows us to estimate not only regression function but also conditional distribution ($\Phi(Y) = I(Y \leq y)$ for any fixed y) and conditional moment ($\Phi(y) = y^r, r > 0$).
Nevertheless, in diverse fields, such as medicine, biology, public health, epidemiology, engineering, economics and demography, the response random variable may be incomplete and subject to random censorship or truncation model. Furthermore, since assuming that the studied data are always independent is unrealistic in practice, there is a growing interest in modelling the dependence structure between pairs of random variables based on incomplete data. A one-parameter family of copulas is often used to model the dependence. Several estimating procedures for the copula parameter and hence for the ensuing marginals have been proposed under different incomplete data schemes. Example of such studies include bivariate censored data Shih & Louis [3]. This work focuses on pairs (Y, T) of truncated variables satisfying $Y \geq T$. The variables are said to be left-truncated if Y is the variable of interest and right truncated otherwise.

This work presents new semi-parametric estimator for the regression function $m(x) = E(Y|X = x)$ when a possible dependency between the truncation time and the variable of interest is modeled with a parametric family of copulas.

2 Main results

Let (Y_i, T_i) for $i = 1, \ldots, N$, be a sample of the pair (Y, T). The unknown distribution functions of Y and T are denoted respectively by F and G, while the survival functions are noted by $\bar{F}$ and $\bar{G}$. Since the lifetime Y_i and the truncation r.v. T_i are both observable only when $Y_i \geq T_i$. We denote $(Y_i, T_i), i = 1, 2, \ldots, n (n \leq N)$ the actual observed sample. On the other side, let X a covariate which takes its values in $\mathbb{R}^d$.
Using a one-parameter family of Archimedean copulas with generating functions $\phi_\alpha, \alpha \in \mathbb{R}$ such as Clayton's [1] family, gives

$$\pi(y,t) = \mathbb{P}(T \leq t, Y > y / Y > T) = \frac{1}{c}\phi_\alpha^{-1}[\phi_\alpha(\bar{F}(y)) + \phi_\alpha(G(t))],\ y \geq t,$$

where ϕ^{-1} is a Laplace transform and c is a normalising constant.
Basing on the idea of Derrar et al. [2], a proposed estimator of the regression function under left truncation model defined by

$$m(x) = E[Y/X = x, Y > T]$$

is given by

$$m_n(x) = \frac{\sum_{i=1}^n Y_i \hat{G}^{-1}(Y_i) K_d(h^{-1}(x - X_i))}{\sum_{i=1}^n \hat{G}^{-1}(Y_i) K_d(h^{-1}(x - X_i))}$$

where

$$\hat{G}(x) = \phi_{\hat{\alpha}}^{-1}\left(-\sum_{t_i > x}\left[\phi_{\hat{\alpha}}\{\hat{c}\hat{\pi}(t_i)\} - \phi_{\hat{\alpha}}\{\hat{c}\hat{\pi}(t_i) - \hat{c}\frac{1}{n}\}\right]\right)$$

Theorem 2.1. *Under standard assumptions, we have*

$$m_n(x) - m(x) \longrightarrow_{n \to +\infty} 0 \qquad a.s.$$

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Partially observed Conditional mean field optimal control

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Abstract
In this paper, we study an optimal control problem with partial information for a system with regime-switching in the conditional mean field setting. when the control domain is convex, the sufficient and necessary maximal principles are derived .

Keywords:Keywords: Stochastic maximum principle, Optimal control, Partial information, Markov regime switching , Conditional mean field
MSC 2010 subject classifications, 93E20, 60H10.

1 Introduction

There are two main methods to solved stochastic optimal control problems, the dynamic programming and the maximum principle, the first bearing the name of Bellman and the second of Pontriagin's, the maximum principle also called the necessary conditions of optimality, it inspired from the theory of optimal process, this principle has the idea that if an control optimizes the cost function of the system studied then it also optimizes a functional called the Hamiltonian

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The Boundary integral method for the Laplace equation with oblique conditions

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Abstract
The aim of this work is to show how the usual Boundary Integral Equation techniques can be extended to deal with harmonic problems with oblique boundary conditions. Based on the Green formula, we express the solution in terms of the boundary data. The essential idea is to apply the Grenn Ostrogradsky theorem to translate the tangential derivative to the fundamental solution. The integral equation of second kind Fredholm type is obtained. By Fredholm's theorem, the existence and uniqueness of the solution is established.

Keywords: Boundary integral equation, Pseudo differential operator, oblique boundary conditions.
2020 Mathematics Subject Classification: Primary 64J10, 73V10, 74S15, 74S30.

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Title

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Abstract
A toy model for some dynamical systems $X(t)$ is Brownian motion constrained in some domain D of $\mathbb{R}^d$. There are many ways to make a free Brownian motion $B(t)$ remain inside D. One way is to apply to a penalization force at the boundary so that the Brownian particle bounces back inside D as soon as it reaches the boundary. The constraining instantaneous force at the boundary is called the boundary local time $L(t)$, whereby the decomposition $X(t) = B(t) + L(t)$. An efficient limit theorem for $L(t)$ is given allowing a concrete way to simulate the constrained dynamics inside a ball $D = B(0, R)$ for example. There are many applications for our result. It can be used for example in global constrainded optimization since a recuurent problem in this field is that iterratively genrated serach point have to remain in D; another application is in PDEs with Neumann boundary condition.

Keywords: Reflecting Brownian motion, Boundary local time, Box counting.
2020 Mathematics Subject Classification: Primary 35-xx, 60F99, 60J55; Secondary 60K37.

1 Introduction

Consider the SDE $dX(t) = dB(t)+dL(t)$ with $X(0) = x \in B(0, R)$. Our aim is to find a normalisation $H(n)$ for the number $N(n, t)$ of dyadic intervals $I_{n,k} = [k2^{-n}, (k + 1)2^{-n}]$, $k \geq 0$, that contain a time $s \leq t$ at which $X(s)$ is at the boundary so that the quantity $H(n)N(n, t)$ has a non-trivial limit as $n \to \infty$.

2 Main results

It turns out that $H(n) = C/\sqrt{2^n}$ where c is some constant and the limit is in fact the boundary local time $L(t)$. Our arguments evolve around those used in [1], like for example working in local time scales. But there are novel difficulties due to the boundary which is not a single point as in [1] and then hitting probabilities involve *non* Markovian processes. We have, among other things, to use the trace of $X(t)$ on the boundary and study its large time behaviour.

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Theory and application of the power Ailamujia distribution

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Abstract
Statistical modeling is constantly in demand for simple and flexible probability distributions. We are helping to meet this demand by proposing a new two-parameter extension of the Ailamujia distribution, called power Ailamujia distribution. It aims to extend the adaptability of the classic Ailamujia distribution through the use of the power transform. We show the interests of this new extension by exploring its theoretical properties and applied capacities. More precisely, we point out the pliant properties of the corresponding probability density and hazard rate functions, and prove some results on stochastic orders, quartile function, moments (raw, incomplete and probability weighted), stress-strength parameter and Tsallis entropy. New classes of distributions based on the power Ailamujia distribution are also presented. Then, we investigate the power Ailamujia model to analyze two kinds of data: complete data and data in presence of censorship. In particular, a goodness-of-fit statistical test allowing the processing of right-censored data is developed. The potential of the new model is demonstrated by its application to four data sets, two being related to the Covid-19 pandemic.

Keywords: Ailamujia distribution; Power distribution; Moments; Stress-strength parameter; Entropy; Data analysis, Covid-19 pandemic.
2020 Mathematics Subject Classification: 60E05, 62E15, 62F10.

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Existence, stability, and numerical simulations of a fractal-fractional Hepatitis B virus model

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Abstract
Mathematical models for the spread of Hepatits B which are based on fractional derivatives have proved to be quite accurate and realistic. In this paper, we extend the integer order model used in studying the dynamic of that virus to a fractal fractional model with a power law type kernel; the new operators transcend the constraints of other fractional-order derivatives and increase the accuracy of the description. We use the Schauder (Banach, respectively) fixed point theorem to prove the existence (uniqueness, repectively) of solutions of these systems. We discuss the stability of the solutions in Ulam's concept, and then we present a numerical scheme based on Newton's polynomials. Finally, we run a numerical simulation of the system to examine the stability and the convergence of the solutions graphically.The latter shows the close correlation between the mathematical model and the real description of the disease prevalence under the influence of asymptomatic carriers; at smaller fractal dimensions, the growth and decay rates in the six groups of the system are slower compared to the larger values. The accuracy of the results suggests that combining the use of the fractional fractal operators and the new numerical scheme would be useful in studying more complex models. .

Keywords: Forecasting Heptitis B, Fractal-fractional derivative, Fractionel Adms Bshorth method, Fixed point technique.

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Analysis and Synthesis of Uncertain singular Fractional-Order Linear Continuous-Time Systems

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Abstract
This paper deals with the robust admissibility problem of singular fractional-order continuous time systems. It is based on new admissibility conditions of singular fractional-order systems expressed in a set of strict Linear Matrix Inequalities ($\mathcal{LMI}$s). Then, a state feedback controller is designed for checking the robust admissibilily of the closed-loop system .

Keywords: first keyword, second keyword, third keyword,.....
2020 Mathematics Subject Classification: Primary xxxxx, xxxxx, xxxxx.

1 Introduction

In the field of singular fractional-order linear time invariant (SFO LTI) systems, the problem of the robust analysis and synthesis is more difficult because not only the stability must be ensured but also the regularity and the elimination of the impulsiveness of the system are required, it is known as the admissibility problem. But to singular fractional order systems, only few papers give the relevant results. This motivates us to devote our attention to the problem of the robust admissibility for uncertain singular fractional-order systems (SFOS). Based on the results derived in [4], necessary and sufficient conditions in terms of $\mathcal{LMI}$s for the robust admissibility are given. Then, based on the obtained result, a state feedback control is developed to get the robust admissibility of the closed-loop system. Numerical examples are presented to illustrate the proposed approach.
Notation. For real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). The notation X^{T} represents the transpose of the matrix X, spec(X) is the spectrum of all eigenvalues of X, Sym $\{X\}$ denotes the expression $X + X^{\mathsf{T}}$ and $\otimes$ stands for the Kronecker product. Matrices, if not explicitly stated, are assumed to have appropriate dimensions. $\Theta = \begin{bmatrix} \sin(\alpha\frac{\pi}{2}) & -\cos(\alpha\frac{\pi}{2}) \\ \cos(\alpha\frac{\pi}{2}) & \sin(\alpha\frac{\pi}{2}) \end{bmatrix}$

Key Words: *Singular Fractional-Order Systems, Robust Admissibility, State Feedback Control, Linear Matrix Inequalities.*

2 Main Results

2.1 Robust Admissibility Analysis

Considering the following uncertain SFO linear continuous system described by

$$\begin{array}{rcl} ED^{\alpha}x(t) & = & (A+\Delta A)x(t)+(B+\Delta B)u(t), 1 \leq \alpha < 2, \\ y(t) & = & Cx(t). \end{array} \tag{2.1}$$

$\Delta A = MF(\sigma)N_1$ and $\Delta B = MF(\sigma)N_2$ are time-invariant matrices representing norm-bounded parameter uncertainties. The uncertain matrix $F(\sigma) \in \mathbb{R}^{h\times q}$ satisfies

$$F(\sigma)^{\intercal}F(\sigma) \leq I_q \tag{2.2}$$

Theorem 2.1. *The unforced system of the uncertain singular system* (2.1) *is admissible for all allowable uncertainties* ΔA *if and only if*
there exist matrices $P \in \mathbb{R}^{n\times n} > 0, Q \in \mathbb{R}^{(n-r)\times n}$ *and a scalar* $\epsilon > 0$ *satisfying the following* $\mathcal{LMI}$

$$\begin{bmatrix} \mathrm{Sym}\left\{\Theta \otimes (A(PE^T+SQ)\right\} + \epsilon I_2 \otimes MM^T & * \\ \Theta \otimes (N_1(PE^T+SQ)) & -\epsilon I_{2q} \end{bmatrix} < 0 \tag{2.3}$$

where $S \in \mathbb{R}^{n\times(n-r)}$ *is any matrix of full column rank and satisfies* $ES = 0$.

2.2 State feedback synthesis of uncertain SFOS

Considering the uncertain singular fractional-order system in (2.1) under the state feedback control $u(t) = Kx(t)$, we obtain the following closed-loop system

$$\begin{array}{rcl} ED^{\alpha}x(t) & = & ((A+BK)+(\Delta A+\Delta BK))\,x(t) \\ y(t) & = & Cx(t) \end{array} \tag{2.4}$$

Theorem 2.2. *The uncertain closed-loop SFO system in* (2.4) *is admissible if and only if there exist matrices* $P \in \mathbb{R}^{n\times n} > 0$, $Q \in \mathbb{R}^{(n-r)\times n}$, *a gain matrix* $K \in \mathbb{R}^{m\times p}$ *and two scalars* $\varepsilon_1 > 0, \varepsilon_2 > 0$ *such that*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^T & \Gamma_{22} \end{bmatrix} < 0 \tag{2.5}$$

where

$$\begin{aligned} \Gamma_{11} &= \mathrm{Sym}\left\{\Theta \otimes A_{oc}\left(PE^T+SQ\right)\right\} + (\varepsilon_1+\varepsilon_2)(I_2 \otimes MM^T) \\ \Gamma_{12} &= \begin{bmatrix} I_2 \otimes (N_1\left(PE^T+SQ\right))^T & I_2 \otimes (N_2K\left(PE^T+SQ\right))^T \end{bmatrix} \\ \Gamma_{22} &= \mathrm{diag}(\varepsilon_1,\varepsilon_1,\varepsilon_2,\varepsilon_2) \otimes I_q \\ A_{oc} &= A+BK \end{aligned}$$

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Periodic solutions for a class of perturbed sixth-order autonomous differential equations

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Abstract
The Averaging theory is a classical tool for the study of the dynamics of nonlinear differential systems with periodic forcing. The averaging theory has a long history that begins with the classical work of Lagrange and Laplace. In this work, we use the averaging theory of first order to study the periodic solutions of the perturbed sixth-order autonomous differential equation

$$x^{(6)} + (1 + p^2 + q^2)\dddot{x} + (p^2 + q^2 + p^2q^2)\ddot{x} + p^2q^2x = \varepsilon F(x, \dot{x}, \ddot{x}, \dddot{x}, \ddddot{x}, x^{(5)}),$$

where p and q are rational numbers different from 1, 0, -1 and $p \neq q$, ε is a small enough and $F \in C^2$ is a nonlinear autonomous function. we present some applications to illustrate our main results.

Keywords: Periodic orbit, Sixth–order differential equation, Averaging theory.
2020 Mathematics Subject Classification: Primary 34C25, 34C29, 37G15.

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New uniqueness results for a class of multi-terms fractional differential equations in terms of ψ-Caputo fractional derivative via the ψ-fractional Bielecki-type norm

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Abstract: In this paper, we study the uniqueness of solutions for a new class of multi-terms fractional differential equations in terms of ψ-Caputo fractional derivative. Our approach is based on the employment of the Banach fixed point theorem along with the ψ-fractional Bielecki-type norm. Lastly, we offer some examples to confirm the validity of our main results.

Keywords: ψ-Caputo operator, multi-terms, uniqueness of solutions, Banach' fixed point theorem, the ψ-fractional Bielecki-type norm
2010 Mathematics Subject Classification: 34A08; 26A33.

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On the number of limit cycles in a class of planar differential systems

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Abstract
This paper investigates the number of limit cycles which can bifurcate from the periodic orbits of the linear center $\dot{x} = y$, $\dot{y} = -x$, when it is perturbed inside the class of all polynomial differential systems of arbitrary degree. Using averaging theory of first and second order, we estimate the lower bounds for the maximum number of limit cycles of these systems.

Keywords: Limit cycles, Averaging theory, Polynomial differential systems.
2020 Mathematics Subject Classification: 34C29, 34C25, 47H11.

1 Introduction

The second part of the 16th Hilbert problem [6] is related to the least upper bound on the number of limit cycles of polynomial vector field having a fixed degree. Until now the problem still remains to be unsolved even though a lot of work to be done in recent decades. A limit cycle of planar differential system is a periodic orbit in the set of all isolated periodic orbits of the differential system. It commonly occur when modelling both the natural and technological sciences. At the end of the 1920s Van Der Pol, Lienard and Andronov proved that a closed trajectory of a self-sustained oscillation occurring in a vacuum tube circuit was a limit cycle as considered by Poincaré.
A classical way to to investigate limit cycles is perturbing a planar differential system which has a center. Many of the results on the limit cycles of polynomial differential systems have been obtained by considering limit cycles which bifurcate from a a linear center $\dot{x} = y$, $y = -x$ inside a class of polynomial differential systems ([7]) which are sometimes called medium amplitude limit cycles. Of course, the number of medium amplitude limit cycles gives a lower bound for the maximum number of limit cycles that a polynomial differential system can have.
There exist several methods to study the number of limit cycles that bifurcate from the periodic annulus of a center, such as the Poincaré return map, the Poincaré-Melnikov integrals, the Abelian integrals, the inverse integrating factor, and the averaging theory (see [8], [3]). In the plane all of them are essentially equivalent.
In this work, we are interested in studying the existence of periodic orbits obtained by perturbing the linear center $\dot{x} = y$, $\dot{y} = -x$ inside the class of general polynomial differential systems expressed by

$$\begin{aligned}\dot{x} &= y-\varepsilon(\varphi_1(x)y+\psi_1(x)y^2)-\varepsilon^2(\varphi_2(x)y+\psi_2(x)y^2),\\ \dot{y} &= -x-\varepsilon(f_1(x)+g_1(x)y+h_1(x)y^2+k_1(x)y^3)\\ &\quad -\varepsilon^2(f_2(x)+g_2(x)y+h_2(x)y^2+k_2(x)y^3),\end{aligned} \tag{1.1}$$

where φ_i, ψ_i have degree m and f_i, g_i, h_i, k_i, have degree n for each $i=1,2$ and ε is a small parameter.

Note that when $\psi_i(x)=\varphi_i(x)=0$ and $k_i(x)=d_0$ for each $i=1,2$, we obtain the generalized Kukles systems.

$$\begin{aligned}\dot{x} &= y,\\ \dot{y} &= -x-(g(x)+f(x)y+g(x)y^2+d_0y^3),\end{aligned} \tag{1.2}$$

- In [**?**], the authors studied using the averaging theory of first and second order the number of limit cycles of system (1.2).

- When $\psi_i(x)=\varphi_i(x)=0$ and f_i, g_i, h_i, k_i, have degree n_1, n_2, n_3, n_4, respectively, the authors of [2], studied the system (1.1) using averaging theory of first and second order.

- Also, by using averaging theory of first and second order, Mellahi, Boulfoul and Makhlouf in [4] studied a class of polynomial Kukles systems of the form

$$\begin{aligned}\dot{x} &= y-l(x)y,\\ \dot{y} &= -x-f(x)-g(x)y-h(x)y^2-d_0y^3,\end{aligned} \tag{1.3}$$

where $l(x), f(x), g(x), h(x)$ and $k(x)$ are polynomials of given degree.

- In 2022, Debz, Boulfoul and Berkane [5] studied the maximum number of limit cycles for the generalized polynomial Kukles differential systems

$$\begin{aligned}\dot{x} &= -y+l(x)y^{2\alpha},\\ \dot{y} &= x-f(x)y^{2\alpha}-g(x)y^{2\alpha+1}-h(x)y^{2\alpha+2}-d_0y^{2\alpha+3},\end{aligned} \tag{1.4}$$

where l, f, k and h have degree m, n_1, n_2, n_3 respectively and α is a positive integer.

2 Main results

The main contribution of this paper is summed up in the following theorem.

Theorem 2.1. *For $|\varepsilon|>0$ sufficiently small, the maximum number of limit cycles of the generalized polynomial differential system* (1.1) *bifurcating from the periodic orbits of the linear center $\dot{x}=y$, $\dot{y}=-x$*

(a) *using the averaging theory of first order is*

$$\lambda_1 = \max\left\{\left[\frac{n}{2}\right]+1, \left[\frac{m+1}{2}\right]\right\}. \tag{2.1}$$

(b) *using the averaging theory of second order is*

$$\lambda_2 = \max\left\{\left[\frac{m+1}{2}\right]+\left[\frac{m}{2}\right], \left[\frac{n+1}{2}\right]+\left[\frac{n}{2}\right]+1, \left[\frac{n+1}{2}\right]+\left[\frac{m+1}{2}\right], \left[\frac{n}{2}\right]+\left[\frac{m}{2}\right]+1\right\}.$$

For the proof see [1].

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